Electroweak Theory*)

——Framework of On-Shell Renormalization and Study of Higher-Order Effects——

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The electroweak theory (the Weinberg-Salam theory) is reviewed emphasizing its aspect of a renormalizable gauge field theory with spontaneous symmetry breaking. The on-shell renormalization procedure is developed where all the renormalization constants are fixed on the mass shell of gauge bosons, fermions and Higgs bosons. It is applied to the calculation of radiative corrections to leptonic processes $\nu_\mu e \to \nu_\mu e$, $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$, $\nu_\mu e \to \mu \nu_e$ and $\mu \to e \bar{\nu}_e \nu_\mu$. The experimental significance of the radiative corrections and the effect of the corrections to the values of physical masses of $W^\pm$ and Z are discussed. The relation among different renormalization procedures is clarified.

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Referral Guide of This Paper:

For details on the on-shell renormalization formalism, refer to

$$\left(\boxed{1} - \boxed{\begin{matrix}2.1\\2.2\\2.3\end{matrix}}\right) - \boxed{\begin{matrix}2.4\\2.5\end{matrix}} - \boxed{3} \longrightarrow \boxed{7} - \boxed{8} \quad \text{and} \quad \boxed{G}$$

For some practical calculations in the Weinberg-Salam theory, refer to

$$\left(\boxed{1} - \boxed{\begin{matrix}2.1\\2.2\\2.3\end{matrix}}\right) \longrightarrow \boxed{4} - \boxed{5} - \boxed{6} - \boxed{7} - \boxed{8} \quad \text{and} \quad \boxed{\begin{matrix}A\\|\\F\end{matrix}}\,.$$

For formulas for calculations in the Weinberg-Salam theory, refer to

$$\boxed{4} \quad \text{and} \quad \boxed{\begin{matrix}A\\|\\F\end{matrix}}\,.$$

For the results of electroweak radiative corrections of $O(\alpha)$ to leptonic processes, refer to

$$\boxed{6} - \boxed{7}\,.$$

Chapter 1

Introduction

§ 1.1 Gauge unification of particle interactions

Classical electrodynamics was brought to completion in the form of the Maxwell equation where the vector potential appears as a fundamental field. As is well-known, the vector potential is regarded as a gauge field according to the local gauge invariance of the theory. The covariant quantization of the theory was performed consistently by introducing the indefinite metric and the theory turned out to be quantum electrodynamics (QED). It is the renormalizable gauge field theory where higher order effects in perturbative expansion for given electrodynamic processes can be systematically calculated. [Tati and Tomonaga 48, Fukuda et al. 49, Schwinger 48, 49a, b, Feynman 48, 49]. Those predictions based on higher order calculations have been extensively tested in various accurate experiments.

The theory of weak interactions originated from Fermi's theory [Fermi 34] on β decays*) and was settled in the effective 4-fermion interactions of the $V-A$ type [Feynman and Gell-Mann 58, Sudarshan and Marshak 58]. Although it was quite successful in the phenomenological sense, the 4-fermion theory was not satisfactory because of its violation of unitarity and of its nonrenormalizability which prevents us from dealing with weak higher order effects in a convincing way. Theories of weak interactions mediated by massive intermediate bose fields instead of the 4-fermion interactions had been proposed to attain universality [Lee et al. 49]. In these theories unitarity is recovered. This type of the theory is, however, still nonrenormalizable and thus there is no improvement of the situation. The improvement could only be possible if the weak vector fields would be regarded as non-Abelian gauge fields [Yang and Mills 54] just like the electromagnetic field as a gauge field in quantum electrodynamics. While the weak vector fields must be massive in order to reproduce the 4-fermion interaction effectively, gauge fields are required to be massless for the gauge invariance.

An attempt to solve this discrepancy and to unify the electromagnetic and weak interactions was proposed by Glashow in the use of $SU(2)\times U(1)$ gauge symmetry [Glashow 61]. A possible mechanism of giving a mass to the gauge field was found by Higgs in the case that the gauge symmetry is

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*) It is interesting to note that Yukawa, as early as Fermi, applied his idea of intermediate bosons to β decays [Yukawa 35]. This is the first attempt to unify strong and weak interactions.

spontaneously broken [Higgs 64, 66]. This is known as the Higgs mechanism. A realistic model of weak interactions was proposed by Weinberg and Salam where the Higgs mechanism was applied to a non-Abelian gauge field theory [Weinberg 67, Salam 68]. It should be noted here that to construct the realistic model of weak interactions it was necessary to unify the theories of weak and electromagnetic interactions in the non-Abelian gauge field theory. According to the proof by 't Hooft such a non-Abelian gauge field theory with spontaneous breakdown of gauge symmetries is renormalizable ['t Hooft 71b]. Experimental evidence for the model proposed by Weinberg and Salam has been accumulated and the model is widely believed to be the real theory of weak and electromagnetic interactions. The theory of this type is often called the electroweak theory.

The strong interaction of hadrons is supposed to originate from the interaction of their constituents, quarks, which is described by the non-Abelian gauge field theory based on the local color gauge symmetry of quarks [Fritzsch et al. 73]. This gauge theory, called quantum chromodynamics (QCD) has many desirable aspects in nonperturbative regime (as a review, see, e.g. [Marciano and Pagels 78]) and has also received strong phenomenological support particularly in short distance processes (for a recent review, see, e.g. [Buras 81]).

Just as QED and the weak interaction theory were unified to be the electroweak theory, the electroweak theory and QCD are expected to be unified as a gauge theory which is called the grand unified theory (GUT) (one of the models is due to [Georgi and Glashow 74]. For recent reviews, see, e.g. [Langacker 81] and [Konuma and Maskawa 81]). Experimental tests of GUT are currently under preparation and information on the validity of GUT will be obtained in the near future. It is, however, also important to continue more phenomenological tests on the electroweak theory and QCD in order to put on a solid basis for these theories which are the basic building blocks of GUT.

In the present review article we try to elucidate the structure of the electroweak theory as a gauge field theory with the intention of testing experimentally its field theoretical aspect.

§ 1.2 Development of the Weinberg-Salam theory

There could be a variety of models for the electroweak theory concerning its detailed structure. The difference of models is typically reflected by the neutral current structure of the theory. According to the experimental analyses of the neutral current structure in recent years, the original model by Weinberg and Salam was finally chosen as a unique model of the electroweak theory.

The Weinberg-Salam model was originally formulated for leptons only.

At that time the charm quark had not yet been discovered while two kinds of neutrinos were known, and the lepton-quark correspondence was not complete.*) The introduction of a constituent with charm quantum number resolved this tantalizing situation [Maki 64a, b, Hara 64, Bjorken and Glashow 64]. In order to incorporate quarks with the Weinberg-Salam model, it is also necessary to fulfill the lepton-quark correspondence. The incorporation of the charm quark was crucial to explain the absence of the strangeness-changing neutral current owing to the GIM mechanism [Glashow et al. 70]. It is now possible to accommodate in the Weinberg-Salam model any number of generations of leptons and quarks in the sequential form. We assign the left-handed (right-handed) quarks and leptons to the doublets (singlets) of the group $SU(2)\times U(1)$. In the present paper we confine ourselves to the case of 3 generations.

$$
\begin{array}{ccc}
\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L & \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L & \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \\
\nu_{eR} & \nu_{\mu R} & \nu_{\tau R} \\
e_R & \mu_R & \tau_R \\
\begin{pmatrix}u\\ d'\end{pmatrix}_L & \begin{pmatrix}c\\ s'\end{pmatrix}_L & \begin{pmatrix}t\\ b\end{pmatrix}_L \\
u_R & c_R & t_R \\
d_R & s_R & b_R
\end{array}
$$

Here suffixes R and L refer to the right- and left-handed projections $(1\pm\gamma_5)/2$ respectively and d' and s' represent the Cabibbo-mixed states [Cabibbo 63]. We neglect the Kobayashi-Maskawa mixing [Kobayashi and Maskawa 73] throughout the present paper. Since, in general, neutrinos may acquire masses, we have included ν_{eR}, $\nu_{\mu R}$ and $\nu_{\tau R}$ in the above tabulation. Though t-quark has not yet been observed, it has also been included.

The group $SU(2)\times U(1)$ reflects the symmetry of electroweak interactions and may be regarded as a local gauge group. The requirement of the $SU(2)\times U(1)$ local gauge invariance leads to the existence of the gauge fields which turn out to be the photon A and the weak bosons $W^{\pm}$ and Z. To make the weak bosons massive we utilize the Higgs mechanism in connection with the spontaneous breakdown of the $SU(2)\times U(1)$ symmetry. The most economical way of doing this is to introduce a doublet of the Higgs scalar field, Φ, the vacuum expectation value of which is given by

$$\langle\Phi\rangle=\begin{pmatrix}0\\ v\end{pmatrix}.$$

*) The lepton-quark correspondence originated from the lepton-hadron correspondence [Gamba et al. 59, Katayama et al. 62, Maki et al. 62].

By this choice of the Higgs field, the $SU(2)\times U(1)$ symmetry is broken down to the $U(1)$ symmetry leaving only the electric charge conserved.

The model described above is, in a sense, minimal as a model of electroweak interactions. One may, of course, consider a model with more parameters. As mentioned before, it is possible to choose a proper model by examining the neutral current structure of the models. By careful analyses of neutrino neutral-current processes (for a review of experimental data, see, e.g. [Hioki 78, Baltay 79]), it was concluded that the Weinberg-Salam model is the unique solution in the quark sector as far as the first generation of quarks is concerned [Sehgal 77, Abbott and Barnett 78a, 78b, Komatsu 78, Hung and Sakurai 79]. The experiment on the parity violation in polarized electron-deuteron scattering [Prescott et al. 78] gave important information on the interference of the weak neutral current and electromagnetic current. By the analysis of this data the Weinberg-Salam model was uniquely chosen in the electron sector [Konuma and Oka 78, Abbott and Barnett 78b]. In Fig. 1. 1 we present experimentally allowed regions for effective coupling constants g_V and g_A of the electron neutral current, where g_V and g_A are defined through the effective Hamiltonian for neutral current interactions:

$$H_{\text{eff}}=(G_{\mathrm{F}}/\sqrt{2})\,\bar{\nu}\gamma_\mu(1-\gamma_5)\nu\bar{e}\gamma^\mu(g_V-g_A\gamma_5)e\,,$$

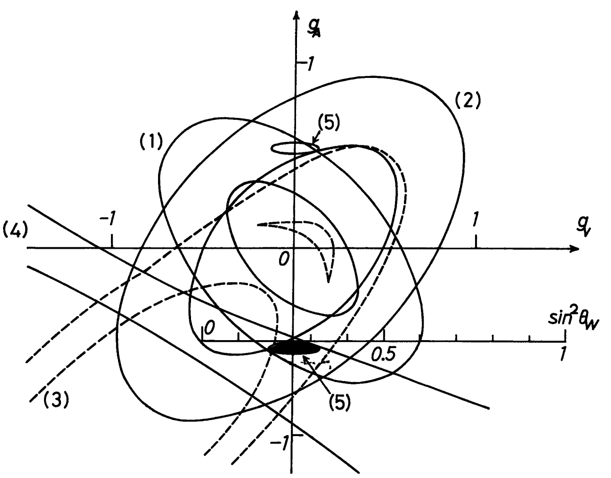


Fig. 1. 1. Experimentally allowed strengths of the neutral current couplings of the electron.
(1) Area between two ellipses is allowed from data of $\nu_\mu e\to\nu_\mu e$ scattering.
(2) Allowed area from data of $\bar{\nu}_\mu e\to\bar{\nu}_\mu e$.
(3) Area surrounded by three dashed curves is consistent with data of reactor neutrino scattering $\bar{\nu}_e e\to\bar{\nu}_e e$.
(4) Allowed region from polarized electron deuteron scattering $\vec{e}d\to eX$.
(5) Two small regions are consistent with data of $e^+e^-\to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$ at PETRA.
The black region is the solution consistent with all the above experimental data and includes the values in the Weinberg-Salam theory with $\sin^2\theta_W\simeq0.25$.

where ν and e represent the neutrino and electron fields respectively. For the Weinberg-Salam model

$$g_V = -\frac{1}{2} + 2\sin^2\theta_W; \quad g_A = -\frac{1}{2}.$$

Experimental constraints set by $\nu_\mu e \to \nu_\mu e$, $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$, $\bar{\nu}_e e \to \bar{\nu}_e e$ and polarized $\vec{e}d \to eX$ data single out the Weinberg-Salam model with $\sin^2\theta_W \simeq 0.25$. Recent experimental analyses at DESY in $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$ and $e^+e^- \to$hadrons have added new information to the validity of the Weinberg-Salam theory in the energy region which covers 3 generations of leptons [Davier 82].

These tests of the model, however, have been restricted to the tree level, i.e., the field-theoretical aspect of the model has never been tested. In order for the Weinberg-Salam model to be established as electroweak theory, there are still more hurdles to be cleared. The following is a list of some of further important experimental tests of the Weinberg-Salam theory.

1) Aspect of the model$\cdots$e.g., Observation of $W^\pm$ and Z.
2) Aspect of the renormalizable field theory$\cdots$e.g., Experimental check of radiative corrections.
3) Aspect of the spontaneous breakdown of the $SU(2)\times U(1)$ gauge symmetry$\cdots$e.g., Observation of the Higgs boson.

In this review article we concentrate our attention on the second aspect of the above list. For this purpose we develop a convenient renormalization procedure and apply it to some practical calculations of radiative corrections to leptonic processes.

§ 1.3 Field-theoretical aspect of the Weinberg-Salam theory

Electroweak theory is a non-Abelian gauge field theory based on the local $SU(2)\times U(1)$ gauge symmetry. The difficulty lies in the fact that the quantization and renormalization procedures in non-Abelian gauge field theories are not straightforward generalization of those in Abelian gauge field theories like QED.

It was found by Feynman in 1962 that, in the covariant quantization of non-Abelian gauge field theories, one needs ghost fields even at one-loop level by the requirement of unitarity and gauge invariance [Feynman 63]. This can be easily seen by calculating the one-loop self-energy part of gauge bosons corresponding to the Feynman diagrams illustrated in Fig. 1.2. In fact it does not satisfy the gauge invariance and its discontinuity does not fulfill the uni-

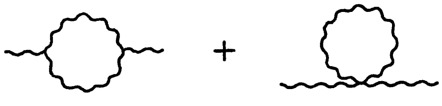


Fig. 1. 2. The Feynman diagram for the one-loop self-energy part of the gauge boson.

tarity requirement. This difficulty may be eliminated by introducing a contribution of a ghost field which obeys an anticommutation relation although it is a scalar field. Feynman's idea was generalized to multi-loops by de Witt, but the origin of the ghost was still mysterious [de Witt 67]. In 1967 Faddeev and Popov gave a beautiful formulation for the quantization of non-Abelian gauge field theories based on the path-integral method [Faddeev and Popov 67]. According to their formulation the appearance of the ghost (Faddeev-Popov ghost) in the covariant quantization of non-Abelian gauge field theories is now clearly understood and the covariant Feynman rules are systematically written down.

In order to study the structure of non-Abelian gauge field theories, however, it is more convenient and more powerful to use the canonical operator formalism than the path integral formalism. In 1978 Kugo and Ojima succeeded in completing the manifestly covariant quantization in the canonical operator formalism, which guarantees the unitary S-matrix, based on the Becchi-Rouet-Stora (BRS) invariance [Kugo and Ojima 78, 79, Becchi et al. 75, 76]. Their formulation may be regarded as a natural generalization of the Nakanishi-Lautrap formalism in QED [Nakanishi 66, Lautrap 67]. In the present review article we follow the Kugo-Ojima formulation to quantize electroweak theory.

Once the problem of quantization is settled, the perturbative calculation can be systematically performed. In calculating higher order effects, however, one encounters ultraviolet divergences in the loop integrations and the renormalization procedure is required to be developed. It was found by 't Hooft and Lee and Zin-Justin that non-Abelian gauge field theories are renormalizable even with the spontaneous symmetry breaking ['t Hooft 71b, Lee and Zin-Justin 72a, b, c]. Although the renormalizability of the electroweak theory is obvious according to the works of 't Hooft and Lee and Zin-Justin, it is still necessary to formulate the renormalization procedure in order to obtain the finite part in divergent amplitudes for a specific electroweak process. There have been proposed several renormalization procedures to deal with electroweak radiative corrections. None of them, however, are performed entirely on the mass shell. We develop, in the present paper, the on-shell renormalization scheme.*) Since there exists some confusion in understanding the processes of calculating electroweak radiative corrections on the basis of various renormalization schemes, we will discuss the comparison of the schemes proposed so far (Chapter 7).

One of the main sources of the confusion lies in the complication due to the existence of many independent parameters. The free parameters existing in the Weinberg-Salam theory are g and g' ($SU(2)$ and $U(1)$ gauge coupling

*) The original idea of the on-shell renormalization scheme was given in [Aoki 79]. See also [Aoki and Hioki 79, Aoki et al. 81, Inoue et al. 80, Bardin et al. 82].

constants respectively), μ^2 and λ (parameters in the Higgs potential) and f_i (the Higgs-fermion Yukawa-coupling constant where i refers to the type of fermions). There is another important parameter v, the vacuum expectation value of the Higgs field. This parameter v is not an independent parameter and should be fixed at the minimum point of the Higgs potential and hence is given in terms of the above free parameters. Since this set of independent parameters $(g, g', \mu^2, \lambda, f_i)$ is inconvenient, it is usually transformed to other sets which consist of physical parameters. Some examples of the set of physical parameters are

$$\begin{array}{llllll} (1) & e\,, & \sin\theta_W\,, & g\,, & m_i\,, & m_\phi\,, \\ (2) & e\,, & \sin\theta_W\,, & M_W\,, & m_i\,, & m_\phi\,, \\ (3) & e\,, & M_Z\,, & M_W\,, & m_i\,, & m_\phi\,, \end{array}$$

where e is the electromagnetic coupling constant, θ_W the Weinberg angle, and M_W, M_Z, m_i and m_ϕ are masses of the W- and Z- boson, the fermion of type i and the Higgs boson ϕ respectively. These various choices of the set of independent parameters are equivalent and related to each other by simple relations at the tree level. However, their relations become very complicated once higher order corrections are taken into account. It is this stage where the proper choice of the set of independent parameters is important.

Several authors have adopted set (1) or (2). While M_W and M_Z are well-defined as pole positions of Green functions, the definition of the parameter $\sin\theta_W$ is rather obscure except at the tree level where $\tan\theta_W = g'/g$. When higher order corrections are taken into account, there is no more unique and a priori definition of $\sin\theta_W$. Hence one must introduce the definition of $\sin\theta_W$ by hand. Moreover, unlike physical masses, $\sin\theta_W$ is not directly measurable (see § 3.1).*) For this reason we do not adopt $\sin\theta_W$ as one of the independent parameters. In our renormalization scheme we choose set (3) as independent parameters where, except for e the exact value of which is known in QED experiments, all the parameters are masses which are physical observables. Since W, Z and Higgs bosons have not been observed, the values of M_W, M_Z and m_ϕ are still unknown. However, if all these mass parameters are fixed, our scheme will be considered to be superior. A detailed explanation of our scheme will be given in Chapter 3, where we will set up all the renormalization conditions on the mass shell.

§ 1.4 Plan of the paper

This paper reviews the field theoretical structure of electroweak theory,

*) One may define $\sin\theta_W$ such that $\cos\theta_W = M_W/M_Z$ in terms of physical masses M_W and M_Z. The scheme with this definition, however, turns out to be equivalent to the on-shell scheme.

presents the on-shell renormalization scheme and deals with some applications of the scheme to higher-order calculations in leptonic processes.

Chapter 2 contains the review of the general framework in electroweak theory. Starting from the gauge principle, we derive the Lagrangian for electroweak theory (§ 2.1), to which the Faddeev-Popov ghost term is added in connection with the covariant quantization (§ 2.2) and the Higgs field term in connection with the spontaneous symmetry breakdown (§ 2.3). The generalized Ward-Takahashi identities are derived (§ 2.4) and the renormalizability of the theory governed by the above Lagrangian is then proved (§ 2.5) by the full use of these identities.

Chapter 3 consists of the description of the on-shell renormalization scheme which we believe is the most convenient and useful. After an introductory remark on renormalization schemes in general (§ 3.1), the on-shell renormalization scheme is introduced and on-shell renormalization conditions are settled (§ 3.2). A further detailed discussion on two-point functions is given in § 3.3. The universality of the electric charge, the on-shell coupling constant of the photon, is then proved carefully (§ 3.4).

Chapter 4 presents a collection of useful tools to perform higher order calculations. The full Lagrangian given in § 2.3 is rearranged explicitly in terms of physical fields (§ 4.1) and the Feynman rules derived from this Lagrangian are tabulated (§ 4.2). All the counter terms necessary to carry out the on-shell renormalization are presented in terms of the physical parameters (§ 4.3).

Chapter 5 is devoted to the detailed description of applications of our renormalization scheme to the calculation of radiative corrections to leptonic processes. We first make a short review on leptonic processes in the Weinberg-Salam theory (§ 5.1). After the brief explanation of the calculational procedure (§ 5.2), the neutral (§ 5.3) and charged (§ 5.4) current processes are taken up for the calculation of radiative corrections.

In Chapter 6 we discuss the numerical results. The values of the *W*- and *Z*-boson masses are discussed on the basis of the results in Chapter 5 (§ 6.1). The numerical estimates of the total cross sections with one-loop corrections are presented (§ 6.2).

Chapter 7 deals with discussions and comparison of calculational devices adopted by various groups. All the approaches are in principle equivalent, but have quite different appearances. Hence this chapter may help understanding relations among different approaches. The discussion is given in the conventional (§ 7.1) and renormalization-group (§ 7.2) approaches separately.

Chapter 8 is devoted to the concluding remarks.

Finally we would like to suggest that readers who are mainly interested in practical calculations in higher orders may skip Chapters 2 and 3 and start from Chapter 4. It is recommended that serious readers who wish to see the detail of the on-shell renormalization scheme read through Chapters 2 and 3. [See the Referral Guide of This Paper placed after contents.]

Notations

We follow the notations of the text book by Bjorken and Drell in most cases [Bjorken and Drell 65]. Thus the metric is $g^{\mu\nu}=(1, -1, -1, -1)$ and γ_5 is defined by $\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3$. The inner product of a γ matrix with vector p is denoted by Feynman's slash: $\gamma_\mu p^\mu \equiv \not{p}$.

See § 4.2 for a slight difference between our notations and those by Bjorken and Drell.

Chapter 2

Structure of Electroweak Theory

In this chapter we shall briefly describe the formal structure of electroweak theory. Starting with the gauge principle, we shall derive the classical Lagrangian for electroweak theory and perform quantization in covariant gauge following the canonical operator formalism. We shall then introduce the Higgs mechanism to give masses to gauge bosons and fermions. The renormalizability of the theory will be shown by the use of generalized Ward-Takahashi identities.

§ 2.1 Gauge principle

It has been well-known that the weak interaction is described by the effective current-current Lagrangian of the following form,

$$\mathcal{L}_{\mathrm{W}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} j_\mu{}^\dagger j^\mu , \tag{2·1}$$

where j_μ is the weak charged current. Here, for simplicity, we shall consider only one generation of leptons and quarks, and further restrict our argument to leptons only. A full generalization to include quarks and multi-generations is straightforward and will be given in § 2.3. In the present case, the current takes the form

$$j_\mu = \bar{\psi}_e \gamma_\mu (1-\gamma_5) \psi_\nu \tag{2·2}$$

with ψ_e and ψ_ν the fields of the electron and the electron-neutrino respectively. On the other hand, the electromagnetic interaction of charged leptons is expressed by the Lagrangian

$$\mathcal{L}_{\mathrm{EM}} = e\, j_\mu{}^{\mathrm{em}} A^\mu , \tag{2·3}$$

where A^μ is the photon field and $j_\mu{}^{\mathrm{em}}$ is the electromagnetic current with the form

$$j_\mu{}^{\mathrm{em}} = \bar{\psi}_e \gamma_\mu \psi_e . \tag{2·4}$$

Thus the weak and electromagnetic interactions are describable in terms of the currents of leptons, j_μ and $j_\mu{}^{\mathrm{em}}$.

Rearranging the structure of j_μ and $j_\mu{}^{\mathrm{em}}$, one easily finds that they can be reexpressed in terms of currents with definite $SU(2) \times U(1)$ transformation properties. In fact, we have

$$j_\mu = 2(J_\mu{}^1 - iJ_\mu{}^2), \tag{2·5}$$

$$j_\mu{}^{\mathrm{em}} = J_\mu{}^0 - J_\mu{}^3, \tag{2·6}$$

where $J_\mu{}^0$ and $J_\mu{}^n$ $(n=1, 2, 3)$ are given by

$$J_\mu{}^0 = \bar{\psi}_e \gamma_\mu \frac{1+\gamma_5}{2}\psi_e + \frac{1}{2}\bar{\psi}\gamma_\mu \frac{1-\gamma_5}{2}\psi,$$

$$J_\mu{}^n = \bar{\psi}\gamma_\mu \frac{1-\gamma_5}{2}\frac{\tau^n}{2}\psi \tag{2·7}$$

with $\psi = (\psi_\nu, \psi_e)$ and $\tau^n/2$ the $SU(2)$ generator in the fundamental representation. The currents $J_\mu{}^0$ and $J_\mu{}^n$ may be identified with those corresponding to the $U(1)$ and $SU(2)$ symmetries respectively. Under the $SU(2)$ symmetry the left-handed (right-handed) fermion $(1-\gamma_5)\psi/2$ $((1+\gamma_5)\psi/2)$ can be classified in a doublet (singlet) representation. This observation suggests that the weak and electromagnetic interactions maintain the $SU(2)\times U(1)$ symmetry. In the following we shall take the $SU(2)\times U(1)$ symmetry as a basis of our argument and shall determine the Lagrangian of electroweak theory by regarding the $SU(2)\times U(1)$ symmetry as a local gauge symmetry.

The algebra of the $SU(2)\times U(1)$ symmetry is generated by T^n and Y (generators of the $SU(2)\times U(1)$ group) with the following commutation relations,

$$[T^l, T^m] = i\varepsilon^{lmn}T^n, \tag{2·8}$$

$$[T^n, Y] = 0, \tag{2·9}$$

where ε^{lmn} is the antisymmetric tensor ($\varepsilon^{123}=1$ and $\varepsilon^{213}=-1$, etc.) and the repeated index should be summed over. For later convenience we introduce indices a, b, c,$\cdots$ which run over 0, 1, 2 and 3, and define

$$T^0 \equiv Y/2. \tag{2·10}$$

Equations (2·8) and (2·9) now read

$$[T^a, T^b] = if^{abc}T^c, \tag{2·11}$$

where $f^{0bc}=0$, and $f^{abc}=\varepsilon^{abc}$ for $\{a, b, c\} = \{1, 2, 3\}$. It should be noted that the eigenvalue of Y in each representation of $SU(2)\times U(1)$ is determined so that the relation $Q = T^3 + Y/2$ holds where Q is the charge operator.

The leptons e and ν_e constitute a singlet and doublet in $SU(2)\times U(1)$ as has already been mentioned in § 1.2. They form the following sets (the neutrino is assumed to be left-handed),

$$R = \psi_{e\mathrm{R}}, \qquad L = \begin{pmatrix} \psi_{\nu\mathrm{L}} \\ \psi_{e\mathrm{L}} \end{pmatrix}, \tag{2·12}$$

where suffixes R and L refer to the right- and left-handed components of field ψ respectively:

$$\psi_{R,L} = \frac{1 \pm \gamma_5}{2} \psi . \tag{2·13}$$

We now introduce the local gauge transformation based on the $SU(2) \times U(1)$ symmetry. The transformation property of the field ψ reads

$$\delta\psi(x) = \theta^a(x) \delta^a \psi(x) , \tag{2·14}$$

where the θ^a's are gauge-transformation parameters and

$$\delta^a \psi(x) = i g^a T^a \psi(x) \quad \text{(no summation on } a) \tag{2·15}$$

with

$$g^a = \begin{cases} g & \text{for} \quad a = 1, 2, 3 , \\ g' & \text{for} \quad a = 0 . \end{cases} \tag{2·16}$$

Here g and g' are $SU(2)$ and $U(1)$ gauge coupling constants respectively. Applying the gauge principle we may derive the classical Lagrangian in a standard manner (see, e.g. [Abers and Lee 73]). The Lagrangian acquires the following expression,

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_F , \tag{2·17}$$

$$\mathcal{L}_G = -\frac{1}{4} F^a_{\mu\nu} F_a{}^{\mu\nu} , \tag{2·18}$$

$$\mathcal{L}_F = i \bar{L} \not{D}_L L + i \bar{R} \not{D}_R R , \tag{2·19}$$

where

$$F^a_{\mu\nu} = \partial_\mu W_\nu{}^a - \partial_\nu W_\mu{}^a + g^a f^{abc} W_\mu{}^b W_\nu{}^c , \tag{2·20}$$

(no summation on a)

$$D_{L\mu} = \partial_\mu - i g^a T^a W_\mu{}^a , \tag{2·21}$$

$$D_{R\mu} = \partial_\mu - i g' T^0 W_\mu{}^0 \tag{2·22}$$

with $W_\mu{}^a$ the $SU(2) \times U(1)$ gauge field. Note that $T^0 = -1/2$ for L (-1 for R). The transformation rule of $W_\mu{}^a$ is given by

$$\delta W_\mu{}^a = \partial_\mu \theta^a + g^a f^{abc} \theta^b W_\mu{}^c . \quad \text{(no summation on } a) \tag{2·23}$$

Obviously $W_\mu{}^a$ for $a = 1, 2, 3$ belongs to the adjoint representation of $SU(2)$.

The gauge fields and matter fields described by the Lagrangian (2·17) are all massless while, in the real world, almost all fermions are massive and the gauge bosons mediating weak interaction are required to be very heavy.

Thus the Lagrangian (2·17) which has been obtained by the simple application of the gauge principle is unrealistic in the present form. Before going into this problem, we shall consider the problem of quantization of the theory in § 2.2.

§ 2. 2 Quantization

We would like to perform the covariant quantization of the theory specified by the Lagrangian (2·17). For this purpose we define the variable canonically conjugate to the field $W_\mu{}^a$,

$$\Pi_\mu{}^a \equiv \frac{\partial \mathcal{L}}{\partial \dot{W}^{a\mu}} = -F^a_{0\mu}\,.$$

According to this definition, the 0-th component of $\Pi_\mu{}^a$ vanishes: $\Pi_0{}^a = 0$. This means that the propagator of gauge fields is singular for the system governed by the local-gauge invariant Lagrangian. Under this circumstance it is impossible to make the covariant quantization consistently.

The singularity disappears if we do not respect the local gauge invariance by adding a gauge fixing term, and we can perform the covariant canonical quantization. At this stage we need to introduce the Faddeev-Popov ghost [Faddeev and Popov 67], in particular, in the covariant quantization of non-Abelian gauge field theory. In this scheme, however, some of the states corresponding to gauge fields become unphysical, i.e., acquire negative norm. Accordingly we have to restrict the states by subsidiary conditions.

Here we first explain the quantization based on the path integral formulation (see, e.g. [Abers and Lee 73, Faddeev and Slavnov 80]) to show how the Faddeev-Popov ghost comes into play. We then perform the covariant canonical quantization in the operator formulation [Kugo and Ojima 78].

To the Lagrangian (2·17) we add the covariant gauge fixing term,

$$\frac{1}{2\alpha}(\partial^\mu W_\mu{}^a)^2, \tag{2·24}$$

where α is the so-called gauge parameter. With this gauge fixing term the theory is now nonsingular and hence we can write down the Feynman rule for the theory. One may then calculate Feynman diagrams which involve gauge-field loops. For example, the proper one-loop bare self-energy part (Fig. 1.2) of the field $W_\mu{}^a$ can be easily calculated using the dimensional regularization to regulate the ultraviolet divergence. It reads in the Feynman gauge ($\alpha=1$),

$$\begin{aligned}\Pi^{ab}_{\mu\nu}(q) = (\delta_{ab} - \delta_{a0}\delta_{b0})\frac{g^2}{16\pi^2}\Big[&\left\{\frac{19}{6}\left(\frac{1}{\varepsilon} - \gamma - \ln\frac{-q^2}{4\pi\mu^2}\right) + \frac{58}{9}\right\} q^2 g_{\mu\nu} \\ &- \left\{\frac{11}{3}\left(\frac{1}{\varepsilon} - \gamma - \ln\frac{-q^2}{4\pi\mu^2}\right) + \frac{67}{9}\right\} q_\mu q_\nu\Big],\end{aligned} \tag{2·25}$$

where μ^2 is the regularization mass scale squared, $\varepsilon=(4-D)/2$ with D the space-time dimensions and γ is the Euler constant. In Eq. (2·25) only gauge-field loop has been taken into account. Apparently we see that Eq. (2·25) violates gauge invariance, i.e., $q^\mu \Pi^{ab}_{\mu\nu}(q)\neq 0$. This situation does not change even if we take account of the contribution of fermion loops, since it is by itself gauge invariant. One can also show that it does not satisfy unitarity in each order of perturbation theory. These inconsistencies may be traced back to the fact that polarizations of gauge fields are not necessarily physical in the covariant gauge. To recover gauge invariance and unitarity, Feynman in 1963 devised a procedure in which one adds a contribution to $\Pi^{ab}_{\mu\nu}$ from a hypothetical scalar particle which is anticommuting and belongs to the adjoint representation of $SU(2)$ [Feynman 63]. In fact, if one takes into account the contribution of the Feynman diagram in Fig. 2.1 with a proper coupling type and an overall negative sign due to the assumed anti-commutation property, one finds

$$\Pi^{ab}_{\mu\nu}(q)=\delta_{ab}\frac{g^2}{72\pi^2}\left[15\left(\frac{1}{\varepsilon}-\gamma-\ln\frac{-q^2}{4\pi\mu^2}\right)+31\right](q^2 g_{\mu\nu}-q_\mu q_\nu), \qquad (2\cdot 26)$$

which is obviously gauge-invariant. This procedure can also be applied to the Feynman amplitude with many external lines and may be extended to the case of multiloop amplitudes [de Witt 67].

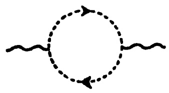


Fig. 2.1. The contribution of the hypothetical scalar particle to the self-energy part of the gauge boson.

A clear-cut formulation to justify this heuristic procedure was given by Faddeev and Popov using the Feynman path-integral method [Faddeev and Popov 67]. Let us consider a transition amplitude defined by the path integral,

$$Z=\int \mathcal{D}\phi \exp\left(i\int d^4x \mathcal{L}\right), \qquad \left(\mathcal{D}\phi \equiv \prod_i \mathcal{D}\phi^i\right) \qquad (2\cdot 27)$$

where the ϕ^i's represent all the involved fields and index i expresses Lorentz and group indices altogether. We set the gauge fixing condition,

$$F_i{}^a \phi^i = f^a, \qquad (2\cdot 28)$$

where F_i^a is an operation on ϕ^i which may or may not be a differentiation and f^a is a given function. For example, if $F_i^a=\partial_\mu$ and $f^a=0$, Eq. (2·28) reads $\partial^\mu W_\mu{}^a=0$. With this gauge fixing the path integral (2·27) must be performed on the restricted paths which satisfy Eq. (2·28). This constraint may be explicitly taken into account by using the functional $\Delta[\phi^i]$ which is defined by

$$\Delta[\phi^i] \equiv \left[\int \mathcal{D}g\,\delta(F_i{}^a\phi_g{}^i - f^a)\right]^{-1}, \tag{2·29}$$

where $\mathcal{D}g$ is an invariant measure of group element g in local $SU(2)\times U(1)$ which has property $\mathcal{D}(gg') = \mathcal{D}(g)$ and $\phi_g{}^i$ is a transform of ϕ^i under the group element g. Apparently from Eq. (2·29) we see that $\Delta[\phi^i]$ is invariant under the transformation g:

$$\Delta[\phi^i] = \Delta[\phi_g{}^i]. \tag{2·30}$$

Using Eqs. (2·29) and (2·30), Eq. (2·27) is rewritten as

$$Z = \int \mathcal{D}g \int \mathcal{D}\phi \Delta[\phi^i]\delta(F_i{}^a\phi^i - f^a)\exp\left(i\int d^4x\mathcal{L}\right), \tag{2·31}$$

where we have taken into account the gauge invariance of $\mathcal{D}\phi^i$ and $\mathcal{L}$. Since the integrand in Eq. (2·31) does not depend on g, one may factor out $\int\mathcal{D}g$ which is an infinite constant. By this factorization one obtains the well-defined transition amplitude Z. Equation (2·31) shows that the application of the gauge fixing condition enforces the measure of the path integral to be deformed resulting in an appearance of $\Delta[\phi^i]$.

The presence of $\Delta[\phi^i]$ in Eq. (2·31) is an obstacle to derive ordinary Feynman rules. In order to circumvent this difficulty, we try to exponentiate $\Delta[\phi^i]$ and redefine the Lagrangian. For this purpose we derive an explicit expression of $\Delta[\phi^i]$ in Eq. (2·31). Noting that $\phi_g{}^i \simeq \phi^i + \theta^a\delta^a\phi^i$ and $\mathcal{D}g \simeq \mathcal{D}\theta$ for $g\simeq 1$, we find, under the condition $F_i{}^b\phi^i = f^b$,

$$\begin{aligned}\Delta[\phi^i] &= \left[\int \mathcal{D}\theta\,\delta(\delta^a(F_i{}^b\phi^i)\theta^a - f^b)\right]^{-1} \\ &= \det(\delta^a(F_i{}^b\phi^i)).\end{aligned} \tag{2·32}$$

Hence in Eq. (2·31) we may effectively replace $\Delta[\phi^i]$ by the determinant in Eq. (2·32). We here introduce auxiliary fields c^a and $\bar{c}^a$ which are scalar fields following anticommutation rules and belonging to the adjoint representation. The field possesses the property of the Grassmann number. With the help of these fields c and $\bar{c}$, we can rewrite $\det(\delta^a(F_i{}^b\phi^i))$ as follows,

$$\det(\delta^a(F_i{}^b\phi^i)) = \int \mathcal{D}\bar{c}\,\mathcal{D}c\,\exp\left[-i\int d^4x\,\bar{c}^a\delta^a(F_i{}^b\phi^i)c^b\right]. \tag{2·33}$$

Note here that the anticommutation property of the fields c^a and $\bar{c}^a$ is necessary to obtain $\det(\delta^a(F_i{}^b\phi^i))$ in Eq. (2·33) instead of $1/\det(\delta^a(F_i{}^b\phi^i))$. The fields c^a and $\bar{c}^a$ are called Faddeev-Popov ghosts (FP ghosts) [Faddeev and Popov 67]. The delta function $\delta(F_i{}^a\phi^i - f^a)$ in Eq. (2·31) can also be exponentiated if we average it over the parameter f^a with the weight $\exp[-i(f^a)^2/2\alpha]$:

$$\int \mathcal{D}f\,\delta(F_i{}^a\phi^i - f^a)\exp\left(-i\int d^4x(f^a)^2/2\alpha\right)$$

$$= \int \mathcal{D}B\exp\left[i\int d^4x\left(B^aF_i{}^a\phi^i + \frac{\alpha}{2}B^aB_a\right)\right], \tag{2·34}$$

where we have introduced an auxiliary field B^a to linearize the gauge term. We finally find

$$Z = \int \mathcal{D}g\int \mathcal{D}\phi\,\mathcal{D}\bar{c}\,\mathcal{D}c\,\mathcal{D}B\exp\left(i\int d^4x\mathcal{L}'\right), \tag{2·35}$$

where the modified Lagrangian $\mathcal{L}'$ reads

$$\mathcal{L}' = \mathcal{L} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}, \tag{2·36}$$

$$\mathcal{L}_{\mathrm{GF}} = B^aF_i{}^a\phi^i + \frac{\alpha}{2}(B^a)^2, \tag{2·36a}$$

$$\mathcal{L}_{\mathrm{FP}} = -\bar{c}^a\delta^a(F_i{}^b\phi^i)c^b. \tag{2·36b}$$

In the covariant global symmetric gauge,

$$F_i{}^a\phi^i = \partial^\mu W_\mu{}^a \quad \text{and} \quad \delta^a(F_i{}^b\phi^i) = \partial^\mu D_\mu{}^{ab}, \tag{2·37}$$

where

$$D_\mu{}^{ab} = \delta^{ab}\partial_\mu + g^a f^{abc}W_\mu{}^c. \tag{2·37a}$$

Performing the integration by parts, we obtain

$$\mathcal{L}_{\mathrm{FP}} = (\partial^\mu\bar{c}^a)D_\mu{}^{ab}c^b. \tag{2·38}$$

The Lagrangian $\mathcal{L}'$ in Eq. (2·36) together with Eq. (2·38) is the basic Lagrangian in the covariant gauge from which the Feynman rules follow.

The above procedure of the quantization can be recapitulated by an operator formulation. As was observed before, in the course of quantization one was forced to introduce the gauge fixing term and then the local gauge invariance was violated. At the same time, however, the Faddeev-Popov ghost term was necessitated. The final Lagrangian thus obtained has a new global symmetry under the transformation including Faddeev-Popov ghost fields. This was found by Becchi, Rouet and Stora and is called the BRS transformation [Becchi et al. 75, 76].

We can now reverse the argument: We start with the BRS transformation and determine $\mathcal{L}_{\mathrm{FP}}$ so that the total Lagrangian is invariant under the BRS transformation. The BRS transformation is defined by the following set of infinitesimal transformations,

$$\delta^{\mathrm{BRS}}\psi(x) = i\lambda g^a c^a(x)T^a\psi(x),$$

$$\delta^{\mathrm{BRS}} W_\mu{}^a(x) = \lambda D_\mu{}^{ab} c^b(x),$$
$$\delta^{\mathrm{BRS}} \bar{c}^a(x) = \lambda B^a(x),$$
$$\delta^{\mathrm{BRS}} c^a(x) = -\frac{\lambda}{2} g^a f^{abc} c^b(x) c^c(x), \quad \text{(no summation on } a\text{)}$$
$$\delta^{\mathrm{BRS}} B^a(x) = 0\,, \tag{2·39}$$

where λ is a parameter independent of x which anticommutes with $c(x)$, $\bar{c}(x)$ and $\psi(x)$. Note that the transformations of $\psi(x)$ and $W_\mu{}^a(x)$ are obtained from their local gauge transformations (2·14) and (2·23) by replacing $\theta^a(x)$ by $\lambda c^a(x)$. We require that the total Lagrangian $\mathcal{L}' = \mathcal{L} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}$ be invariant under the BRS transformation (2·39) leaving $\mathcal{L}_{\mathrm{FP}}$ unknown. Obviously $\mathcal{L}\,(=\mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{F}})$ is invariant under (2·39) as it has originally the local gauge invariance. On the other hand, $\mathcal{L}_{\mathrm{GF}}$ transforms under (2·39) such that

$$\delta^{\mathrm{BRS}} \mathcal{L}_{\mathrm{GF}} = \lambda B^a \delta^a (F_i{}^b \phi_i) c^b\,. \tag{2·40}$$

To maintain $\delta^{\mathrm{BRS}} \mathcal{L}' = 0$, we must compensate Eq. (2·40) by $\delta^{\mathrm{BRS}} \mathcal{L}_{\mathrm{FP}}$. By taking the form (2·36b) as $\mathcal{L}_{\mathrm{FP}}$, we find

$$\delta^{\mathrm{BRS}} \mathcal{L}' = -\bar{c}^a \delta^{\mathrm{BRS}} (\delta^b (F_i{}^a \phi_i) c^b), \tag{2·41}$$

which vanishes. In fact, by applying Jacobi's identity for operator O,

$$\frac{\delta(\delta^a O)}{\delta \phi_i} \delta^b \phi_i - \frac{\delta(\delta^b O)}{\delta \phi_i} \delta^a \phi_i = -g^a f^{abc} \delta^c O\,, \tag{2·42}$$

we can show that

$$\delta^{\mathrm{BRS}} (\delta^b (F_i{}^a \phi_i) c^b) = -\lambda \frac{g}{2} f^{abc} \delta^c (F_i{}^a \phi_i) c^d c^b$$
$$+ \delta^b (F_i{}^a \phi_i) \left(-\lambda \frac{g}{2}\right) f^{bcd} c^c c^d = 0\,.$$

Thus the Lagrangian $\mathcal{L}'$ with the form (2·36) is the one which satisfies our requirement.

Following the usual procedure through the Lagrangian $\mathcal{L}'$, we can define canonically conjugate momenta and set up equal-time canonical commutation relations. For the FP ghost fields anticommutation relations are assumed and the Fermi statistics is adopted. According to the Noether theorem, there exists a conserved current $J_\mu{}^{\mathrm{BRS}}$ as $\mathcal{L}'$ is invariant under the BRS transformation. The charge Q_{B} defined by

$$Q_{\mathrm{B}} = \int d^3x J_0{}^{\mathrm{BRS}}(x) \tag{2·43}$$

generates the BRS transformation (2·39). For the Lagrangian $\mathcal{L}'$ in the covariant global symmetric gauge (2·37), Q_B takes the form

$$Q_B = \int d^3x \left(B^a \overleftrightarrow{\partial_0} c^a + g^a f^{abc} B^a A_0{}^b c^c - \frac{1}{2} g^a f^{abc} \partial_0 \bar{c}^a c^b c^c \right). \qquad (2\cdot 44)$$

A remarkable property of the BRS charge Q_B is the nilpotency

$$Q_B{}^2 = 0 \,. \qquad (2\cdot 45)$$

This property may be checked directly by using Eq. (2·44) and suggests that applying the BRS transformation twice, one gets the vanishing result:

$$\delta^{\mathrm{BRS}}(\delta^{\mathrm{BRS}} O) = 0 \,. \qquad (2\cdot 46)$$

The proof of $\delta^{\mathrm{BRS}}\mathcal{L}' = 0$ in Eq. (2·41) is in fact an example of this property.

The physical state $|\mathrm{phys}\rangle$ in this formalism is the state satisfying

$$Q_B |\mathrm{phys}\rangle = 0 \,. \qquad (2\cdot 47)$$

Using the nilpotency of Q_B, one can show that the space spanned by $|\mathrm{phys}\rangle$ has the positive semidefinite norm [Kugo and Ojima 78, 79]. According to this fact, the unitarity of the physical S-matrix, the S-matrix projected on the physical space, is guaranteed. Thus a consistent quantization of the theory is maintained in the covariant operator formalism.

It is quite important to note that the FP ghosts c^a and $\bar{c}^a$ and the auxiliary field B^a (the scalar component of $W_\mu{}^a$) are combined with the longitudinal component $W_L{}^a$ of $W_\mu{}^a$ and only the combination of them with zero norm can appear in the physical space. Hence c^a, $\bar{c}^a$, B^a and $W_L{}^a$ effectively decouple from the physical states. According to this formalism, we can also make a lucid explanation of the Higgs mechanism to be described in § 2. 3. In this case, four fields c^a, $\bar{c}^a$, B^a and χ^a (Nambu-Goldstone mode) are combined to disappear from the physical space and three remaining components of the gauge field represent a massive spin-1 particle [Nambu and Jona-Lasinio 61, Goldstone 61]. The mechanism in which four unphysical components of fields may appear in the physical space only as a combination of zero-norm states is called the quartet mechanism. One may find all possible asymptotic states by constructing the representation of the algebra of Q_B and Q_c (conserved charge of the ghost number). In the gauge theory there appear only two kinds of particles: the physical particles (singlet) and the quartets which are unobservable [Nakanishi 79].

§ 2. 3 Higgs mechanism

As stressed at the end of § 2. 1, gauge fields cannot acquire masses in a gauge invariant (or, strictly speaking, BRS invariant) Lagrangian. Also fermion mass terms are not included in the Lagrangian (2·36), because $\bar{\psi}_L\psi_R$ is a doublet representation of $SU(2)$ and gauge non-invariant. To make the theory realistic we need important ideas in quantum field theory, i.e., the spontaneous symmetry breakdown and the Higgs mechanism.

The idea of spontaneous symmetry breakdown was introduced in elementary particle physics first by Nambu and Jona-Lasinio in connection with the chiral symmetry [Nambu and Jona-Lasinio 61]. Consider a continuous symmetry of Lagrangian $\mathcal{L}$. According to the usual Noether theorem, there exists a conserved charge Q which generates the symmetry transformation,

$$[Q, \phi]=i\delta\phi\,, \tag{2·48}$$

and commutes with the Hamiltonian:

$$[Q, H]=0\,. \tag{2·49}$$

In the case of quantum mechanics where the degrees of freedom are finite, the vacuum (the minimum energy state) is unique and is a simultaneous eigenstate of H and Q. On the contrary, in quantum field theory with infinite degrees of freedom, it is possible to have degenerate vacuum states since transitions between these vacuum states are possible only by an infinite number of operations. Here all the states are constructed on one vacuum which is no longer an eigenstate of the charge Q:*)

$$Q|0\rangle\neq 0\,. \tag{2·50}$$

In other words, the symmetry of the Lagrangian is realized as the degeneration of vacuums and it is not manifest in the real world constructed on one vacuum. This is the spontaneous symmetry breakdown. The remnant of the symmetry is, however, observed through the scalar mode called the Nambu-Goldstone (NG) particle which, roughly speaking, corresponds to the state (2·50). Taking account of the commutativity (2·49), we find that this mode is a massless excitation.

In order to describe spontaneous symmetry breakdown, it is most convenient to introduce scalar fields which transform as in Eq. (2·48). We assume that the vacuum expectation value of $\delta\phi$ does not vanish:

$$\langle 0|\delta\phi|0\rangle\neq 0\,, \tag{2·51}$$

which implies Eq. (2·50) and hence spontaneous symmetry breakdown. To have the non-vanishing vacuum expectation value, we may add to $\mathcal{L}$ a potential term of $\delta\phi$ which assures the stability of this vacuum. We may regard this scalar as an elementary field or a tool of effective expression of the spontaneous

*) Strictly speaking, the charge Q is ill-defined in this case, but, for simplicity, we use Q in the present argument.

symmetry breakdown. In the Hilbert space constructed on this vacuum, it is convenient to expand $\delta\phi$ around its vacuum expectation value and separate it into classical and quantum-fluctuation parts. Thus we obtain the effective Lagrangian on this vacuum.

In the case that the symmetry is a local symmetry, that is, gauge symmetry, the Nambu-Goldstone mode is absorbed into the gauge boson as its longitudinal component and consequently the gauge boson turns out to be massive. This is called the Higgs mechanism [Higgs 64, 66]. The above phenomenon is now explained in the following. The gauge invariant kinetic term of scalar ϕ which is called the Higgs scalar takes the form,

$$|D_\mu\phi|^2 \equiv |(\partial_\mu - igT^a A_\mu{}^a)\phi|^2 , \tag{2·52}$$

where T^a is the generator in the representation of ϕ. We suppose that the symmetry generated by charge Q^a is spontaneously broken:

$$Q^a|0\rangle \neq 0 . \tag{2·53}$$

The above equation is represented in terms of scalar ϕ such that

$$\langle\delta^a\phi\rangle = iT^a\langle\phi\rangle \neq 0 . \tag{2·54}$$

The gauge boson $A_\mu{}^a$ corresponding to Q^a has, on this vacuum, the mass matrix generated from (2·52):

$$\mathcal{L}_{\text{mass}} = M_{ab}A_\mu{}^a A^{b\mu} , \tag{2·55}$$

$$M_{ab} \equiv \langle\phi^*\rangle T^a T^b \langle\phi\rangle . \tag{2·56}$$

The mechanism of the absorption of the Nambu-Goldstone particle (corresponding to the field $\delta\phi$) is understood in a covariant gauge as the fact that the NG mode becomes a member of an unphysical quartet.

Fermions can also have masses through spontaneous symmetry breakdown. If scalar ϕ interacts with fermions in a gauge invariant way:

$$\mathcal{L}_{\text{int}} = \sum \bar{\psi}\psi\phi , \tag{2·57}$$

the following mass terms emerge,

$$\mathcal{L}_{\text{mass}} = \sum \bar{\psi}\psi\langle\phi\rangle . \tag{2·58}$$

Now let us discuss the Weinberg-Salam model. A common expectation based on the phenomenology is that the weak interaction is mediated by massive gauge bosons, while the electromagnetic interaction is described by the photon which is a massless gauge field. Thus the $SU(2)\times U(1)$ symmetry of the Lagrangian (2·36) must be broken spontaneously in the following manner,

$$SU(2)\times U(1)\to U(1)_{\rm EM}\,.$$

The symmetry breaking of this pattern is made possible by introducing an adequate Higgs scalar which develops a vacuum expectation value as follows:

$$T^a\langle\phi\rangle\neq 0\,,\quad (a=0,1,2,3)$$
$$Q\langle\phi\rangle=0\,,\tag{2·59}$$

where Q is the electromagnetic charge generator defined by

$$Q=T^3+T^0\,.\tag{2·60}$$

On this vacuum only one gauge boson which couples to Q remains massless and others acquire masses.

In order to make fermions massive, it is necessary to introduce $SU(2)$ doublet Higgs fields, because the $\bar{\psi}_{\rm L}\psi_{\rm R}$ term belongs to the doublet representation and Yukawa interactions $\bar{\psi}_{\rm L}\psi_{\rm R}\phi$ must be gauge invariant.

Taking account of the above argument, we introduce one $SU(2)$ doublet Higgs field for which the eigenvalue of T^0 is 1/2. As potential terms of the Higgs field we take the form,

$$\begin{aligned}V(\phi)&\equiv-\mu^2\Phi^\dagger\Phi+\lambda(\Phi^\dagger\Phi)^2\\&=\lambda\left(\Phi^\dagger\Phi-\frac{v^2}{2}\right)^2-\frac{\mu^4}{4\lambda}\,,\end{aligned}\tag{2·61}$$

where v is introduced to denote the minimum point of the potential $V(\phi)$. The vacuum expectation value of Φ satisfies

$$\langle\Phi^\dagger\rangle\langle\Phi\rangle=\frac{1}{2}v^2\,.\tag{2·62}$$

Taking a suitable $SU(2)$ phase convention, we may write in general,

$$\langle\Phi\rangle=\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}.\quad (v:\ \text{real})\tag{2·63}$$

This convention corresponds to our representation of charge generator Q, (2·60), and it is easily verified that the symmetry generated by Q is not broken on this vacuum:

$$Q\langle\Phi\rangle=\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}=0\,.\tag{2·64}$$

It should be noted here that a single Higgs doublet field necessarily led to the breakdown of $SU(2)\times U(1)$ down to $U(1)$. This remaining $U(1)$ symmetry is interpreted as that of QED in any convention with an appropriate redefinition of the charge generator.

The expression of Higgs Φ on this vacuum is given in terms of hermitian fields ϕ, χ_1, χ_2 and χ_3 such that

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} i\chi_1 + \chi_2 \\ v + \phi - i\chi_3 \end{pmatrix}. \tag{2·65}$$

The Lagrangian for the Higgs field reads

$$\mathcal{L}_{\mathrm{H}} = (D_{\mathrm{L}}{}^{\mu}\Phi)^{\dagger}(D_{\mathrm{L}\mu}\Phi) + \mu^2\Phi^{\dagger}\Phi - \lambda(\Phi^{\dagger}\Phi)^2 . \tag{2·66}$$

Through $\mathcal{L}_{\mathrm{H}}$ the gauge boson mass matrix is generated as follows:

$$\mathcal{L}_{\text{mass}} = M_{ab} W_{\mu}{}^{a} W^{b\mu} ,$$

$$M_{ab} = \begin{array}{c|cccc} & W_1 & W_2 & W_3 & W_0 \\ \hline W_1 & g^2 & 0 & 0 & 0 \\ W_2 & 0 & g^2 & 0 & 0 \\ W_3 & 0 & 0 & g^2 & -gg' \\ W_0 & 0 & 0 & -gg' & g'^2 \end{array} \times \frac{v^2}{8} . \tag{2·67}$$

By diagonalizing the above mass matrix, we obtain

$$\mathcal{L}_{\text{mass}} = M_W^2 W_{\mu}{}^{+} W^{-\mu} + \frac{1}{2} M_Z^2 Z_{\mu} Z^{\mu} , \tag{2·68}$$

$$M_W = \frac{1}{2} g v , \quad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v ,$$

$$W_{\mu}{}^{\pm} = (W_{\mu}{}^{1} \mp i W_{\mu}{}^{2}) / \sqrt{2} ,$$

$$Z_{\mu} = (g W_{\mu}{}^{3} - g' W_{\mu}{}^{0}) / \sqrt{g^2 + g'^2} ,$$

$$A_{\mu} = (g' W_{\mu}{}^{3} + g W_{\mu}{}^{0}) / \sqrt{g^2 + g'^2} , \tag{2·69}$$

where $W_{\mu}{}^{\pm}$, Z_{μ} and A_{μ} represent the charged weak bosons, the neutral weak boson and the photon respectively. There is no mass term corresponding to the photon because it couples to the charge of the remaining symmetry, Q.

The corresponding Nambu-Goldstone bosons of $W_{\mu}{}^{\pm}$ and Z_{μ} are represented by $\chi^{\pm}\,(=(\chi_1 \mp i\chi_2)/\sqrt{2})$ and χ_3, and they are all massless as easily seen in $\mathcal{L}_{\mathrm{H}}$ (2·66). These NG bosons form three "quartets" with B fields (scalar polarizations of gauge fields) and FP-ghosts and antighosts. These "quartets" are all unphysical and can appear only with zero norm in the physical Hilbert space. This is guaranteed by the physical state condition (2·47) (see §§ 2.2 and 3.3). There are also mixing terms of the form $\partial_{\mu}\chi \cdot W^{\mu}$ in $\mathcal{L}_{\mathrm{H}}$. In the 't Hooft gauge which we adopt in the following, they are all cancelled away by the corresponding ones coming from the gauge fixing terms (see § 4.1).

Thus three components of Higgs field Φ are unphysical NG bosons and there remains the field ϕ which is the physical Higgs boson. This particle has mass,

$$m_\phi = \sqrt{2}\,\mu\,, \tag{2·70}$$

and is observable though it has not yet been found.

The Yukawa interaction of fermions with Higgs field ϕ is given by

$$\mathcal{L}_{\mathrm{M}} = -f\bar{L}\Phi R + \mathrm{h.c.}\,, \tag{2·71}$$

which generates fermion mass terms,

$$\mathcal{L}_{\mathrm{mass}} = -f(v/\sqrt{2})\cdot\bar{\psi}_e\psi_e\,.$$

The Yukawa coupling f is a free parameter in our Lagrangian and hence the electron mass is an input parameter as in QED.

We finally obtain the full Lagrangian by adding $\mathcal{L}_{\mathrm{H}}$ (2·66) and $\mathcal{L}_{\mathrm{M}}$ (2·71) to $\mathcal{L}'$ of Eq. (2·36),

$$\mathcal{L} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{M}}\,. \tag{2·72}$$

This is our basic Lagrangian with which we start our argument on renormalization and application to the calculation of higher order effects in leptonic processes.

Here we study the physical contents of the electroweak interactions described by our Lagrangian in the tree approximation. The term of our Lagrangian corresponding to the photon coupling is

$$\mathcal{L}_{\mathrm{EM}} = (gg'/\sqrt{g^2+g'^2})\,A^\mu\bar{\psi}_e\gamma_\mu\psi_e + \cdots\,, \tag{2·73}$$

where only the photon-fermion coupling is explicitly shown. In order for this interaction to reproduce QED, we identify the electromagnetic coupling constant as follows,

$$e \equiv gg'/\sqrt{g^2+g'^2}\,. \tag{2·74}$$

Next we consider effective four-fermion interactions in the low-energy limit mediated by massive gauge bosons. There are two kinds of these effective four-fermion interactions. One is the charged current interaction mediated by the $W^\pm$ boson:

$$\mathcal{L}^{\mathrm{eff}}_{\mathrm{charged}} = (g^2/8M_W^2)\,j_\mu^\dagger j^\mu\,, \tag{2·75}$$

where j_μ is defined in Eq. (2·5). Comparing it with Eq. (2·1), we obtain the following relation:

$$G_F/\sqrt{2}=g^2/8M_W^2=1/2v^2\,, \tag{2·76}$$

which determines the vacuum expectation value, the mass scale of the symmetry breakdown.

The other is the neutral current interaction mediated by the Z boson, which is parametrized as follows:

$$\mathcal{L}^{\text{eff}}_{\text{neutral}}=\sqrt{2}\,G_F j_\mu^{\text{NC}} j^{\text{NC}\mu}\,, \tag{2·77}$$

where

$$j_\mu^{\text{NC}}=\bar{\psi}\{T^3(1-\gamma_5)-2\sin^2\theta_W Q\}\psi\,, \tag{2·78}$$

$$\sin\theta_W=g'/\sqrt{g^2+g'^2}\,. \tag{2·79}$$

The angle θ_W is the rotation angle between (W^3, W^0) and (Z, A) and is called the Weinberg angle. We can express it by the masses of $W^\pm$ and Z:

$$\sin^2\theta_W=1-M_W^2/M_Z^2\,. \tag{2·80}$$

This neutral current interaction is very important to determine the electroweak theory among many gauge models as mentioned in § 1. 2. We should notice that the neutral current is parametrized in a simple way with one unknown parameter $\sin^2\theta_W$, but this simple parametrization is possible only in the case of tree approximation (see Chapter 7).

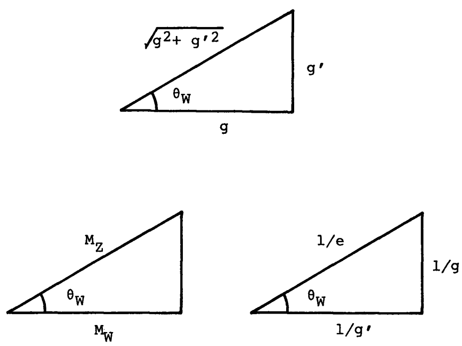


Fig. 2. 2. Relations among tree parameters g ($SU(2)$ coupling), g' ($U(1)$ coupling), θ_W (the Weinberg angle), M_W and M_Z (gauge boson masses) and the electromagnetic coupling e.

Since the beginning of § 2. 1, we have restricted ourselves to the case of single generation of leptons. The final form of our Lagrangian (2·72) includes only the electron and electron-neutrino as a fermion family. We are now in a position to extend our scheme to the more realistic case of multi-generations of leptons and quarks.

As has already been mentioned in § 1. 2, leptons and quarks constitute singlets and doublets in $SU(2)\times U(1)$. We represent fermions in the following way:

$$R_i=(\psi_i)_R\,,\quad R_I=(\psi_I)_R\,,\quad L_I=\begin{pmatrix}\psi_I\\ \psi_I'\end{pmatrix}_L, \tag{2·81}$$

where i and $I(=1,2,3)$ are indices representing 3 generations of leptons and quarks, ψ_i and ψ_I correspond to the usual up $(T^3=+1/2)$ and down $(T^3=-1/2)$ components of leptons and quarks and suffixes R and L represent the right-handed and left-handed components of field ψ:

$$\psi_{R,L}=\frac{1\pm\gamma_5}{2}\psi\,. \tag{2·82}$$

The primes of ψ_i' and ψ_I' express the mixing among leptons or quarks,

$$\psi_i'=U_{iI}\psi_I\,,\quad \psi_I'=U_{Ii}^{-1}\psi_i \tag{2·83}$$

with U the mixing matrix. If the neutrinos are massless, then $U=1$ for leptons (this corresponds to the case of no mixing in the minimum Higgs scheme). For later convenience we introduce a doublet

$$L_i=\begin{pmatrix}\psi_i'\\ \psi_i\end{pmatrix}_L. \tag{2·84}$$

It should be noted that L_i and L_I is related by

$$L_i=U_{iI}L_I\,. \tag{2·85}$$

The full Lagrangian corresponding to Eq. (2·72) can be derived in the present general case just in the same way as before. The resulting Lagrangian is given by

$$\mathcal{L}=\mathcal{L}_G+\mathcal{L}_F+\mathcal{L}_{GF}+\mathcal{L}_{FP}+\mathcal{L}_H+\mathcal{L}_M\,, \tag{2·86}$$

where $\mathcal{L}_G$, $\mathcal{L}_{GF}$, $\mathcal{L}_{FP}$ and $\mathcal{L}_H$ are the same as the previous ones, i.e., (2·18), (2·36a), (2·36b) and (2·66) respectively. Only the parts of the Lagrangian which include fermions are subject to change,

$$\mathcal{L}_F=i\sum_I\bar{L}_I\not{D}_L L_I+i\sum_{n=i,I}\bar{R}_n\not{D}_R R_n\,, \tag{2·87}$$

$$\mathcal{L}_M=-\sum_i f_i\bar{L}_i\Phi R_i-\sum_I f_I\bar{L}_I(i\tau_2\Phi^*)R_I+\text{h.c.}\,, \tag{2·88}$$

where f_i and f_I are the Yukawa coupling constants. Owing to the nonvanishing vacuum expectation value of the Higgs field Φ, fermions acquire masses through the Lagrangian (2·88). The masses of fermions are given in the tree approximation by

$$m_{i,I}=f_{i,I}v/\sqrt{2}\ . \tag{2·89}$$

If we use Eq. (2·76) with the value of G_F determined by the experimental data on the muon decay width in the tree approximation, i.e.,

$$G_F=1.166\times10^{-5}\mathrm{GeV}^{-2}\ , \tag{2·90}$$

we obtain

$$v/\sqrt{2}=174\ \mathrm{GeV}\ . \tag{2·91}$$

This is a typical mass scale at which the symmetry $SU(2)\times U(1)$ breaks down to $U(1)_{\mathrm{EM}}$ spontaneously. Using Eqs. (2·69), (2·80) and (2·91), we find

$$M_W=37.3/|\sin\theta_{\mathrm{W}}|\ ,\quad (\mathrm{GeV})$$

$$M_Z=74.6/|\sin 2\theta_{\mathrm{W}}|\ .\quad (\mathrm{GeV}) \tag{2·92}$$

Recent experimental data on $\nu_\mu e$ and $\nu_\mu N$ scattering and polarized eN scattering suggest (assuming the massless neutrino) that

$$\sin^2\theta_{\mathrm{W}}\simeq0.235\ , \tag{2·93}$$

giving

$$M_W\simeq77\ \mathrm{GeV}\ ,$$

$$M_Z\simeq88\ \mathrm{GeV}\ . \tag{2·94}$$

The above arguments on parameters M_W, M_Z, G_F and $\sin^2\theta_{\mathrm{W}}$ are restricted to the tree level. If one takes into account higher order effects and performs the necessary renormalization, one finds that those relations like Eqs. (2·69) (2·70), (2·74), (2·76) and (2·89) may be modified. A detailed discussion of the higher order effects will be given in Chapter 7.

As we have seen, the experimental information on e, G_F and $\sin^2\theta_{\mathrm{W}}$ determines all the parameters relevant to all effective four-fermion weak interactions (at the tree level). Thus the gauge theory of electroweak interactions proves itself to have higher predictive power.

§ 2.4 Ward-Takahashi identities

The Ward-Takahashi (WT) identities are the relations among Green functions given by symmetries of the theory. Originally the WT identities were discovered in QED [Ward 50, Takahashi 57]. In these days generalized ones are usually referred to. There are an infinite number of these WT identities. In this section we restrict ourselves to derivation of the WT identities based on the BRS symmetry which has been stated in the previous section.

The BRS symmetry plays an essential role to construct the operator formalism of gauge theories as a physically consistent quantum field theory in a covariant way. For example, consider the unitarity of physical S-matrix. It is guaranteed in terms of the quartet mechanism as explained in § 2.2. This mechanism states that unphysical quartet members appear in the physical Hilbert space only as zero-norm states. This is equivalent to the existence of the appropriate WT identities of Green functions including unphysical particles in the final state although the following type of the WT identities is not explicitly used in the proof of the quartet mechanism [Kugo and Ojima 78].

We derive the WT identities for the effective action given by the BRS symmetry. These identities will be used to prove the renormalizability (§ 2.5), the consistency of the renormalization conditions (§ 3.3) and the charge universality (§ 3.4).

Let us define the generating functional $W[J, K]$ of Green functions.

$$\exp iW[J, K] \equiv \langle 0|\mathrm{T} \exp iS[J, K]|0\rangle, \tag{2·95}$$

$$S[J, K] \equiv \int d^4x [J_i(x)\hat{\phi}_i(x) + K_i(x)\delta^{\mathrm{BRS}}\hat{\phi}_i(x)], \tag{2·96}$$

where $\hat{\phi}_i(x)$ stand for all field operators*) in the theory and $J_i(x)$ and $K_i(x)$ are c-number source currents with the same commutativity as that of corresponding $\hat{\phi}_i(x)$ and $\delta^{\mathrm{BRS}}\hat{\phi}_i(x)$. It should be mentioned that the differentiation with respect to anticommuting η is understood to be the so-called left-differentiation, which corresponds to the distribution formula:

$$D_\eta(AB) = (D_\eta A)B + (-1)_\eta{}^A A(D_\eta B),$$

where $(-1)_\eta{}^A$ is $+1$ (-1) when A is commutative (anticommutative) with η.

Differentiation of the above $W[J, K]$ with respect to J_i leads to a connected Green function of the field $\hat{\phi}$:

$$\frac{1}{i^{n-1}} \left.\frac{\delta^n W[J, K]}{\delta J_1 \delta J_2 \cdots \delta J_n}\right|_{J=K=0} = \langle 0|\mathrm{T}\hat{\phi}_1 \cdots \hat{\phi}_n|0\rangle^c. \tag{2·97}$$

The proof of this equation is given in Appendix G. As a next step, following the usual procedure, we define classical fields ϕ_i in terms of W,

$$\phi_i \equiv \frac{\delta}{\delta J_i} W[J, K]. \tag{2·98}$$

We define the effective action $\Gamma[\phi, K]$ as the Legendre transform of $W[J, K]$ with respect to J:

$$\Gamma[\phi, K] \equiv W[J[\phi, K], K] - J_i[\phi, K]\phi_i. \tag{2·99}$$

*) In the previous sections, ϕ_i has been used for field operators. In this section, ϕ_i stand for classical fields and field operators are written as $\hat{\phi}_i$.

It should be noticed that the source J is a functional of ϕ and K. In fact, with the help of Γ the source J is expressed as

$$J_i = -(+)\frac{\delta\Gamma}{\delta\phi_i} \quad \text{for commuting (anticommuting) field.} \qquad (2\cdot100)$$

By successive differentiations with respect to ϕ, we obtain one-particle irreducible Green functions:

$$\frac{1}{i}\frac{\delta^2\Gamma[\phi, K]}{\delta\phi_i\delta\phi_j}\bigg|_{\phi=\langle\hat{\phi}\rangle, K=0} \langle 0|\mathrm{T}\hat{\phi}_i\hat{\phi}_k|0\rangle^c = \delta_{jk}\,, \qquad (2\cdot101)$$

$$i\frac{\delta^n\Gamma[\phi, K]}{\delta\phi'_{i_1}\cdots\delta\phi'_{i_n}}\bigg|_{\phi=\langle\hat{\phi}\rangle, K=0} \langle 0|\mathrm{T}\hat{\phi}_{i_1}\hat{\phi}_{i_1'}|0\rangle^c \cdots \langle 0|\mathrm{T}\hat{\phi}_{i_n}\hat{\phi}_{i_n'}|0\rangle^c$$

$$= \langle 0|\mathrm{T}\hat{\phi}_{i_1}\cdots\hat{\phi}_{i_n}|0\rangle^c_{\text{proper}} \quad \text{for} \quad n \geqq 3\,. \qquad (2\cdot102)$$

See Appendix G for the proof of these formulas. In these formulas the ordering of operators should be kept carefully so that the correct sign is obtained for anticommuting fields.

Let us derive the WT identity for the effective action Γ. We start with a trivial relation:

$$\langle 0|\Big[i\lambda Q_{\mathrm{B}}, \mathrm{T}\exp iS[J, K]\Big]|0\rangle = 0\,, \qquad (2\cdot103)$$

the validity of which is clear if we remember the physical state condition (2·47),

$$Q_{\mathrm{B}}|0\rangle = 0\,.$$

Noticing Eq. (2·46),

$$\delta^{\mathrm{BRS}}(\delta^{\mathrm{BRS}}\hat{\phi}_i) = 0\,,$$

we obtain

$$\int d^4x J_i(x)\langle 0|\mathrm{T}\delta^{\mathrm{BRS}}\hat{\phi}_i(x)\exp iS[J, K]|0\rangle = 0\,, \qquad (2\cdot104)$$

which is expressed in terms of the effective action Γ as

$$\int d^4x \frac{\delta\Gamma}{\delta\phi_i(x)}\frac{\delta\Gamma}{\delta K_i(x)} \equiv \frac{\delta\Gamma}{\delta\phi_i}\cdot\frac{\delta\Gamma}{\delta K_i} = 0\,, \qquad (2\cdot105)$$

where use has been made of Eq. (2·100) and

$$\frac{\delta W}{\delta K_i}\bigg|_{J\text{ fixed}} = \frac{\delta\Gamma}{\delta K_i}\bigg|_{\phi\text{ fixed}}\,. \qquad (2\cdot106)$$

Equation (2·105) is a goal of our derivation. We write all fields in Eq. (2·105) explicitly:

$$\frac{\delta\Gamma}{\delta W_\mu{}^a}\cdot\frac{\delta\Gamma}{\delta K_a{}^\mu}+\frac{\delta\Gamma}{\delta\psi_i}\cdot\frac{\delta\Gamma}{\delta K_{\psi_i}}+\frac{\delta\Gamma}{\delta\bar{\psi}_i}\cdot\frac{\delta\Gamma}{\delta K_{\bar{\psi}_i}}+\frac{\delta\Gamma}{\delta\Phi_i}\cdot\frac{\delta\Gamma}{\delta K_{\Phi_i}}$$

$$+\frac{\delta\Gamma}{\delta\bar{c}}B+\frac{\delta\Gamma}{\delta c^a}\cdot\frac{\delta\Gamma}{\delta K_{c^a}}=0\,, \tag{2·107}$$

where we have used the following relation:

$$\frac{\delta\Gamma}{\delta K_{\bar{c}}}=\frac{\delta W[J,K]}{\delta J_B}=B\,. \tag{2·108}$$

The identity (2·107) can be changed to a more useful form. We start with the equations of motion for fields $\hat{B}^a$ and $\hat{c}^a$,

$$F_i{}^a\hat{\phi}_i+\alpha\hat{B}^a=0\,, \tag{2·109}$$

$$\delta^b(F_i{}^a\hat{\phi}_i)\hat{c}^b=0\,, \tag{2·110}$$

where we have taken the same gauge fixing Lagrangian as that in § 2. 3:

$$\mathcal{L}_{\mathrm{GF}}=F_i{}^a\hat{\phi}_i\hat{B}^a+\frac{\alpha}{2}\hat{B}^a\hat{B}^a\,.$$

We rewrite the trivial relations

$$\langle 0|\mathrm{T}(F_i{}^a\hat{\phi}_i+\alpha\hat{B}^a)\exp iS[J,K]|0\rangle=0\,, \tag{2·111}$$

$$\langle 0|\mathrm{T}(\delta^b(F_i{}^a\hat{\phi}_i)\hat{c}^b)\exp iS[J,K]|0\rangle=0\,. \tag{2·112}$$

The covariant combination $\partial^\mu\hat{W}_\mu{}^a$ should appear in the terms $F_i{}^a\hat{\phi}_i$ because the canonical conjugate momentum of $\hat{W}_0{}^a$, $\hat{\Pi}_{\hat{W}_0{}^a}\equiv\delta\hat{\mathcal{L}}/\delta\dot{\hat{W}}_0{}^a$, can be obtained from the gauge fixing terms, $\hat{B}\cdot F_i{}^a\hat{\phi}_i$. We extract the differential operator out of the T-product taking into account the equal time commutator terms between $\hat{W}_0{}^a$ and $\exp i\,S[J,K]$. Noting that only B field, i.e., $\hat{\Pi}_{\hat{W}_0{}^a}$ in $\mathrm{S}[J,K]$ does not commute with $\hat{W}_0{}^a$ at equal time, we obtain from Eq. (2·111)

$$(F_i{}^a\phi_i+\alpha B^a)\langle 0|\mathrm{T}\exp iS[J,K]|0\rangle-\langle 0|\mathrm{T}[\hat{W}_0{}^a,\exp iS[J,K]]_{\mathrm{ETC}}|0\rangle$$

$$=(F_i{}^a\phi_i+\alpha B^a+J_B{}^a)\langle 0|\mathrm{T}\exp iS[J,K]|0\rangle$$

$$=\left(F_i{}^a\phi_i+\alpha B^a-\frac{\delta\Gamma}{\delta B^a}\right)\langle 0|\mathrm{T}\exp iS[J,K]|0\rangle$$

$$=0\,. \tag{2·113}$$

Similarly, in Eq. (2·112) we notice that in the terms $\delta^b(F_i{}^a\hat{\phi}_i)\hat{c}^b$ there are differential terms of the canonical conjugate momentum of $\hat{c}^a$, $\partial^\mu\hat{\Pi}_{\hat{\bar{c}}^a}$, which contribute as equal time commutator terms. Then we obtain

$$F_i{}^a\langle 0|\mathrm{T}\delta^{\mathrm{BRS}}\hat{\phi}_i \exp iS[J,K]|0\rangle - \langle 0|\mathrm{T}[\hat{\Pi}_{\hat{c}^a}, \exp iS[J,K]]_{\mathrm{ETC}}|0\rangle$$

$$= \left(F_i{}^a\frac{\delta W}{\delta K_i} + J_{\bar{c}^a}\right)\langle 0|\mathrm{T}\exp iS[J,K]|0\rangle$$

$$= \left(F_i{}^a\frac{\delta\Gamma}{\delta K_i} + \frac{\delta\Gamma}{\delta\bar{c}^a}\right)\langle 0|\mathrm{T}\exp iS[J,K]|0\rangle$$

$$= 0\,. \tag{2·114}$$

In this way Eqs. (2·111) and (2·112) are changed to

$$F_i{}^a\phi_i + \alpha B^a = \frac{\delta\Gamma}{\delta B^a}\,, \tag{2·115}$$

$$F_i{}^a\frac{\delta\Gamma}{\delta K_i} = -\frac{\delta\Gamma}{\delta\bar{c}^a}\,. \tag{2·116}$$

Although these relations are obtained by using the equations of motion, they should be regarded as kinematical relations based on the BRS invariance since they are related only with $\mathcal{L}_{\mathrm{GF}}$ and $\mathcal{L}_{\mathrm{FP}}$.

It is more convenient to introduce the modified effective action $\tilde{\Gamma}$ defined by

$$\tilde{\Gamma} \equiv \Gamma - \int \mathcal{L}_{\mathrm{GF}} d^4x\,, \tag{2·117}$$

where $\mathcal{L}_{\mathrm{GF}}$ is understood as described in terms of classical fields. $\tilde{\Gamma}$ is independent of B:

$$\frac{\delta\tilde{\Gamma}}{\delta B^a} = \frac{\delta\Gamma}{\delta B^a} - (F_i{}^a\phi_i + \alpha B^a) = 0\,, \tag{2·118}$$

where use has been made of Eq. (2·115). Substituting $\tilde{\Gamma}$ into Eq. (2·116), we obtain

$$F_i{}^a\frac{\delta\tilde{\Gamma}}{\delta K^i} + \frac{\delta\tilde{\Gamma}}{\delta\bar{c}^a} = 0\,. \tag{2.119}$$

Let us rewrite Eq. (2·107) by using $\tilde{\Gamma}$:

$$\frac{\delta\tilde{\Gamma}}{\delta W_\mu{}^a}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_a{}^\mu} + \frac{\delta\tilde{\Gamma}}{\delta\psi_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\psi_i}} + \frac{\delta\tilde{\Gamma}}{\delta\bar{\psi}_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\bar{\psi}_i}} + \frac{\delta\tilde{\Gamma}}{\delta\Phi_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\Phi_i}} + \frac{\delta\tilde{\Gamma}}{\delta c^a}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{c^a}}$$

$$= -\frac{\delta}{\delta\phi_i}\int\mathcal{L}_{\mathrm{GF}}d^4x\cdot\frac{\delta\tilde{\Gamma}}{\delta K_i} - \frac{\delta\tilde{\Gamma}}{\delta\bar{c}}\cdot B\,.$$

The right-hand side terms vanish according to Eq. (2·119) as follows:

$$\frac{\delta}{\delta\phi_i}\int\mathcal{L}_{\mathrm{GF}}d^4x\cdot\frac{\delta\tilde{\Gamma}}{\delta K_i}+\frac{\delta\tilde{\Gamma}}{\delta\bar{c}^a}\cdot B=B\cdot\left(F_i{}^a\frac{\delta\tilde{\Gamma}}{\delta K_i}+\frac{\delta\tilde{\Gamma}}{\delta\bar{c}^a}\right)=0\,.$$

The final form of the WT identity is

$$\frac{\delta\tilde{\Gamma}}{\delta W_\mu^a}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_a{}^\mu}+\frac{\delta\tilde{\Gamma}}{\delta\psi_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\psi_i}}+\frac{\delta\tilde{\Gamma}}{\delta\bar{\psi}_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\bar{\psi}_i}}$$
$$+\frac{\delta\tilde{\Gamma}}{\delta\Phi_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\Phi_i}}+\frac{\delta\tilde{\Gamma}}{\delta c^a}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{c^a}}=0\,. \tag{2·120}$$

We differentiate Eq. (2·120), for example, with respect to ϕ and set ϕ to their vacuum expectation values: $\phi=\langle\hat{\phi}\rangle$. Each obtained term is a product of two one-particle irreducible Green functions:

$$\sum_j\frac{\delta\tilde{\Gamma}}{\delta\phi_i\cdots\delta\phi_j}\bigg|_{\phi=\langle\hat{\phi}\rangle,K=0}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_j\delta\phi_{j+1}\cdots\delta\phi_k}\bigg|_{\phi=\langle\hat{\phi}\rangle,K=0}=0\,. \tag{2·121}$$

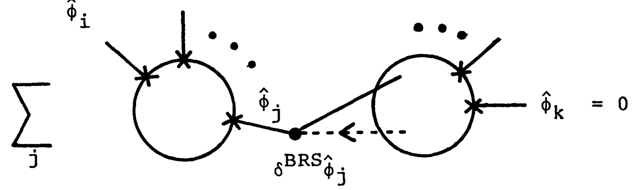


Fig. 2.3. Graphical representation of a Ward-Takahashi identity (2·121).

We express Eq. (2·121) graphically as in Fig. 2.3. In this graph the black dot at the center represents an operator product $\delta^{\mathrm{BRS}}\hat{\phi}_j$ and the right half of the diagram stands for the Green function into which the operator product $\delta^{\mathrm{BRS}}\hat{\phi}_j$ is inserted. The dotted line represents the ghost propagator. In the case that $\delta^{\mathrm{BRS}}\hat{\phi}_j$ contain linear terms of fields, special attention is required. For gauge fields, for example, the right half of Eq. (2·121) is

$$\langle 0|\mathrm{T}\delta^{\mathrm{BRS}}\hat{W}_\mu{}^a\cdots|0\rangle^c_{\mathrm{proper}}=\langle 0|\mathrm{T}\,(gf^{abc}\hat{W}_\mu{}^b\hat{c}^c+\partial_\mu\hat{c}^a)\cdots|0\rangle^c_{\mathrm{proper}}\,.$$

As it stands, the contribution of the second term on the right-hand side is not included in Fig. 2.3. We understand, however, that the diagram in the figure represents the whole contribution including that of $\partial_\mu\hat{c}^a$ for simplicity of graphical expression.

In the following sections, these WT identities, Eqs. (2·118) ~ (2·120) play an important role in proving the renormalizability and charge universality and in analyzing consistency of the renormalization conditions.

§ 2.5 Renormalizability

It was proved first by 't Hooft that non-Abelian gauge theories are renormalizable ['t Hooft 71b, c]. Further contributions have clarified the structure of gauge theories [Lee and Zinn-Justin 72a, b, c, Slavnov 72, Taylor 71, t' Hooft and Veltman 72b, Fujikawa et al. 72, for review articles, see Abers and Lee 73, Lee 76]. In this section general theory of the renormalizability is briefly reviewed and the Ward-Takahashi (WT) identities are derived which constitute the essential part of the renormalizability of gauge theories.

Consider a theory described by a Lagrangian $\mathcal{L}(g_0, \phi_0)$ where g_0 and ϕ_0 generically represent bare coupling constants (including masses) and bare fields respectively. The effective action Γ of the theory defined in § 2.4 is calculated by the loop expansion. This Γ as a functional of g_0 and ϕ_0 includes divergences. The renormalizability of the theory dictates that the effective action is a finite functional Γ_R of g_R and ϕ_R,

$$\Gamma_R(g_R, \phi_R) = \Gamma(g_0, \phi_0), \qquad (2\cdot122)$$

where g_R and ϕ_R are defined by an appropriate transformation of g_0 and ϕ_0. We introduce renormalization constants Z through the transformation,

$$g_0 = {}_gZ g_R, \qquad (2\cdot123a)$$

$$\phi_0 = Z_3^{1/2}\phi_R, \qquad (2\cdot123b)$$

and assume the loop expansion of the above renormalization constants,

$$Z = 1 + Z^{(1)} + Z^{(2)} + \cdots . \qquad (2\cdot124)$$

These $Z^{(n)}$'s generate counter terms for the effective action Γ_R in the loop expansion. The problem is whether all divergent integrals in Γ_R can be cancelled out by available counter terms loop by loop.

In each order of the loop expansion counter terms play a role to subtract polynomials in momentum variable from Feynman amplitudes. The resultant renormalized Feynman integrals are written in the closed form, where subtraction is expressed by Taylor operators such as

$$1 - \sum_i \tau_i, \quad \tau_i \equiv (\partial_p)^i|_{p=\mu} .$$

By the use of this closed form for renormalized Feynman integrals, it was strictly proved that if sufficient counter terms are prepared to subtract all superficial divergences, all renormalized Feynman integrals are convergent and finite, that is, the theory is renormalizable [Bogoliubov and Parasiuk 57, Hepp 66, Zimmermann 69]. Here the superficial degree of divergence of a Feynman integral is the degree of divergence obtained from purely dimensional analysis.

Renormalizability requires first of all that the number of types of superficially divergent graphs is finite, otherwise we must prepare an infinite number of (bare) parameters to renormalize the theory. This requirement is satisfied only in the case where all coupling constants g_0 in $\mathcal{L}$ have non-negative dimension in the momentum scale [Sakata et al. 51, 52]. Gauge theories satisfy the condition: Gauge coupling constants are dimensionless. If we introduce scalar fields, the Yukawa couplings with fermions and scalar self couplings up to quartic ones are allowed.

Secondly the consistency should be proved between the degrees of freedom of counter terms and independent divergences of amplitudes. In gauge theories (with simple Lie group) all gauge coupling constants, for example, are g for three-vertices and g^2 for four-vertices. This uniqueness of couling constants is nothing but the gauge symmetry which gives the theory a very strong predictive power. The uniqueness of various (bare) couplings indicates that the counter terms are strongly constrained. In order to absorb all divergences in amplitudes into constrained counter terms corresponding relations should exist among divergent amplitudes. These relations are given by the WT identities derived in § 2. 4.

In the Weinberg-Salam (WS) theory, for example, the electron mass is dynamically related to the Higgs coupling constant with the electron. This is one of the above-mentioned constraints. Especially in the gauge theory with spontaneous symmetry breakdown, the renormalizability of the theory is non-trivial if we are involved in the perturbation calculation on the symmetry breaking vacuum.

Let us review briefly the proof of the renormalizability of gauge theories by the use of the induction. We define the operation $*$ by

$$F * G \equiv \frac{1}{2}\left(\frac{\delta F}{\delta\phi_i}\cdot\frac{\delta G}{\delta K_i}+\frac{\delta G}{\delta\phi_i}\cdot\frac{\delta F}{\delta K_i}\right), \tag{2·125}$$

where ϕ_i and K_i are relevant fields and corresponding BRS sources respectively. The WT identity (2·120) is expressed as follows by this operation,

$$\tilde{\Gamma} * \tilde{\Gamma} = 0 . \tag{2·126}$$

We renormalize ϕ_i and K_i properly so that the WT identity (2·126) holds in the expression with renormalized fields and BRS sources.

In the loop expansion, we write

$$\tilde{\Gamma}_{\mathrm{R}} \equiv \sum_{i=0}^{\infty} \Gamma_{\mathrm{R}}{}^{(i)} . \tag{2·127}$$

We also expand the action $\tilde{S}$ with source terms,

$$\tilde{S} \equiv \sum_{i=0}^{\infty} \tilde{S}^{(i)} , \tag{2·128}$$

where $\tilde{S}^{(i)}$ ($i \geq 1$) are counter terms. We suppose that $\tilde{\Gamma}_{\mathrm{R}}$ is a finite functional to the n-loop order with counter terms to the same order:

$$\widetilde{\Gamma}_{\mathrm{R}}{}^{(j)}[\sum_{i=0}^{n}\widetilde{S}^{(i)}]=\text{finite for } j\leq n\,. \tag{2·129}$$

The $(n+1)$-loop part of the identity (2·126) is written as

$$\widetilde{\Gamma}_{\mathrm{R}}{}^{(0)}[\widetilde{S}^{(0)}]*\widetilde{\Gamma}_{\mathrm{R}}{}^{(n+1)}[\sum_{i=0}^{n+1}\widetilde{S}^{(i)}]+[\text{finite functionals}]=0\,. \tag{2·130}$$

Taking out the divergent part of Eq. (2·130), we have

$$\widetilde{S}^{(0)}*\widetilde{\Gamma}_{\mathrm{div}}^{(n+1)}[\sum_{i=0}^{n+1}\widetilde{S}^{(i)}]=0\,, \tag{2·131}$$

where we have used the fact that the effective action to the tree approximation is the tree action,

$$\widetilde{\Gamma}_{\mathrm{R}}{}^{(0)}=\widetilde{S}^{(0)}\,. \tag{2·132}$$

The $(n+1)$-loop part $\widetilde{\Gamma}_{\mathrm{div}}^{(n+1)}[\sum_{i=0}^{n+1}\widetilde{S}^{(i)}]$ consists of two kinds of terms,

$$\widetilde{\Gamma}_{\mathrm{div}}^{(n+1)}[\sum_{i=0}^{n+1}\widetilde{S}^{(i)}]=\widetilde{\Gamma}_{\mathrm{div}}^{(n+1)}[\sum_{i=0}^{n}\widetilde{S}^{(i)}]+\widetilde{S}^{(n+1)}\,. \tag{2·133}$$

The first term on the right-hand side of Eq. (2·133) is amplitudes calculated with counter terms up to n-loop order and its inner divergences have been eliminated. The second term, $S^{(n+1)}$, is the $(n+1)$-loop counter terms to cancel out superficial divergences. By substituting Eq. (2·133) into Eq. (2·131), we obtain

$$\widetilde{S}^{(0)}*\widetilde{\Gamma}_{\mathrm{div}}^{(n+1)}[\sum_{i=0}^{n}\widetilde{S}^{(i)}]=0\,. \tag{2·134}$$

We note here that the relation

$$\widetilde{S}^{(0)}*\widetilde{S}^{(n+1)}=0\,, \tag{2·135}$$

holds since $\widetilde{S}^{(n+1)}$ is a local gauge invariant functional. Equation (2·134) is called the renormalization equation [Lee 74].

The general solution of this functional equation is obtained as follows [Jogleker and Lee 76],

$$\widetilde{\Gamma}_{\mathrm{div}}^{(n+1)}[\sum_{i=0}^{n}\widetilde{S}^{(i)}]=a_jG_j+\widetilde{S}^{(0)}*F\,, \tag{2·136}$$

where G_j is a local gauge invariant functional of gauge and matter fields and F is a functional of all fields and BRS sources. We should recall here that it is sufficient for the renormalizability that superficially divergent parts are cancelled out. The superficially divergent part of $\widetilde{\Gamma}$ is the functional with dimension up to four. The solution (2·136) with dimension up to four is shown to have the form of the action with a suitable redefinition of coupling constants, fields and sources. Therefore we can take the $(n+1)$-loop counter terms as

$$S^{(n+1)} \equiv -\Gamma_{\mathrm{div}}^{(n+1)}\Big[\sum_{i=0}^{\prime n} S^{(i)}\Big] \tag{2·137}$$

with appropriate renormalization constants. Thus the divergent part of $\Gamma^{(n+1)}[\sum_{i=0}^{n+1} S^{(n+1)}]$ vanishes. Combining with the fact that in the tree approximation (the zero-loop order) the theory is finite, we have completed the proof of renormalizability to all orders.

We explain below the relation between a symmetric theory and a symmetry broken theory. We show that the theory with spontaneous symmetry breakdown (symmetry broken theory) can be actually renormalized with the same counter terms as those for the symmetric theory. In principle the above proof for the renormalizability holds regardless of the structure of the vacuum, symmetric or symmetry broken, since we deal with the effective action which simultaneously describes systems with all possible vacua. However, the renormalization condition (2·137) actually requires symmetric calculation of the perturbation theory. In the symmetric calculation of a symmetry broken theory, the propagator of the corresponding Higgs field ϕ ($\langle\phi\rangle\neq 0$), that is, the propagator expanded from the point $\langle\phi\rangle=0$, has negative mass squared. Therefore, it is non-trivial whether a consistent symmetric and mass-independent renormalization procedure exists.

Symmetric and mass-independent renormalization schemes are obtained by several authors. [Weinberg 73, 't Hooft 73, Kugo 77, see, also, Collins and Macfarlane 74]. The most convenient one is the minimal subtraction scheme by 't Hooft. Consider a gauge theory with a Higgs potential Lagrangian,

$$\mathcal{L}_{\mathrm{V}} = -\frac{1}{2}m_0^2\phi_0^2 - \frac{1}{4!}\lambda_0\phi_0^4 . \tag{2·138}$$

In this theory we define renormalization constants as usual. For m_0^2 we define

$$m_0^2 = Z_m m^2 . \tag{2·139}$$

In the minimal subtraction scheme we subtract only pole terms at $D=4$ in the D-dimensional regularization. In this scheme all dimensionless renormalization constants including Z_m are determined as follows:

$$Z_i = \sum_{n=1}^{\infty} C_n \frac{1}{(D-4)^n} , \tag{2·140}$$

where C_n is a function of renormalized coupling constants (dimensionless). By using this scheme, we renormalize symmetric ($m^2 \geq 0$) and symmetry broken ($m^2 < 0$) theory simultaneously with common counter terms. The difficulty which may be caused by the negative mass squared does not occur. In a usual mass-dependent renormalization scheme a negative mass squared of the propagator brings about the difficulty that renormalization constants which should be real are actually estimated to be complex.

By using the above symmetric and mass-independent renormalization scheme, we obtain the effective action for the symmetry broken theory ($m^2<0$) as follows. First of all, we calculate the effective action in the symmetric theory with a positive m^2 and make it finite by renormalization. Then we set m^2 to be a negative value. We find the $\phi_{\min}$ which gives the absolute minimum of the effective potential. Finally, we reexpand the effective action with $\tilde{\phi}=\phi-\phi_{\min}$ and obtain the effective action of the symmetry broken theory expressed by the true excitation mode of fields. If it is necessary, we may impose an additional finite renormalization on the obtained effective action. Of course, no difficulty takes place in the procedure of this additional renormalization.

The renormalization conditions for amplitudes in the minimal subtraction scheme is not clear. We present explicitly a set of renormalization conditions for a symmetric and mass-independent renormalization scheme [Kugo 77]. The point is to renormalize the effective action so that it becomes a finite functional of both fields and mass m^2 with common counter terms. The renormalization condition appropriate for this purpose is, for example,

$$\Gamma^{(2)}(p^2=0,\lambda,m^2=0)=0\,, \tag{2·141a}$$

$$\frac{\partial\Gamma^{(2)}}{\partial m^2}(p^2=0,\lambda,m^2)\big|_{m^2=\mu^2}=1\,, \tag{2·141b}$$

$$\frac{\partial\Gamma^{(2)}}{\partial p^2}(p^2,\lambda,m^2=\mu^2)\big|_{p^2=0}=-1\,, \tag{2·142}$$

$$\Gamma^{(4)}(p^2=0,\lambda,m^2=\mu^2)=-\lambda\,, \tag{2·143}$$

where $\Gamma^{(n)}$ is the n-point function of Higgs ϕ in the momentum representation,

$$\Gamma^{(n)}=\frac{1}{n!}\mathrm{FT}\frac{\delta^n\Gamma}{\delta\phi(x_1)\cdots\delta\phi(x_n)}\,,$$

and μ^2 is a renormalization point. We have not set μ^2 equal to zero on account of the infrared divergences. Here we should notice that a usual mass renormalization condition is split into two conditions (2·141). This is necessary to renormalize the effective action comprehensively as a functional of fields, coupling constants and mass m^2 with mass independent counter terms. In order to satisfy the above renormalization conditions, we must prepare two terms for the mass counter terms; one is proportional to m^2 and the other is a constant independent of m^2. We introduce renormalization constants and separate $\mathcal{L}_{\mathrm{V}}$ into the tree part and the counter-term parts as follows:

$$\phi_0 = Z_\phi^{1/2}\phi\,, \tag{2·144a}$$

$$\lambda_0 = Z_\lambda Z_\phi^{-2}\lambda\,, \tag{2·144b}$$

$$m_0^2 = Z_m Z_\phi^{-1} m^2 + \delta m^2 Z_\phi^{-1}\,, \tag{2·144c}$$

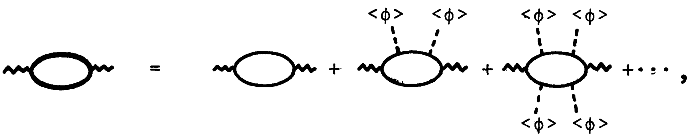
$$\mathcal{L}_V = \mathcal{L}_V^t + \mathcal{L}_V^c\,, \tag{2·145}$$

$$\mathcal{L}_V^t = -\frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4\,, \tag{2·146a}$$

$$\mathcal{L}_V^c = -\frac{1}{2}m^2(Z_m - 1)\phi^2 - \frac{1}{2}\delta m^2\phi^2 - \frac{\lambda}{4!}(Z_\lambda - 1)\phi^4\,. \tag{2·146b}$$

By introducing mass renormalization constants Z_m and δm^2 redundantly as in Eq. (2·144c), we have necessary mass counter terms for the renormalization conditions (2·141). All the renormalization constants (Z_ϕ, Z_λ, Z_m, δm^2) are determined mass-(m^2-) independently. The counter term δm^2 can be identically zero especially in the minimal subtraction scheme.

The relation between the symmetric ($\langle\phi\rangle = 0$) and symmetry broken ($\langle\phi\rangle \neq 0$) theories which is explained above is easily understandable by graphical manner. Consider a fermion loop contribution to the self-energy of a gauge boson in a symmetry broken theory. The fermion is assumed to be massless in a symmetric theory and to acquire mass from a vacuum expectation value of Higgs scalar ϕ. We expand the self-energy graph such that



(2·147)

where the fermion line represents massive (massless) propagator on the left (right)-hand side. The ultraviolet divergences occur only in the first and second terms on the right-hand side of Eq. (2·147). These two divergences correspond to those in the symmetric theory; one corresponds to the two-point function and the other corresponds to the four-point function of gauge bosons and Higgs scalars. Thus counter terms for the symmetric theory also cancel out divergences in the symmetry broken theory. As will be shown below, some kinds of universalities between amplitudes hold in the symmetric theory. In the symmetry broken theory, for example, in Eq. (2·147) only the first and second terms on the right-hand side satisfy these universalities and the rest terms break them. Though the breaking terms are finite, these universalities do not hold in the symmetry broken theory. This point will also be mentioned later.

In the following we give explicit expressions of the WT identities in the form of Green functions which guarantee the renormalizability in a symmetric theory with a simple group. We deal with four examples: a) two-point function of gauge bosons, b) three-point functions of gauge bosons and ghosts, c) three-point functions of gauge bosons and fermions and d) four-point functions of gauge bosons. In a) we show that gauge bosons remain massless with higher order corrections. In b) $\sim$d) we clarify constraints among various vertices which are indispensable for the proof of the renormalizability because all relevant couplings in the Lagrangian are described by unique coupling constant (g and g^2).

a) We begin with the WT identity (2·120):

$$\frac{\delta\tilde{\Gamma}}{\delta W_\nu{}^\alpha}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_\alpha{}^\nu}+\frac{\delta\tilde{\Gamma}}{\delta\psi_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\psi_i}}+\frac{\delta\tilde{\Gamma}}{\delta\bar{\psi}_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\bar{\psi}_i}}$$
$$+\frac{\delta\tilde{\Gamma}}{\delta\Phi_i}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\Phi_i}}+\frac{\delta\tilde{\Gamma}}{\delta c^\alpha}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{c^\alpha}}=0\,, \qquad (2\cdot148)$$

where Greek letters represent group indices of the adjoint representation. We make differentiation of Eq. (2·148):

$$\frac{\delta^2}{\delta W_\mu{}^\beta\delta c^\gamma}\text{Eq. }(2\cdot148)\Big|_0\,,$$

where the symbol $|_0$ represents setting all field variables to zero. Taking account of the ghost number conservation, we have only one term,

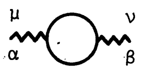
$$\frac{\delta^2\tilde{\Gamma}}{\delta W_\mu{}^\beta\delta W_\nu{}^\alpha}\cdot\frac{\delta^2\tilde{\Gamma}}{\delta K_\alpha{}^\nu\delta c^\gamma}\Big|_0=0\,. \qquad (2\cdot149)$$

In the momentum representation we define invariant amplitudes as follows:

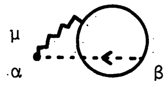
$$\tilde{\Gamma}^{\mu\nu}_{\alpha\beta}(p)\equiv\int d^4x\,e^{ip(x-y)}\frac{\delta^2\tilde{\Gamma}}{\delta W_\mu{}^\alpha(x)\,\delta W_\nu{}^\beta(y)}$$
$$=\delta^{\alpha\beta}\left\{A(p^2)\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)+B(p^2)\frac{p^\mu p^\nu}{p^2}\right\}, \qquad (2\cdot150)$$

$$G^\mu_{\alpha\beta}(p)\equiv\int d^4x\,e^{ip(x-y)}\frac{\delta^2\tilde{\Gamma}}{\delta K_\mu{}^\alpha(x)\,\delta c^\beta(y)}$$
$$=p^\mu\delta^{\alpha\beta}G(p^2)\,. \qquad (2\cdot151)$$

We write them graphically as (see Appendix G),

$\tilde{\Gamma}^{\mu\nu}_{\alpha\beta}(p)\equiv$ 

, (2·152)

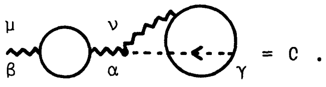
$G^\mu_{\alpha\beta}(p)\equiv$ 

, (2·153)

where the dotted line represents the current of the conserved ghost number and the black dot stands for the local operator product of BRS transform ($\delta^{\mathrm{BRS}} W_\mu{}^a$ in Eq. (2·153)). We note here that a derivative of the effective action Γ with respect to a BRS source K equals that of W defined in (2·95):

$$\frac{\delta\tilde{\Gamma}}{\delta K}=\frac{\delta\Gamma}{\delta K}=\frac{\delta W}{\delta K}\,. \tag{2·154}$$

With the above graphical notations Eq. (2·149) is expressed as



(2·155)

In terms of invariant functions we have from Eq. (2·155)

$$B(p^2)G(p^2)=0\,. \tag{2·156}$$

As will be shown in Eq. (2·169), the function $p^2G(p^2)$ is equal to the inverse propagator of the ghost and it cannot be identically zero. Hence we have the identity,

$$B(p^2)\equiv 0\,. \tag{2·157}$$

Because of this identity the propagator of the gauge boson without gauge fixing term contribution is

$$\tilde{D}^{\mu\nu}_{\alpha\beta}(p^2)=\delta^{\alpha\beta}A^{-1}(p^2)\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right), \tag{2·158}$$

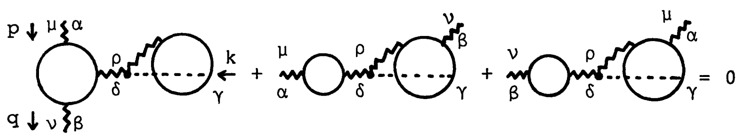
that is, transverse. The pole position of the propagator remains zero because of Eq. (2·157) and another identity,

$$A(0)=B(0)\,. \tag{2·159}$$

This identity (2·159) is nothing but the requirement that the one-particle irreducible graph (the inverse propagator $\tilde{\Gamma}^{\mu\nu}$) does not have any pole at least in the perturbation theory. Thus the gauge boson is massless in any order of the perturbation theory and the second order divergence does not appear in the gauge boson self-energy. We can make the self-energy finite only by adjusting the wave function renormalization constant of the gauge boson field.

b) We proceed to the three-point vertices of gauge bosons and ghosts. The differentiation of the WT identity (2·148) gives the following relations:

$$\frac{\delta^3}{\delta W_\mu{}^\alpha\delta W_\nu{}^\beta\delta c^\gamma}\,\text{Eq. (2·148)}\Big|_0 :$$



(2·160)

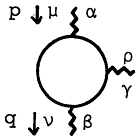
The invariant amplitude of the tree three-vertex of gauge bosons corresponds to $g^{\mu\nu}(p+q)^{\rho}f^{\alpha\beta\gamma}$ term. The relevant part of Eq. (2·160) is written as

$$\begin{aligned} &i\Gamma(P,k)\,g^{\mu\nu}P^{\rho}f^{\alpha\beta\delta}k_{\rho}\delta^{\delta\gamma}G(k) \\ &\quad+\delta^{\alpha\delta}A\left(\frac{P-k}{2}\right)g^{\mu\rho}\cdot g^{\rho\nu}H(P,k)\,f^{\delta\beta\gamma} \\ &\quad+\delta^{\beta\delta}A\left(\frac{P+k}{2}\right)g^{\nu\rho}\cdot g^{\rho\mu}H(-P,k)\,f^{\delta\alpha\gamma}=0\,, \end{aligned} \tag{2·161}$$

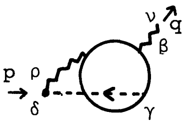
where

$$P=q+p,\quad k=q-p\,, \tag{2·162}$$

and we define the invariant functions Γ and H as follows:



$$\equiv ig^{\mu\nu}P^{\rho}f^{\alpha\beta\gamma}\Gamma(P,k)+[\text{other invariant amplitudes}], \tag{2·163}$$



$$\equiv g^{\rho\nu}f^{\delta\beta\gamma}H(P,k)+[\text{other invariant amplitudes}]. \tag{2·164}$$

With the operation of $\delta/\delta k_{\rho}|_{k_{\rho}=0,P^2=0}$ to Eq. (2·160) we have

$$iG(0)\,\Gamma(0,0)=A'(0)\,H(0), \tag{2·165}$$

where for simplicity we neglect the effect of the infrared divergences contained in the above functions. No form factor other than $g^{\mu\nu}P^{\rho}f^{\alpha\beta\gamma}$ term in (2·163) contributes to Eq. (2·165).

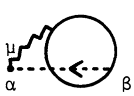
Next we use the second type of the WT identities. We adopt a gauge fixing Lagrangian,

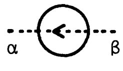
$$\mathcal{L}_{\mathrm{GF}}=\partial_{\mu}W_{\beta}{}^{\mu}B^{\beta}+\frac{\alpha}{2}B_{\beta}B^{\beta}\,, \tag{2·166}$$

which preserves the global symmetry. In this gauge the WT identity (2·119) reads

$$\partial^{\mu}\frac{\delta\widetilde{\Gamma}}{\delta K_{\alpha}{}^{\mu}}+\frac{\delta\widetilde{\Gamma}}{\delta\bar{c}^{\alpha}}=0\,. \tag{2·167}$$

By differentiating Eq. (2·167) with respect to c^{β}, we have

$-ip^{\mu}$ 

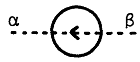
$+$ 

$=0$, (2·168)

that is,

$$p^2 G(p^2) = i\gamma(p^2), \tag{2·169}$$

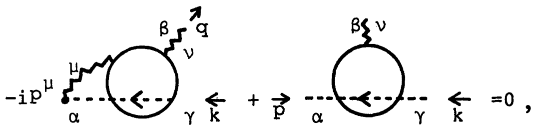
where we define the ghost inverse propagator as

$$\int d^4x e^{ip(x-y)} \frac{\delta^2 \Gamma}{\delta \bar{c}^\alpha(x)\delta c^\beta(y)} \equiv$$  $$\equiv -\delta^{\alpha\beta}\gamma(p^2). \tag{2·170}$$

We obtain a useful relation from Eq. (2·169),

$$G(0) = i\gamma'(0). \tag{2·171}$$

Further differentiation $\delta^2/\delta W_\nu^\beta \delta c^\gamma$ of Eq. (2·167) gives

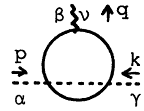


(2·172)

that is,

$$H(P, k) = -\Gamma_c(P, k), \tag{2·173}$$

where Γ_c is defined as



$$= -ip^\nu f^{\alpha\beta\gamma}\Gamma_c(P, k) + [\text{other invariant amplitudes}]. \tag{2·174}$$

Consequently we obtain the identity,

$$\frac{\Gamma(0,0)}{A'(0)} = \frac{-iH(0,0)}{G(0)} = \frac{\Gamma_c(0,0)}{\gamma'(0)}, \tag{2·175}$$

where we have used the relations (2·165), (2·171) and (2·173).

The corresponding renormalized functions are the coefficient functions of the renormalized field variables of the effective action:

$$\Gamma_R \equiv Z_3^{3/2}\Gamma, \tag{2·176a}$$

$$A_R \equiv Z_3 A, \tag{2·176b}$$

$$\Gamma_{cR} \equiv Z_3^{1/2} Z_c \Gamma_c, \tag{2·176c}$$

$$\gamma_R \equiv Z_c \gamma, \tag{2·176d}$$

where Z factors are defined as follows:

$$W^\mu \equiv Z_3^{1/2} W_R{}^\mu, \tag{2·177a}$$

$$c \equiv Z_c^{1/2} c_R, \tag{2·177b}$$

$$\bar{c} \equiv Z_c^{1/2} \bar{c}_{\mathrm{R}} . \tag{2·177c}$$

In terms of these renormalized amplitudes the identity (2·175) is rewritten as

$$\frac{\Gamma_{\mathrm{R}}(0,0)}{A_{\mathrm{R}}'(0)} = \frac{\Gamma_{c\mathrm{R}}(0,0)}{\gamma_{\mathrm{R}}'(0)} \left(= \frac{-iH(0,0)}{G(0)} Z_3^{1/2} \right). \tag{2·178}$$

This relation represents that if we can make Γ_{R} finite, which is possible, of course, by adjusting the charge (g) renormalization constant, we obtain the finite $\Gamma_{c\mathrm{R}}$ automatically. In other words, the counter term to makc Γ_{R} finite also makes $\Gamma_{c\mathrm{R}}$ finite. This property is an ingredient of the renormalizability of gauge theories.

Here it should be noted that we have not neglected any finite term in deriving the relation (2·178). Taking the on-shell renormalization conditions (see § 3. 2),

$$A'(0) = \gamma'(0) = -1 , \tag{2·179}$$

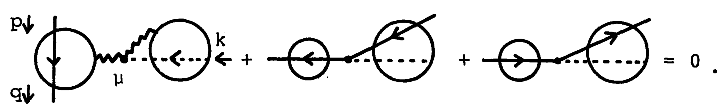
the equality of the on-shell charge (the charge universality) between the cubic self coupling of gauge bosons and the gauge-ghost coupling is proved:

$$\Gamma_{\mathrm{R}}(0,0) = \Gamma_{c\mathrm{R}}(0,0) \left(= \frac{iH(0,0)}{G(0)} Z_3^{1/2} \right). \tag{2·180}$$

This is also called the Slavnov-Taylor identity [Slavnov 72 and Taylor 71]. This relation is not a direct result of the renormalizability of the theory, since it shows an equality including all finite contributions. In this sense the renormalization equation (2·134) does not assure the charge universality, which is no more than closed relations among infinite parts.

c) Next we investigate gauge couplings of fermions. By a similar differentiation of the WT identity (2·148), we obtain

$$\frac{\delta^a}{\delta\bar{\psi}_i \delta\psi_j \delta c^a} \text{Eq. (2·148)}\Big|_0 :$$

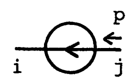


(2·181)

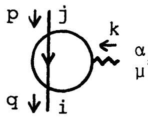
We rewrite it as follows:

$$\Gamma_F{}^{\mu}(P,k) \cdot k_{\mu} G(k^2) = K\left(\frac{P+k}{2}\right) H_F(P,k) + \bar{H}_F(P,k) K\left(\frac{P-k}{2}\right), \tag{2·182}$$

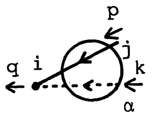
where we have used invariant functions defined by



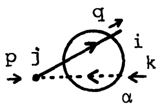
$\equiv \delta^{ij} K(p)$, (inverse propagator) (2·183)



$$\equiv T^{\alpha}_{ij}\Gamma_F{}^{\mu}(P, k), \tag{2·184}$$



$$\equiv T^{\alpha}_{ij}H_F(P, k), \tag{2·185a}$$



$$\equiv T^{\alpha}_{ij}\overline{H}_F(P, k). \tag{2·185b}$$

Here T^{α}_{ij} is generator matrices of the representation of the fermion. In the above definitions we have supposed that different multiplets of fermions do not mix with each other for simplicity of the argument. It should be noted that K and $\overset{(-)}{H}_F$ are matrices with spinor indices and are not commutable with each other. We define renormalized quantities as follows:

$$\psi = Z_F{}^{1/2}\psi_{\mathrm{R}}, \quad \overline{\psi} = Z_F{}^{1/2}\overline{\psi}_{\mathrm{R}}, \tag{2·186}$$

$$K_{\mathrm{R}} = Z_F K, \tag{2·187}$$

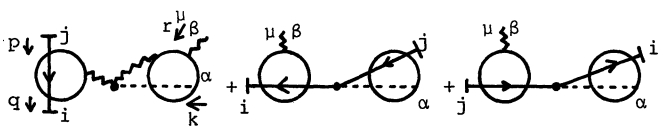
$$\Gamma^{\mu}_{F\mathrm{R}} = Z_3{}^{1/2} Z_F \Gamma_F{}^{\mu}. \tag{2·188}$$

By analysing the structure of K and $\overset{(-)}{H}_F$ and using the on-shell renormalization conditions for K_{R} (see § 3. 2. 2), we obtain the final result,

$$\Gamma^{\mu}_{F\mathrm{R}}(P^2 = 4m_F{}^2, k=0) = -\gamma^{\mu}\overline{H}_F(P^2 = 4m_F{}^2, k=0)\, Z_3{}^{1/2}/G(0). \tag{2·189}$$

We differentiate furthermore the WT identity (2·148) in order to deduce information on $\overline{H}_F$:

$$\left.\frac{\delta^4}{\delta\overline{\psi}_i\delta\psi_j\delta c^{\alpha}\delta W_{\mu}{}^{\beta}}\mathrm{Eq.}\ (2\cdot 148)\right|_0 :$$



$$= 0\ , \tag{2·190}$$

where momenta k and r are set equal to zero and all amplitudes are sandwitched by on-shell spinors, which are represented by the symbol ⊣. Taking corresponding form factors, we obtain

$$T^{\gamma}_{ij}f^{\gamma\beta\alpha}\gamma^{\nu}\cdot g^{\nu\mu}H + T^{\beta}_{ik}\gamma^{\mu}\cdot T^{\alpha}_{kj}H_F + T^{\alpha}_{ik}\overline{H}_F\cdot T^{\beta}_{kj}\gamma^{\mu} = 0\,. \tag{2·191}$$

Equation (2·191) is reduced to

$$H(0, 0) = i\overline{H}_F(P^2 = 4m_F{}^2, k=0), \tag{2·192}$$

where use has been made of commutators of generator matrices:

$$[T^{\alpha}, T^{\beta}] = if^{\alpha\beta\gamma}T^{\gamma}\,. \qquad (2\cdot193)$$

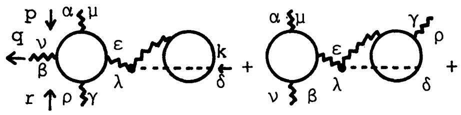
From Eqs. (2·178), (2·189) and (2·192), we have

$$\Gamma^{\mu}_{F\mathrm{R}}(P^2=4m_F^2,\, k=0) = \gamma^{\mu}\frac{iH(0,0)}{G(0)}Z_3^{1/2} = \gamma^{\mu}\Gamma_{\mathrm{R}}(0,0) = \gamma^{\mu}\Gamma_{c\mathrm{R}}(0,0)\,. \qquad (2\cdot194)$$

This identity assures the renormalizability for gauge couplings of fermions and the on-shell charge universality among gauge couplings of fermions, of ghosts and of gauge bosons themselves.

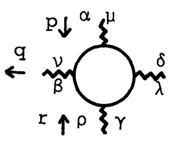
d) Finally, we study the four-point function of gauge bosons. We differentiate the WT identity (2·148):

$$\frac{\delta^4}{\delta W_{\mu}{}^{\alpha}\delta W_{\nu}{}^{\beta}\delta W_{\rho}{}^{\gamma}\delta c^{\delta}}\ \text{Eq. (2·148)}\Big|_0 :$$



+ [permutation] =0 , (2·195)

where all particles are on their mass shell. We define the invariant amplitudes of the four point function:



$$\equiv g^{\mu\rho}g^{\nu\lambda}f^{\alpha\beta\omega}f^{\gamma\delta\omega}\Gamma_4(p,q,r) + [\text{other invariant amplitudes}]. \qquad (2\cdot196)$$

By extracting $f^{\alpha\beta\omega}f^{\gamma\delta\omega}g^{\mu\rho}(p+q)^{\nu}$ term in Eq. (2·195), we obtain

$$\Gamma_4(0,0,0)G(0) = i\Gamma(0,0)H(0,0)\,. \qquad (2\cdot197)$$

With the aid of Eq. (2·165) and the definition of $\Gamma_{4\mathrm{R}}$,

$$\Gamma_{4\mathrm{R}} = Z_3^2\Gamma_4\,, \qquad (2\cdot198)$$

we have the identity,

$$\Gamma_{4\mathrm{R}}(0,0,0) = -\{\Gamma_{\mathrm{R}}(0,0)\}^2/A_{\mathrm{R}}'(0)\,. \qquad (2\cdot199)$$

Thus $\Gamma_{4\mathrm{R}}(0,0,0)$ is renormalized to be g_{R}^2 under the on-shell renormalization conditions: $\Gamma_{\mathrm{R}}(0,0)=g_{\mathrm{R}}$ and $A_{\mathrm{R}}'(0)=-1$ (Eq. (2·179)).

We have shown how the identities assuring the renormalizability are obtained from the WT identities. Those identities express the on-shell charge universality as a by-product in a gauge theory without spontaneous symmetry breakdown. Two properties of the theory, the renormalizability and the charge universality, are essentially independent. Actually in a symmetry broken theory the charge universality is broken though the renormalizability holds. The difference between these properties is in the fact that the renormalizability is related to the infinite part of amplitudes, while the charge universality is the property of full amplitudes with all finite terms (see also § 3.4).

Chapter 3

On-Shell Renormalization in Electroweak Theory. I

We start with a general discussion on renormalization procedure and, at the same time, explain problems associated with choice of renormalization schemes. We then restrict ourselves to the on-shell renormalization scheme in the Weinberg-Salam theory. A detailed discussion will be given on the derivation of on-shell conditions. Special attention will be paid to two-point functions which exhibit complicated structure. Finally we give a proof of the universality of the electric charge by the full use of our on-shell scheme.

§ 3.1 Renormalization schemes

We would like to present a general procedure in renormalization calculations. In this process several problems arise: There are some ambiguities in defining renormalized parameters in the theory and also in giving numerical values to these parameters from the knowledge of experimental data.

Suppose that we calculate physical quantities, F, in a renormalizable field theory which includes a bare parameter g_0 (coupling constant) among others. Physical quantities, F, expressed in terms of g_0 are divergent in general and the renormalization is necessary.*) The definition of the renormalized coupling constant g, however, is not unique. In order to give an explicit expression for F in terms of g, we must fix the scheme. The numerical values of F are predicted by determining the value of g. To give the value of g we choose some reliable experimental data.

If the theory can be solved exactly, this apparent ambiguity in defining g does not affect physical predictions. We have, however, only perturbative method at hand and we have to truncate the series at some order. In this case the physical predictions, in general, may be affected by the ambiguities in g (e.g., "scheme dependence problem" in the perturbative QCD). Moreover there appears another ambiguity in determining the numerical value of g from the experimental data. For example, suppose that we have two independent physical quantities, $\sigma(g)$ and $\Gamma(g)$, and experimental data on σ and Γ are given. The value of g determined perturbatively by using the data on σ may be different from that for Γ by the amount which is interpreted as higher order effect.

Let us now specialize to the Weinberg-Salam theory in which several

*) We here mention only the ultraviolet (UV) divergences. Infrared divergences will be discussed in practical calculation (Chapter 5).

independent parameters appear. Different choices of set of independent parameters give rise to the apparent difference in expressions of the Lagrangian. For example, several authors use the set of parameters,*) g ($SU(2)$ coupling), $\sin\theta_W$ and M_W. It is also possible to choose the set e, M_W and M_Z, which is our choice in this paper. (Accordingly, the Feynman rules are expressed by e, M_W and M_Z as shown in § 4.2.)

With a set of parameters suitably chosen, one can calculate a relevant S-matrix element as a function of these parameters. To eliminate ultraviolet divergences which may appear in the course of the calculation, one applies an appropriate renormalization scheme. As stated before, the choice of the renormalization scheme is rather arbitrary. The so-called MS ['t Hooft 73], $\overline{\text{MS}}$ [Bardeen et al. 78] and MOM [Celmaster and Gonsalves 79] schemes in quantum chromodynamics are some of the typical examples. If we know an exact solution of the theory, different choices of the schemes do not cause any difference in the resulting S-matrix element although its expression as a function of the renormalized parameters may differ. In truncated perturbation theories, however, different choices of the schemes can lead to different physical predictions.

In the Weinberg-Salam theory it is often a standard manner to use, as in QED, the fine structure constant $\alpha=e^2/4\pi$ defined in the Thomson limit as an expansion parameter. This means that we choose a specific scheme to define α, i.e., the on-shell scheme. As we have more parameters other than α, i.e., M_W and M_Z, we must specify the prescription to split bare parameters M_{W0} and M_{Z0} into renormalized parts and counter terms, $M_{W0}^2=M_W{}^2+\delta M_W{}^2$ and $M_{Z0}^2=M_Z{}^2+\delta M_Z{}^2$. We adopt the scheme which is straightforward generalization of the one commonly used in QED: We determine, for example, counter terms $\delta M_W{}^2$ and Z_W (W field renormalization constant) so that the transverse part of the renormalized W boson self-energy, $A_R{}^W(q^2)$, behaves as

$$A_R{}^W(q^2) \quad \text{and} \quad \frac{d}{dq^2}A_R{}^W(q^2)\to 0 \quad \text{as} \quad q^2\to M_W{}^2\,. \tag{3·1}$$

With this condition, the renormalized mass M_W is identical to the pole position of the W propagator, i.e., the physical mass. For this reason our scheme is called the on-shell scheme.

The S-matrix element is now obtained perturbatively as a function of renormalized parameters e, M_W and M_Z in a specific renormalization scheme (the on-shell scheme in our case). To obtain the physical prediction on the S-matrix element, we must determine the values of these parameters with the help of given experimental data. The values of e, M_W and M_Z can be de-

*) The WS theory with one Higgs doublet includes five kinds of independent parameters. As two parameters other than those shown here, m_f (fermion mass) and m_ϕ (Higgs mass) are usually used.

termined if we have three pieces of independent experimental information. (The value of e is actually determined in the Thomson limit without recourse to the perturbative argument. Thus the value of e is exact and this scheme is a preferred scheme in the W-S theory as well as in QED.) It is this stage where an ambiguity in predicting radiative corrections to the S-matrix element creeps in as far as we use the perturbation theory. Of course this ambiguity does not exist if we know an exact solution. As a set of experimental data, α^{exp}, Γ^{exp} and R^{exp} are often utilized, where Γ is the total width for the decay $\mu\to e\bar{\nu}\nu$ and R is a ratio of neutral current cross section to charged current one or a ratio of cross sections of neutrino and antineutrino processes (e.g., $R=\sigma(\bar{\nu}e\to\bar{\nu}e)/\sigma(\nu e\to\nu e)$).*) Assume that we perform the calculation to n-loop order and determine the values of M_W and M_Z using the data Γ^{exp} and R^{exp}. We denote those values by $M_W{}^{(n)}$ and $M_Z{}^{(n)}$, and Γ and R calculated to n-loop order by $\Gamma^{(n)}$ and $R^{(n)}$ respectively. We have

$$\Gamma^{(n)}(M_W{}^{(n)}, M_Z{}^{(n)}) = \Gamma^{\mathrm{exp}}\,, \tag{3·2}$$

$$R^{(n)}(M_W{}^{(n)}, M_Z{}^{(n)}) = R^{\mathrm{exp}}\,. \tag{3·3}$$

If the masses of W and Z bosons will be measured directly in the near future, we may also choose the set of parameters $M_W{}^{\mathrm{exp}}$ and $M_Z{}^{\mathrm{exp}}$ as input data instead of Γ^{exp} and R^{exp} (another obvious parameter α is neglected), and substitute $M_W{}^{\mathrm{exp}}$ and $M_Z{}^{\mathrm{exp}}$ directly into the renormalized parameters M_W and M_Z. In our previous papers [Aoki et al. 80 and 81], we have taken the latter set**) and given the numerical results. In this article we will consider both sets and give corresponding results. Another set we analyze in this article is the one e^{exp}, Γ^{exp} and M_Z^{exp}. This set is interesting in the sense that the high accuracy of the mass determination of Z boson is expected in coming experiments.

We have given a brief summary concerning the renormalization schemes. We show it schematically in Table 3.1.

Finally we wish to stress that our scheme in which parameters α, M_W and M_Z are used and the subtraction of UV divergence is made on mass shell, is a natural extension of the renormalization in QED, and is very convenient and physical since the parameters $M_W{}^2$ and $M_Z{}^2$ directly correspond to the pole positions of gauge boson propagators, that is, the physical masses. (For example, if one chooses another scheme with the Weinberg angle, θ_{W}, as one of the parameters, there appears an ambiguity in defining the angle θ_{W} since it is not the physical quantity. This point will be discussed in § 7.1.)

*) We have put the suffix "exp" to α, Γ and R to stress that they express experimental values.

**) Since W and Z bosons are not yet observed, we have assumed suitable values for $M_W{}^{\mathrm{exp}}$ and $M_Z{}^{\mathrm{exp}}$.

Table 3.1. Flow chart for renormalization procedure.

Choose a set of independent parameters
(Present case; e_0, M_{W0}, M_{Z0}, m_{f0}, $m_{\phi 0}$)

⇩

Separate the bare parameters and fields into renormalized parts and counter terms.
(Introduce renormalization constants.)[a]
$e_0=Ye$, $M_{W0}^2=M_W^2+\delta M_W^2$, $M_{Z0}^2=M_Z^2+\delta M_Z^2$, $\cdots$
$\psi_0=Z_\psi^{1/2}\psi$, $W_{0\mu}=Z_W^{1/2}W_\mu$, $\cdots$.

⇩

Choose a subtraction scheme to fix the counter terms.

⇩

Calculate relevant amplitudes with a suitable regularization of UV divergences.

⇩

Eliminate all divergences by the above counter terms and subtraction conditions.

⇩

We obtain the finite S-matrix element as a function of the renormalized parameters, e, M_W, M_Z, m_f and m_ϕ. But numerical values of the renormalized parameters are not yet fixed.

⇩

Choose input data in order to fix the values of the renormalized parameters.[b]
(Present cases; 1) α, M_W, M_Z, m_f, m_ϕ, 2) α, Γ, R, m_f, m_ϕ, 3) α, Γ, M_Z, m_f, m_ϕ)

⇩

We can predict for various physical quantities as functions of the input data.

a) The full list of counter terms is given in §4.3.
b) $\Gamma\equiv\Gamma(\mu\to e\nu\bar{\nu})$, $R\equiv\sigma(\bar{\nu}_\mu e\to\bar{\nu}_\mu e)/\sigma(\nu_\mu e\to\nu_\mu e)$.

§ 3.2 On-shell renormalization condition

In this section we present a general discussion about the on-shell renormalization conditions without special recourse to the Weinberg-Salam theory. We start with the case of scalar fields. We recapitulate the reduction formula for the S-matrix in order to clarify the relation between the S-matrix element and the corresponding Green function. The on-shell renormalization conditions are defined and their explicit forms are given in the case with particle mixing. We proceed to the case of Dirac fields where non-commutativity of propagators should be taken into account. Finally the case of vector bosons is studied, where the mixing with scalars including Nambu-Goldstone bosons makes a little complication.

3.2.1 *Scalar fields*

First we would like to recapitulate the framework of dealing with the S-matrix for scalar fields in the case without mixing. The S-matrix element is given by, in the Heisenberg picture,

$$S = \langle q_1, \cdots, q_m : \text{out} | k_1, \cdots, k_n : \text{in} \rangle, \tag{3·4}$$

where $|\text{in}\rangle$ and $|\text{out}\rangle$ are in and out asymptotic states respectively which are eigenstates of the total Hamiltonian and of other conserved charges including total momentum. These asymptotic states are constructed by repeated applications of creation operators $a^{\dagger}_{\substack{\text{in}\\\text{out}}}(k_i)$ to the vacuum $|0\rangle$. These creation operators form corresponding asymptotic fields:

$$\phi_{\substack{\text{in}\\\text{out}}}(x) = \int d^3k [a_{\substack{\text{in}\\\text{out}}}(k) f_k(x) + a^{\dagger}_{\substack{\text{in}\\\text{out}}}(k) f_k{}^*(x)] \, , \tag{3·5}$$

where $f_k(x)$ are normalized solutions of the Klein-Gordon equation. The asymptotic field satisfies a free field equation:

$$(\Box + m^2) \phi_{\text{as}}(x) = 0 \, , \tag{3·6}$$

where ϕ_{as} represents ϕ_{in} or ϕ_{out}. The Feynman propagator for ϕ_{as} reads

$$\begin{aligned} \Delta_{\text{as}}(p^2) &\equiv i \int \langle 0 | \mathrm{T} \phi_{\text{as}}(x) \phi_{\text{as}}(y) | 0 \rangle d^4(x-y) \, e^{-ip(x-y)} \\ &\equiv \mathrm{FT} \; i \langle 0 | \mathrm{T} \phi_{\text{as}}(x) \phi_{\text{as}}(y) | 0 \rangle \\ &= \frac{1}{m^2 - p^2 - i\varepsilon} \, , \end{aligned} \tag{3·7}$$

where FT denotes the Fourier transformation defined above.

This asymptotic field is related to the Heisenberg field as

$$\phi(x) \to \tilde{Z}^{1/2} \phi_{\substack{\text{in}\\\text{out}}}(x) \, , \quad t \to \mp \infty \, . \tag{3·8}$$

It should be noted that the limit in Eq. (3·8) is the so-called weak limit. This relation is the well-known LSZ asymptotic condition [Lehmann et al. 55]. The factor $\tilde{Z}^{1/2}$ appearing in Eq. (3·8) is interpreted as an amplitude that $\phi(x)$ creates the one-particle state out of the vacuum:

$$\begin{aligned} \langle p : \text{as} | \phi(x) | 0 \rangle &= \tilde{Z}^{1/2} \langle p : \text{as} | \phi_{\text{as}}(x) | 0 \rangle \\ &= \tilde{Z}^{1/2} f_k{}^*(x) \, . \end{aligned} \tag{3·9}$$

Using this relation, the spectral representation of ϕ propagator [Umezawa and Kamefuchi 51, Källén 52, Lehmann 54],

$$\Delta(p^2) = \int_0^\infty d\sigma \rho(\sigma) \frac{1}{\sigma^2 - p^2 - i\varepsilon}\,, \tag{3·10}$$

is separated into one-particle and continuous-spectrum contributions:

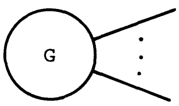
$$\Delta(p^2) = \frac{\tilde{Z}}{m^2 - p^2 - i\varepsilon} + \int_{\text{th}}^\infty d\sigma \rho(\sigma) \frac{1}{\sigma^2 - p^2 - i\varepsilon}\,. \tag{3·11}$$

Note that the propagator $\Delta(p^2)$ is defined analogously by Eq. (3·7) if one replaces ϕ_{as} by ϕ. Equation (3·11) shows that the factor $\tilde{Z}$ defined in the asymptotic condition (3·8) is calculated as a pole residue of the full propagator.

With the aid of the asymptotic condition the S-matrix element defined by Eq. (3·4) is related to the Green function:

$$\begin{aligned} &\langle q_1 \cdots q_m : \text{out} | k_1 \cdots k_n : \text{in} \rangle \\ &= \frac{i^{m+n}}{(\tilde{Z}^{1/2})^{m+n}} \int \prod_{i=1}^n d^4x_i \prod_{j=1}^m d^4y_j \cdot f_{k_i}(x_i) (\overrightarrow{\Box}_{x_i} + m^2) \\ &\quad \times \langle 0 | \mathrm{T} \phi(y_1) \cdots \phi(y_m) \phi(x_1) \cdots \phi(x_n) | 0 \rangle (\overleftarrow{\Box}_{y_j} + m^2) f^*_{q_j}(y_j)\,. \end{aligned} \tag{3·12}$$

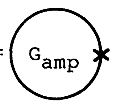
This is called the reduction formula [Lehmann et al. 55]. Schematically the S-matrix element (3·12) is represented by



$$\left. \begin{matrix} (m^2 - k_1^2) \dfrac{1}{\tilde{Z}^{1/2}} \\ \vdots \\ (m^2 - q_m^2) \dfrac{1}{\tilde{Z}^{1/2}} \end{matrix} \right|_{k_1^2, \cdots, q_m^2 = m^2}\,, \tag{3·13}$$

where G is the corresponding Green function in the momentum representation.

Taking account of the expression of the propagator (3·11), we notice that each external-line factor $(m^2 - k_i^2)\tilde{Z}^{-1/2}$ in (3·13) picks out the pole part of the propagator leaving the factor $\tilde{Z}^{1/2}$. Consequently the reduction formula states that the S-matrix element is just the corresponding Green function with the external legs amputated where the external momenta are put on the mass shell and the factors $\tilde{Z}^{1/2}$ are multiplied on each leg:

$S =$  $\tilde{Z}^{1/2}$, (3·14)

where the cross mark stands generally for all amputated legs.

Now we discuss the renormalization conditions. The renormalization conditions are necessary to calculate Green functions, although they have no effect on the representation of the S-matrix itself which is expressed only by Heisenberg fields and well-defined $\tilde{Z}^{1/2}$ factor. In order to follow the renormalization procedure, we introduce a wave-function renormalization constant

$Z^{1/2}$ which is determined by a suitable renormalization condition,

$$\phi(x) \equiv Z^{1/2} \phi_{\text{ren}}(x) . \tag{3·15}$$

It should be noted that Eq. (3·15) is an operator equation and the factor $Z^{1/2}$ has nothing to do with $\tilde{Z}^{1/2}$ introduced before. One can, however, choose the renormalization condition

$$Z^{1/2} = \tilde{Z}^{1/2} . \tag{3·16}$$

With this choice of renormalization scheme the $\tilde{Z}^{1/2}$ factors in Eq. (3·14) can be regarded as usual counter terms for renormalized Green functions. Such a renormalization condition that ensures the equality of $Z^{1/2}$ and $\tilde{Z}^{1/2}$ is called the on-shell renormalization condition. This scheme is simple and convenient noticing that we need $\tilde{Z}^{1/2}$ to obtain S-matrix elements.

Next we discuss the problem of mass renormalization. Although the mass renormalization has no direct effect on the relation between the S-matrix and Green function, the mass shift in Green functions should be taken into account in perturbative calculation. The full propagator takes the form

$$\Delta(p^2) = \frac{1}{m^2 - p^2 - \Pi(p^2)} ,$$

where m is the renormalized mass which is taken to be equal to the physical mass. The proper self-energy part is denoted by $\Pi(p^2)$ in which mass counter term δm^2 is included. Referring to the reduction formula (3·12), we adjust δm^2 so that the pole position of the propagator coincides with the mass squared m^2 loop by loop,

$$\Delta^{-1}(m^2) = 0 . \tag{3·17}$$

By the on-shell condition we mean that, in addition to Eq. (3·16), the condition (3·17) is implicitly imposed.

We define the renormalized propagator by

$$\Delta_{\text{ren}} \equiv \text{FT}\, i\langle 0|\text{T}\phi_{\text{ren}}(x)\phi_{\text{ren}}(y)|0\rangle = \frac{1}{Z}\Delta .$$

The pole part of Δ_{ren} is given by

$$\Delta_{\text{ren}}[\text{pole}] = \frac{1}{Z}\Delta[\text{pole}] = \frac{\tilde{Z}}{Z}\frac{1}{m^2 - p^2} = \frac{\tilde{Z}}{Z}\Delta_{\text{as}} , \tag{3·18}$$

where the mass renormalization has already been taken care of. By requiring Eq. (3·16), we get an equality between $\Delta_{\text{ren}}[\text{pole}]$ with the on-shell condition and Δ_{as}. Now we write down the on-shell renormalization conditions explicitly in terms of the inverse propagator,

$$\Delta_{\text{ren}}^{-1}(m^2) = 0 , \qquad \Delta_{\text{ren}}^{-1\prime}(m^2) = -1 , \tag{3·19}$$

or in terms of the proper self-energy part,

$$\Pi_{\rm ren}(m^2)=0\,, \qquad \Pi'_{\rm ren}(m^2)=0\,. \tag{3·20}$$

We proceed to the case with particle mixing. In this case all particles other than the lightest one are unstable and can decay into the lightest particle with emission of photons. Rigorously speaking, such unstable particles have no corresponding asymptotic field, and their propagators have poles at the position $m_j^2-i\Gamma_j$, where Γ_j is the total width. In the following, however, we adopt the approximation in which we neglect the finite width of particles and consider only the real part of the propagators.

Let us take the sector with N particles. Their asymptotic fields (in the sense of the above approximation) are related to Heisenberg fields with the matrix $\tilde{Z}^{1/2}$ factor:

$$\phi_i(x)\longrightarrow \tilde{Z}_{ij}^{1/2}\,\phi_j{}^{\rm as}(x)\,, \tag{3·21a}$$

$$\tilde{Z}_{ij}^{-1/2}\,\phi_j(x)\longrightarrow \phi_i{}^{\rm as}(x)\,. \tag{3·21b}$$

The propagator of this sector,

$$\Delta_{ij}\equiv i\,{\rm FT}\langle 0|{\rm T}\,\phi_i(x)\phi_j(y)\,|0\rangle\,, \tag{3·22}$$

has N poles, positions of which are m_i^2 $(i=1\cdots N)$, the mass of $\phi_i{}^{\rm as}$. The pole part of the propagator is estimated by using Eq. (3·21):

$$\begin{aligned}\Delta_{ij}[\text{pole}]&=i\,{\rm FT}\cdot\tilde{Z}_{im}^{1/2}\tilde{Z}_{jn}^{1/2}\langle 0|{\rm T}\phi_m{}^{\rm as}(x)\,\phi_n{}^{\rm as}(y)\,|0\rangle\\ &=\tilde{Z}_{im}^{1/2}\tilde{Z}_{jn}^{1/2}\frac{\delta_{mn}}{m_n^2-p^2}\,,\end{aligned} \tag{3·23}$$

which shows the relation between $\tilde{Z}_{ij}$ and the pole structure of the propagator Δ_{ij}.

The S-matrix element with external line i, S_i, is related to the Green function G in such a way as

$$S_i=\;G_{\rm amp}\;-\!\!\oslash\!\!\stackrel{j}{-\!\!\!-\!\!\bullet}\;(m_i^2-p^2)\,\tilde{Z}_{ij}^{-1/2}\,, \tag{3·24}$$

where by suffix amp we mean that the Green function G is amputated. Substituting Eq. (3·23) for the propagator into Eq. (3·24), we obtain

$$\begin{aligned}S_i&=G_{\rm amp}\,{*}^{k}\;\tilde{Z}_{km}^{1/2}\,\tilde{Z}_{jn}^{1/2}\frac{\delta_{mn}}{m_n^2-p^2}(m_i^2-p^2)\,\tilde{Z}_{ij}^{-1/2}\\ &=G_{\rm amp}\,{*}^{k}\;\tilde{Z}_{ki}^{1/2}\,.\end{aligned} \tag{3·25}$$

The wave function renormalization constants are introduced in the matrix form:

$$\phi_i \equiv Z_{ij} \phi_j{}^{\text{ren}} \,. \tag{3·26}$$

The renormalized propagator is defined by

$$\varDelta_{ij}^{\text{ren}} \equiv i \, \text{FT} \langle 0 | \text{T} \, \phi_i{}^{\text{ren}}(x) \phi_j{}^{\text{ren}}(y) \, | 0 \rangle$$

and its pole part reads

$$\begin{aligned} \varDelta_{ij}^{\text{ren}}[\text{pole}] &= Z_{im}^{1/2} Z_{jn}^{1/2} \varDelta_{mn}[\text{pole}] \\ &= Z_{im}^{1/2} Z_{jn}^{1/2} \tilde{Z}_{mk}^{1/2} \tilde{Z}_{nl}^{1/2} \varDelta_{kl}^{\text{as}} \,, \end{aligned} \tag{3·27}$$

where mass renormalization has already been taken into account. Renormalization conditions in which $Z_{ij}^{1/2} = \tilde{Z}_{ij}^{1/2}$ are given by

$$\varDelta_{ij}^{\text{ren}}[\text{pole}] = \frac{\delta_{ij}}{m_i^2 - p^2} \,. \tag{3·28}$$

These are the on-shell renormalization conditions with mixing.

We transform these conditions to those for the inverse propagator. In the region of p^2 very close to m_n^2 the renormalized propagator has a pole in (n, n) component and other components are regular $(O(1))$:

$$\varDelta_{ij}^{\text{ren}} \Big|_{p^2 \to m_n^2} = \begin{array}{c c} & n \\ n & \begin{pmatrix} & \vdots & \\ \cdots & 1/\varepsilon & \cdots \\ & \vdots & \end{pmatrix} \end{array} , \tag{3·29}$$

$$\varepsilon = m_n^2 - p^2 \,. \tag{3·30}$$

The inverse propagator is obtained by inverting the matrix of Eq. (3·29):

$$\varDelta_{ij}^{\text{ren}-1} = (\det \varDelta^{\text{ren}})^{-1} A_{ij} \,, \tag{3·31}$$

where $\det \varDelta^{\text{ren}}$ is the determinant of the matrix $\varDelta_{ij}^{\text{ren}}$ and A_{ij} is the (i, j) cofactor of $\varDelta_{ij}^{\text{ren}}$. The determinant of $\varDelta^{\text{ren}}$ is given by

$$\det \varDelta^{\text{ren}} = \frac{1}{\varepsilon} A_{nn} + O(1) \,, \tag{3·32}$$

and therefore,

$$(\det \varDelta^{\text{ren}})^{-1} = A_{nn}^{-1} \cdot \varepsilon + O(\varepsilon^2) \,. \tag{3·33}$$

On the other hand, the cofactors are of $O(1)$ for $i = n$ or $j = n$ while they are of $O(1/\varepsilon)$ for $i \neq n$ and $j \neq n$.

Hence we obtain

$$\Delta_{ij}^{\text{ren}-1}\Big|_{p^2\to m_n{}^2}=\left(\begin{array}{c|c|c} O(1) & O(\varepsilon) & O(1) \\ \hline \cdots O(\varepsilon)\cdots & \varepsilon & \cdots O(\varepsilon)\cdots \\ \hline O(1) & O(\varepsilon) & O(1) \end{array}\right). \tag{3·34}$$

In other words,

$$\Delta_{ij}^{\text{ren}-1}\Big|_{p^2\to m_n{}^2}=\begin{cases}\varepsilon & \text{for } i=j=n\ , \\ O(\varepsilon) & \text{for } i=n,\ j\neq n \text{ and } i\neq n,\ j=n\ , \\ O(1) & \text{otherwise}\ .\end{cases} \tag{3·35}$$

It is now obvious from Eq. (3·35) that our on-shell renormalization conditions for $\Delta_{ij}^{\text{ren}-1}$ read for all n $(1\cdots N)$:

i) $i=j=n$,

$$\Delta_{nn}^{\text{ren}-1}(m_n{}^2)=0\ , \tag{3·36a}$$

$$\frac{\partial}{\partial p^2}\Delta_{nn}^{\text{ren}-1}(m_n{}^2)=-1\ , \tag{3·36b}$$

ii) $i=n,\ j\neq n$,

$$\Delta_{nj}^{\text{ren}-1}(m_n{}^2)=0\ , \tag{3·37}$$

iii) $i\neq n,\ j=n$,

$$\Delta_{in}^{\text{ren}-1}(m_n{}^2)=0\ . \tag{3·38}$$

Note that we need not impose the condition on $\Delta_{ij}^{\text{ren}-1}$ for $i\neq n$ and $j\neq n$. These conditions ensure the equality $Z_{ij}^{1/2}=\tilde{Z}_{ij}^{1/2}$ and the external $\tilde{Z}_{ki}^{1/2}$ factor in Eq. (3·25) is interpreted as the counter term generated by the replacement of Eq. (3·26) in the Lagrangian.

In the following we argue that the number of renormalization conditions (3·36)$\sim$(3·38) is equal to that of independent renormalization constants by which the renormalized propagator is determined. Not all renormalization conditions are independent of each other. In fact only the conditions on the (i, j) components $(i\geq j)$ are independent since Δ_{ij}^{ren} is symmetric by exchange of i and j. First of all, the condition (3·36a) is satisfied by adjusting the mass counter terms $\delta m_i{}^2$. There remain N conditions (3·36b) and two conditions for all $i>j$: (3·37) and (3·38). The number of the conditions in total amounts to N^2 $(=N+2{}_NC_2)$. These N^2 conditions can be satisfied by the N^2 degrees of freedom of $Z_{ij}^{1/2}$. It should be noticed that the matrix $Z_{ij}^{1/2}$ is not orthogonal but a general real matrix, that is, all N^2 components of it are independent of each other.

Finally, we remark that the above property of $Z_{ij}^{1/2}$ means that the asymptotic fields or states are not related to Heisenberg fields through a general rotation in $O(N)$ space. This fact is one of the most important points to understand and construct the on-shell renormalization scheme. This point is related to the ambiguity in defining $\sin^2\theta_W$ including higher order terms in the Weinberg-Salam theory. There have been many confusions about this point in discussing the renormalization scheme of the Weinberg-Salam theory, which will be discussed later.

3.2.2 *Dirac fields*

For particles with spin 1/2 the derivation of the on-shell renormalization conditions is essentially the same as that for scalar particles except the fact that the propagators have spin indices and do not commute with each other.

Consider the Weyl fields corresponding to irreducible representations of the Lorentz group,

$$\psi_{L,i}(x), \qquad \psi_{R,j}(x)\ ,$$

where we denote the chirality of the Weyl fields by the sufficies L and R. In general the mixing among all the Weyl fields should be taken into account in the renormalization procedure. The right-handed Weyl fields $\psi_{R,j}(x)$ may be changed to the fields with the left-handed chirality such that

$$\psi_{L,j}(x) \equiv C\bar{\psi}_{R,j}(x), \tag{3·39}$$

where C is the charge conjugation matrix. Then all the Weyl fields possess the left-handed chirality and mix with each other.

In practical cases there are conservation laws which partly prohibit the mixings. The electric charge, for example, is assumed to be conserved. This assumption guarantees that only particles with the same charge can mix with each other. In the charged particle sector we adopt the Dirac field representation.*) In this case we define the relevant fields by

$$\psi_i = \psi_{L,i} + \psi_{R,i}\ , \tag{3·40}$$

$$\psi_{L,i} \equiv \frac{1}{2}(1-\gamma_5)\psi_i\ , \tag{3·41a}$$

$$\psi_{R,i} \equiv \frac{1}{2}(1+\gamma_5)\psi_i\ . \tag{3·41b}$$

We assume the following asymptotic conditions,

---

*) It is possible to represent a charged particle by the two-component Weyl field. Such a particle is necessarily massless and is unrealistic. An extension of the formulation to include such a massless charged particle is straightforward (see the last part of this subsection).

$$\psi_{\mathrm{L},i}(x) \to \tilde{Z}^{1/2}_{\mathrm{L},ij}\psi^{\mathrm{as}}_{\mathrm{L},j}(x)\ , \tag{3·42a}$$

$$\psi_{\mathrm{R},i}(x) \to \tilde{Z}^{1/2}_{\mathrm{R},ij}\psi^{\mathrm{as}}_{\mathrm{R},j}(x)\ . \tag{3·42b}$$

We note here that the left- and right-chirality fields do not mix with each other because of the fermion-number conservation which is directly related to the charge conservation.

For neutral particles, e.g., neutrinos, however, the charge conservation has nothing to do with the fermion number conservation. In fact neutrinos can have the Majorana type masses as well as the Dirac type masses. In the case where the fermion number is not a good quantum numbers, we must deal with a full mixing of all Weyl fields. We will comment on the on-shell renormalization conditions in this case at the end of this subsection.

We go back to the case of the Dirac fields described by Eqs. (3·40) and (3·42). The renormalized fields are defined by

$$\psi_{\mathrm{L},i}(x) \equiv Z^{1/2}_{\mathrm{L},ij}\psi^{\mathrm{ren}}_{\mathrm{L},j}(x), \tag{3·43a}$$

$$\psi_{\mathrm{R},i}(x) \equiv Z^{1/2}_{\mathrm{R},ij}\psi^{\mathrm{ren}}_{\mathrm{R},j}(x). \tag{3·43b}$$

If the parity transformation which exchanges L and R chirality is a symmetry of the theory, we have the identity,

$$Z^{1/2}_{\mathrm{L},ij} = Z^{1/2}_{\mathrm{R},ij}\ . \tag{3·44}$$

Because of this relation we need not introduce independent renormalization constants for ψ_{L} and ψ_{R} in QED. In the Weinberg-Salam theory the relation (3·44) does not hold, and $Z^{1/2}_{\mathrm{L},ij}$ and $Z^{1/2}_{\mathrm{R},ij}$ are independent constants.

The on-shell renormalization conditions for the propagator

$$S_{ij} \equiv \mathrm{FT}\ i\langle 0|\mathrm{T}\psi_i(x)\bar{\psi}_j(y)|0\rangle\ , \tag{3·45}$$

are

$$S^{\mathrm{ren}}_{ij}[\mathrm{pole}] = S^{\mathrm{as}}_{ij}\ , \tag{3·46}$$

where the propagator for asymptotic fields is given by

$$S^{\mathrm{as}}_{ij}(p) = \frac{\delta_{ij}}{m_i - \not{p}}\ . \tag{3·47}$$

The renormalized propagator has a pole only in the (n, n) channel at $p^2 = m_n^2$,

$$S^{\mathrm{ren}}_{ij}(p) \xrightarrow[p^2 \to m_n^2]{} \begin{array}{cc} & n \\ n & \begin{pmatrix} & \vdots & \\ \cdots & \dfrac{1}{m_n - \not{p}} & \cdots \\ & \vdots & \end{pmatrix} \end{array}. \tag{3·48}$$

The inverse propagator

$$K^{\text{ren}}_{ij}(p) \equiv \{S^{\text{ren}}(p)\}^{-1}_{ij}\,, \tag{3·49}$$

behaves as

$$K^{\text{ren}}_{ij} \xrightarrow[p^2\to m_n{}^2]{} \begin{array}{cc} & n \\ n & \left(\begin{array}{ccc} & \vdots & \\ & M^{in}(m_n-\not{p}) & \\ & \vdots & \\ \cdots(m_n-\not{p})M^{nj}\cdots & (m_n-\not{p}) & \cdots \\ & \vdots & \end{array}\right) \end{array}. \tag{3·50}$$

In the component form Eq. (3·50) is expressed as

$$K^{\text{ren}}_{ij} \xrightarrow[p^2\to m_n{}^2]{} \begin{cases} (m_n-\not{p}) & \text{for} \quad i=n,\ \ j=n\,, & (3·51\text{a}) \\ M^{in}(m_n-\not{p}) & \text{for} \quad i\neq n,\ \ j=n\,, & (3·51\text{b}) \\ (m_n-\not{p})M^{nj} & \text{for} \quad i=n,\ \ j\neq n\,, & (3·51\text{c}) \\ \text{arbitrary} & \text{for} \quad i\neq n,\ \ j\neq n\,. & (3·51\text{d}) \end{cases}$$

Note that M^{ij} is generally uncommutable with $\not{p}$ because of the existence of γ^5 in it. The ordering in Eqs. (3·50) and (3·51) guarantees that the inverse of K reproduces Eq. (3·48) properly.

In order to write down the on-shell renormalization conditions (3·51) explicitly, it is convenient to introduce on-shell spinors $u(m)$ and $\bar{u}(m)$ which are the solutions of the Dirac equations with mass m:

$$(\not{p}-m)\,u(m)=0\,, \tag{3·52a}$$

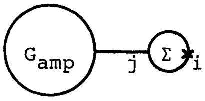
$$\bar{u}(m)\,(\not{p}-m)=0\,, \tag{3·52b}$$

where momentum p in $u(m)$ and $\bar{u}(m)$ is suppressed. Using these $u(m)$ and $\bar{u}(m)$, we can represent the conditions (3·51) in the following form:

$$\left.\begin{array}{l} K^{\text{ren}}_{ij}u(m_j)=0\,, \qquad (3·53\text{a}) \\ \bar{u}(m_i)K^{\text{ren}}_{ij}=0\,, \qquad (3·53\text{b}) \end{array}\right\} \text{ for all } \ i,\ j\,,$$

$$\left.\begin{array}{l} \left\{\dfrac{1}{m_i-\not{p}}K^{\text{ren}}_{ii}\right\}u(m_i)=u(m_i)\,, \qquad (3·54\text{a}) \\ \bar{u}(m_i)\left\{K^{\text{ren}}_{ii}\dfrac{1}{\not{p}-m_i}\right\}=\bar{u}(m_i)\,, \qquad (3·54\text{b}) \end{array}\right\} \text{ for all } \ i\,,$$

where the momentum squared p^2 is set on the mass shell $p^2=m_j^2$ in Eq. (3·53a) and $p^2=m_i^2$ in Eqs. (3·53b) and (3·54). The above expressions (3·54) correspond to the fact that external-line corrections vanish automatically in the on-shell renormalization scheme:



$$u(m_i)=0\,, \tag{3·55}$$

where Σ is the self-energy part.

We split the inverse propagator $K(p)$ into four invariant functions:

$$K_{ij}^{\text{ren}}(p)\equiv K_1^{ij}(p^2)\cdot\mathbf{1}+K_5^{ij}(p^2)\cdot\gamma^5+K_\gamma^{ij}(p^2)\cdot\not{p}+K_{5\gamma}^{ij}(p^2)\cdot\not{p}\gamma^5\,, \tag{3·56}$$

where terms containing $\sigma^{\mu\nu}$ do not appear since only one momentum variable p_μ is available to contract $\sigma^{\mu\nu}$. Using Eq. (3·56), the renormalization conditions (3·53, 3·54) are

$$[i=j] \qquad K_1^{ii}(m_i^2)+m_iK_\gamma^{ii}(m_i^2)=0\,, \tag{3·57a}$$

$$2m_iK_1^{ii\prime}(m_i^2)+K_\gamma^{ii}(m_i^2)+2m_i^2K_\gamma^{ii\prime}(m_i^2)=1\,, \tag{3·57b}$$

$$K_5^{ii}(m_i^2)=0\,, \tag{3·57c}$$

$$K_{5\gamma}^{ii}(m_i^2)=0\,, \tag{3·57d}$$

$$[i\neq j] \qquad K_1^{ij}(m_j^2)+m_jK_\gamma^{ij}(m_j^2)=0\,, \tag{3·58a}$$

$$K_1^{ij}(m_i^2)+m_iK_\gamma^{ij}(m_i^2)=0\,, \tag{3·58b}$$

$$K_5^{ij}(m_j^2)-m_jK_{5\gamma}^{ij}(m_j^2)=0\,, \tag{3·58c}$$

$$K_5^{ij}(m_i^2)+m_iK_{5\gamma}^{ij}(m_i^2)=0\,. \tag{3·58d}$$

The condition (3·57b) is concisely written as

$$\frac{\partial}{\partial\not{p}}\{K_1(p^2)+\not{p}K_\gamma(p^2)\}_{\not{p}=m}=1\,. \tag{3·59}$$

We then discuss the consistency between the numbers of renormalization conditions and renormalization constants. Because of the hermiticity of the effective action Γ which is formally guaranteed by the hermiticity of the Lagrangian,

$$\Gamma^\dagger=\Gamma\,, \tag{3·60}$$

the inverse propagator has the following symmetry:

$$K_{ij}^{\text{ren}}(p)=\gamma^{0\dagger}K_{ij}^{\text{ren}\dagger}(p)\gamma^0\,. \tag{3·61}$$

In deriving the above symmetry we have used the relation

$$K_{ij}^{\text{ren}}(p)=\int d^4(x-y)\,e^{ip(x-y)}\frac{\delta^2\Gamma}{\delta\psi_i^{\text{ren}}(x)\,\delta\bar{\psi}_j^{\text{ren}}(y)}\,. \tag{3·62}$$

It is easy to check that the above hermiticity condition is consistent with the renormalization conditions. The renormalization condition (3·53a) for $i>j$,

$$K_{ij}^{\mathrm{ren}} u(m_j) = 0\,, \tag{3·63}$$

implies

$$\bar{u}(m_j) K_{ji}^{\mathrm{ren}} \gamma^0 = 0\,, \tag{3·64}$$

which is nothing but the condition (3·53b). Thus it is sufficient to impose the renormalization conditions only on K_{ij} for $i \geqq j$. Those counter terms determined in the sector $i \geqq j$ are sufficient to satisfy the renormalization conditions for the sector $i<j$. In the sector $i=j$ we obtain, from the hermiticity condition (3·61),

$$K_1^{ii*} = K_1^{ii}\,, \tag{3·65a}$$

$$K_5^{ii*} = -K_5^{ii}\,, \tag{3·65b}$$

$$K_\gamma^{ii*} = K_\gamma^{ii}\,, \tag{3·65c}$$

$$K_{5\gamma}^{ii*} = K_{5\gamma}^{ii}\,. \tag{3·65d}$$

Equations (3·65) indicate that all diagonal invariant functions are purely real or imaginary. According to these hermiticity conditions, the renormalization conditions (3·53a) and (3·53b) and also [(3·54a) and (3·54b) for the sector $i=j$] are not independent of each other.

Now we are ready for counting the number of independent conditions. In the sector $i=j$ we have four conditions (3·57) since each equation is purely real or imaginary as seen in Eqs. (3·65). The conditions (3·58) for $i>j$ are all complex equations, which correspond to eight real conditions. Hence the number of necessary conditions is $4N$ for $i=j$ and $4N(N-1)$ for $i>j$. Thus it amounts to $4N^2$. Counter terms for the self-energy part are generated by the Z_L and Z_R factors and mass parameters as follows:

$$\not{p}(Z_\mathrm{L}^{1/2\dagger} Z_\mathrm{L}^{1/2})_{ij} \frac{1-\gamma_5}{2} + \not{p}(Z_\mathrm{R}^{1/2\dagger} Z_\mathrm{R}^{1/2})_{ij} \frac{1+\gamma_5}{2}$$

$$- (Z_\mathrm{L}^{1/2\dagger} M Z_\mathrm{R}^{1/2})_{ij} \frac{1+\gamma_5}{2} - (Z_\mathrm{R}^{1/2\dagger} M^\dagger Z_\mathrm{L}^{1/2})_{ij} \frac{1-\gamma_5}{2}\,, \tag{3·66}$$

where M is the bare mass matrix. The general mass matrix M can be diagonalized with real positive eigenvalues by an appropriate biunitary transformation:

$$\{VMU\}_{ij} = \delta_{ij} m_i\,. \tag{3·67}$$

These unitary matrices U and V are absorbed into the definition of the $Z_L{}^{1/2}$ and $Z_R{}^{1/2}$ matrices, and so independent mass parameters are only the m_i's ($i=1\sim N$) in Eq. (3·67). Furthermore the common phase rotation of $Z_L{}^{1/2}$ and $Z_R{}^{1/2}$,

$$Z^{1/2}_{L,ij}\to e^{i\theta_i}\delta_{ik}Z^{1/2}_{L,kj}\,, \tag{3·68a}$$

$$Z^{1/2}_{R,ij}\to e^{i\theta_i}\delta_{ik}Z^{1/2}_{R,kj}\,, \tag{3·68b}$$

does not modify the counter terms (3·66). Consequently the number of degrees of freedom of counter terms is

$$4N^2[Z_L^{1/2}, Z_R^{1/2}]+N[m_i]-N[\theta_i]=4N^2\,, \tag{3·69}$$

which coincides with that of the renormalization conditions. We should notice here that the matrices $Z_L{}^{1/2}$ and $Z_R{}^{1/2}$ are not unitary. This implies that the on-shell states cannot be obtained by the unitary transformation from those states corresponding to the Heisenberg fields. The situation is similar to the case of scalar fields. The above counting is not sufficient for the proof of renormalizability, but one can easily show that all equations of renormalization conditions have a unique solution for counter terms.

Here we discuss the case where the fermion number is not necessarily conserved, for example, the case of neutrinos. In general all fermions are the Majorana particles in this case. We represent these fermions by the four-component Majorana fields which satisfies the self-charge-conjugate condition,

$$C\bar{\psi}_M=\psi_M\,. \tag{3·70}$$

This representation is more convenient than a two-component representation because a trace including both Dirac fields and Majorana fields has to be often calculated. The propagator of (asymptotic) Majorana fields is

$$\begin{aligned} S^{as}_{\psi\bar{\psi}} &\equiv \mathrm{FT}\ i\langle 0|\mathrm{T}\psi^{as}_M(x)\bar{\psi}^{as}_M(y)\,|0\rangle \\ &=\frac{1}{m-\not{p}}\,, \end{aligned} \tag{3·71a}$$

$$\begin{aligned} S^{as}_{\psi\psi} &\equiv \mathrm{FT}\ i\langle 0|\mathrm{T}\psi^{as}_M(x)\psi^{as}_M(y)\,|0\rangle \\ &=-S^{as}_{\psi\bar{\psi}}C\,, \end{aligned} \tag{3·71b}$$

$$\begin{aligned} S^{as}_{\bar{\psi}\bar{\psi}} &\equiv \mathrm{FT}\ i\langle 0|\mathrm{T}\bar{\psi}^{as}_M(x)\bar{\psi}^{as}_M(y)\,|0\rangle \\ &=C^{-1}S^{as}_{\psi\bar{\psi}}\,, \end{aligned} \tag{3·71c}$$

where we have used the Majorana condition (3·70). The on-shell renormalization condition is

$$S^{\text{ren}}_{\psi\bar{\psi}} = S^{\text{as}}_{\psi\bar{\psi}} \,.$$

We only deal with the Dirac-like part of the propagator of Majorana fields (3·71a). The Majorana condition (3·70) for renormalized fields guarantees the correct form for $S^{\text{ren}}_{\psi\psi}$ and $S^{\text{ren}}_{\bar{\psi}\bar{\psi}}$. Hence we obtain the same formulas of on-shell renormalization conditions as those for Dirac fields. The invariant amplitudes of the inverse propagator are, however, constrained by the Majorana condition (3·70) and the number of the independent renormalization conditions is reduced to $2N^2+N$ for the sector of N Majorana particles. On the other hand, the Majorana condition (3·70) for the renormalized fields imposes the relation

$$Z_{\text{L}} = Z_{\text{R}}{}^{*} \,,$$

and the degrees of freedom of the renormalization constants are also $2N^2+N$.

Finally, we comment on the model which we will adopt for the estimation of the radiative corrections in Chapter 5. In the model, neutrinos are described by pure left-handed fields and are to be massless assuming the fermion number conservation. There is no mixing among different generations. This is a special case and the renormalization procedure becomes very simple. The inverse propagator of neutrinos is constrained so that

$$K_{\nu} = K_{\nu}(p^2)\,\not{p}\,(1-\gamma_5)\,.$$

The renormalization constant $Z_i^{1/2}$ is introduced as

$$\nu_{\text{L},i} \equiv Z_i^{1/2} \nu^{\text{ren}}_{\text{L},i} \,.$$

We adjust this $Z_i^{1/2}$ factor so that the renormalized inverse propagator K^{ren}_{ν} takes the form

$$K^{\text{ren}}_{\nu} = -\frac{1}{2}\not{p}\,(1-\gamma_5)\,. \tag{3·72}$$

This is the on-shell renormalization condition for the pure left-handed neutrinos.

3.2.3 *Vector boson sector*

In the vector boson sector mixings take place not only among vector fields but also among vector and scalar fields. Here the scalar fields are Nambu-Goldstone (NG) bosons or physical Higgs particles. In the case of the Weinberg-Salam (WS) theory the neutral gauge boson sector is most complicated where Z and A bosons, NG scalar χ_3 and physical Higgs scalar ϕ take part. If CP is a good symmetry, the physical Higgs $\phi\,(CP=+)$ decouples from the neutral boson sector $(CP=-1)$. On the other hand, as far as we adopt a manifestly renormalizable covariant gauge, NG bosons, which are absorbed into the longitudinal components of gauge bosons, must be taken into account in the calculation of Feynman graphs.

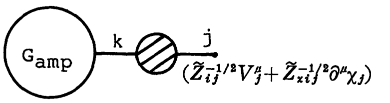
We consider mixing of vector bosons V_i^μ and scalars χ_i. The asymptotic fields of vector bosons are given by

$$\tilde{Z}_{ij}^{-1/2} V_j^\mu(x) + \tilde{Z}_{\chi ij}^{-1/2} \partial^\mu \chi_j(x) \rightarrow V_i^{\mu,\mathrm{as}}(x) \,. \tag{3·73}$$

Here $\partial_\mu \chi_j$ term, i.e., the scalar mode, plays a role of guaranteeing that the asymptotic vector field satisfies the physical polarization condition:

$$\partial_\mu V_i^{\mu,\mathrm{as}}(x) = 0 \,. \tag{3·74}$$

According to the usual reduction formula, the S-matrix element with a V_i^μ vector boson is obtained from the Green function as shown below,



$$\times (M_i^2 - p^2)\, \varepsilon_\mu \,, \tag{3·75}$$

where M_i is the mass of the $V_i^{\mu,\mathrm{as}}$ vector boson and ε_μ is a polarization vector of the emitted (absorbed) vector boson, which satisfies

$$p^\mu \varepsilon_\mu = 0 \,. \tag{3·76}$$

By this physical polarization condition the $\partial^\mu \chi_j$ part of the Green function in Eq. (3·75) does not contribute to the S-matrix element. Similarly the Green function with χ_j on the k side in Eq. (3·75) does not contribute since the $V_j^\mu \chi_k$ transition propagator is proportional to the momentum p^μ, which is contracted with ε_μ. Thus the part of the k-j transition propagator which contributes to the S-matrix element (3·75) is the vector-vector transition part alone.

We decompose the vector-vector propagator into transversal and longitudinal parts:

$$\begin{aligned} \Delta^{\mu\nu} &\equiv \mathrm{FT}(-i)\langle 0|\mathrm{T} V_i^\mu(x)\, V_j^\nu(y)\,|0\rangle \\ &\equiv T(p^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + L(p^2)\frac{p^\mu p^\nu}{p^2} \,, \end{aligned} \tag{3·77}$$

where $T(p^2)$ and $L(p^2)$ are invariant functions. In Eq. (3·75) only T part survives under condition (3·76). The pole part of T is given by

$$T_{ij}[\mathrm{pole}] = \tilde{Z}_{im}^{1/2} \tilde{Z}_{jn}^{1/2} \frac{\delta_{mn}}{M_n^2 - p^2} \,. \tag{3·78}$$

The S-matrix element is expressed as follows:

$$
\begin{aligned}
&G_{\mathrm{amp}}{}^{k}_{\mu}\, \tilde{Z}^{1/2}_{km} \tilde{Z}^{1/2}_{jn} \frac{\delta_{mn}}{M_n{}^2 - p^2} \tilde{Z}^{-1/2}_{ij} (M_i{}^2 - p^2)\, \varepsilon_\mu \\
&= G_{\mathrm{amp}}{}^{k}_{\mu}\, \tilde{Z}^{1/2}_{ki} \varepsilon^\mu \, .
\end{aligned} \tag{3·79}
$$

We introduce the renormalization constant $Z^{1/2}_{ij}$ such that

$$V_i{}^\mu(x) \equiv Z^{1/2}_{ij} V_j{}^{\mu,\mathrm{ren}}(x) . \tag{3·80}$$

The on-shell renormalization conditions in which $Z^{1/2}_{ij} = \tilde{Z}^{1/2}_{ij}$ are

$$T^{\mathrm{ren}}_{ij}[\mathrm{pole}] = \frac{\delta_{ij}}{M_i^2 - p^2} . \tag{3·81}$$

The inverse propagator may be expressed as

$$\Delta^{-1\mu\nu}_{ij}(p) = A_{ij}(p^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + B_{ij}(p^2)\frac{p^\mu p^\nu}{p^2} , \tag{3·82}$$

where

$$A_{ij}(p^2) = T^{-1}_{ij}(p^2), \quad B_{ij}(p^2) = L^{-1}_{ij}(p^2) .$$

The on-shell renormalization conditions in terms of $A_{ij}(p^2)$ read

$[i=j]$

$$A_{ii}(M_i^2) = 0, \quad A_{ii}{}'(M_i^2) = -1 , \tag{3·83a}$$

$[i \neq j]$

$$A_{ij}(M_i^2) = 0, \quad A_{ij}(M_j^2) = 0 . \tag{3·83b}$$

Just as in the case of scalars, the wave function renormalization matrix $Z^{1/2}_{ij}$ is not orthogonal but is a general real matrix. An important point concerning vector-scalar mixing is that the renormalization constants corresponding to the $\tilde{Z}^{1/2}_{\chi ij}$ factor in Eq. (3·73) are unnecessary for the purpose of obtaining the S-matrix element. Hence it is sufficient to introduce the renormalization constants only for vector-vector mixing part. Another point is that the on-shell renormalization conditions are imposed only on $A_{ij}(p^2)$. The counter terms for $B_{ij}(p^2)$ which are already fixed by the above conditions on $A_{ij}(p^2)$, actually cancel the infinity of $B_{ij}(p^2)$ and the full renormalized inverse propagator is made finite automatically if the theory is renormalizable.

Finally, we would like to make some comments. We can easily generalize the former results to the case of fields with arbitrary spin. First of all, we need not consider explicitly mixing of lower-spin fields such as scalar mixing in the vector sector. The contribution of the lower spin fields vanishes because of the on-shell polarization condition. Just to obtain the S-matrix element, we need not mix lower-spin fields.

Next, the on-shell renormalization conditions are imposed on the part of the propagator which survives by the application of on-shell polarization.

Schematically the on-shell renormalization conditions read for all i and j (no summation for repeated indices)

$$\langle i|K_{ij}=0\,, \tag{3·84a}$$

$$K_{ij}|j\rangle=0\,, \tag{3·84b}$$

$$\langle i|\{K_{ii}S_i^{\mathrm{T}}\}=1\,, \tag{3·84c}$$

$$\{S_i^{\mathrm{T}}K_{ii}\}|i\rangle=1\,, \tag{3·84d}$$

where K_{ij} is the inverse propagator, S_i^{T} is the diagonalized tree propagator and $|i\rangle$ denotes the on-shell particle state (wave function). The momentum variables in Eq. (3·84) are understood to be on the mass shell of the corresponding state.

We can interpret the conditions of Eq. (3·84) from another point of view: Equation (3·84a) shows that the matrix $K_{ij}(p^2)$ has eigenvalue zero at $p^2=m_i^2$ and the state vector $\langle i|$ is the left eigenstate for it. Equation (3·84c) represents normalization of this eigenvector. We will explain this interpretation of the on-shell renormalization conditions further in § 3. 3. 2.

There are many cases where we cannot adopt the on-shell renormalization scheme defined above. In a theory with some constraints independent mass parameters of all physical particles cannot be supplied by all independent parameters in the Lagrangian. In other words, there may be some relations in tree mass parameters and these relations may not hold with higher order corrections. The so-called pseudo-Goldstone particle, which is massless in the tree approximation and becomes massive through higher loop corrections, is an example of such a constrained system. In these cases we cannot make the tree masses physical ones. One may regard the difference between tree and physical masses as counter terms. This procedure, however, destroys the loop expansion of the effective action and necessarily breaks the renormalizability of the theory. We must adopt the scheme in which the particle masses (the pole positions of the propagator) get corrected loop by loop. In the standard Weinberg-Salam theory the number of the independent bare parameters in the Lagrangian is sufficient to supply the full degrees of freedom of physical particle masses, and hence we perform the renormalization procedure in the on-shell scheme without any problem.

§ 3.3 Two-point functions in the Weinberg-Salam theory

In the preceding section we have studied the on-shell renormalization scheme and proposed the renormalization conditions. In this section we investigate the structure of two-point functions in the Weinberg-Salam (WS) theory. First dealing with the charged gauge boson sector, we see the manifestation of the quartet mechanism which assures the unitarity of the physical S-matrix and we prove the renormalizability of this sector explicitly. Then we proceed to the neutral gauge boson sector, where we prove that the on-shell renormalization conditions can be imposed consistently with the aid of the Ward-Takahashi (WT) identities [Aoki 79].

Although in this and the next section we make sure of the validity and the consistency of our renormalization scheme, readers who are interested only in explicit renormalization calculation may jump over these sections (§§ 3.3 and 3.4).

3.3.1 *Charged gauge boson sector*

We begin with the WT identities for the effective action, Eq. (2·120) derived in § 2.4,

$$\frac{\delta\tilde{\Gamma}}{\delta W^a_\mu}\cdot\frac{\delta\tilde{\Gamma}}{\delta K^\mu_a}+\frac{\delta\tilde{\Gamma}}{\delta\Phi}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_\phi}+\frac{\delta\tilde{\Gamma}}{\delta\psi}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_\psi}+\frac{\delta\tilde{\Gamma}}{\delta\bar{\psi}}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\bar{\psi}}}+\frac{\delta\tilde{\Gamma}}{\delta c^a}\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{c^a}}=0\,, \tag{3·85}$$

where $\tilde{\Gamma}$ is defined as before in Eq. (2·117):

$$\tilde{\Gamma}\equiv\Gamma-\int\mathcal{L}_{\mathrm{GF}}\,d^4x\,, \tag{3·86}$$

and $W_\mu{}^a$, ϕ, ψ ($\bar{\psi}$) and c^a represent (bare) gauge fields, scalars, fermions and ghost fields respectively. The corresponding BRS source terms are K^μ_a, K_ϕ, K_ψ ($K_{\bar{\psi}}$) and K_{c^a} respectively.

In the charged boson sector relevant fields are charged gauge bosons (W^+_μ, W^-_μ), charged Nambu-Goldstone (NG) bosons (χ^+,χ^-) and charged ghosts and antighosts ($c^+, c^-, \bar{c}^+, \bar{c}^-$). The ghosts decouple from the W-χ sector because of the ghost number conservation. We define two-point functions as follows:

$$\begin{aligned}\tilde{\Gamma}_{\mu\nu}&\equiv\mathrm{FT}\left.\frac{\delta^2\tilde{\Gamma}}{\delta W^{+\mu}(x)\,\delta W^{-\nu}(y)}\right|_0\\&=A(p^2)\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+B(p^2)\frac{p_\mu p_\nu}{p^2}\\&=A(p^2)\,T_{\mu\nu}+B(p^2)\,L_{\mu\nu}\,,\end{aligned} \tag{3·87}$$

$$\widetilde{\Gamma}^{+-}_{W_\mu \chi} \equiv \mathrm{FT} \frac{\delta^2 \widetilde{\Gamma}}{\delta W^{+\mu}(x)\delta\chi^-(y)}\bigg|_0$$

$$\equiv i p_\mu C^{+-}(p^2), \tag{3·88}$$

$$\widetilde{\Gamma}_{\chi\chi} \equiv \mathrm{FT} \frac{\delta^2 \widetilde{\Gamma}}{\delta\chi^+(x)\delta\chi^-(y)}\bigg|_0$$

$$\equiv p^2 F(p^2), \tag{3·89}$$

$$\widetilde{\Gamma}^{+-}_{c\bar{c}} \equiv \mathrm{FT} \frac{\delta^2 \widetilde{\Gamma}}{\delta c^+(x)\delta\bar{c}^-(y)}\bigg|_0$$

$$\equiv \gamma^{+-}(p^2), \tag{3·90}$$

where FT denotes Fourier transform defined in Eq. (3·7) and the notation $|_0$ represents that each field is set to its vacuum expectation value respectively. In Eq. (3·89) we have written the factor p^2 explicitly; this factorization will be justified in Eq. (3·99b). Taking account of the hermiticity of the effective action, we obtain the relation:

$$\{C^{-+}(p^2)\}^* = C^{+-}(p^2), \tag{3·91}$$

$$\{\gamma^{-+}(p^2)\}^* = \gamma^{+-}(p^2), \tag{3·92}$$

both sides of which are purely real quantities in CP invariant case.

In the tree approximation these invariant functions take the values as

$$A(p^2) = M_W^2 - p^2,$$

$$B(p^2) = M_W^2,$$

$$C^{+-}(p^2) = C^{-+}(p^2) = M_W,$$

$$F(p^2) = 1, \tag{3·93}$$

where M_W is the charged gauge boson mass in the tree level. In order to write the explicit form of $\gamma(p^2)$ we need to fix the gauge. For example, in the 't Hooft gauge which will be defined in Eq. (3·103), they are given by

$$\gamma^{+-}(p^2) = \gamma^{-+}(p^2) = \alpha M_W^2 - p^2. \tag{3·94}$$

Two-point functions (3·87)$\sim$(3·90) are expressed in the following matrix form:

$$
\tilde{\Gamma}^{(2)}=
\begin{array}{c||c|c|c|c}
 & W_\nu^- & \chi^- & c^- & \bar{c}^- \\
\hline\hline
W_\mu^+ & A(p^2)T_{\mu\nu}+B(p^2)L_{\mu\nu} & ip_\mu C^{+-}(p^2) & 0 & 0 \\
\hline
\chi^+ & -ip_\mu C^{-+}(p^2) & p^2F(p^2) & 0 & 0 \\
\hline
c^+ & 0 & 0 & 0 & \gamma^{+-}(p^2) \\
\hline
\bar{c}^+ & 0 & 0 & -\gamma^{-+}(p^2) & 0
\end{array}
\,, \tag{3·95}
$$

where the minus signs attached to C^{-+} and γ^{-+} appear on account of the change of order of differentiation.

In order to obtain the relations among invariant functions imposed by gauge invariance we differentiate the WT identity (3·85) and set all the fields to their vacuum expectation values which are zero in this case.

$$\frac{\delta^2}{\delta W_\mu{}^+\delta c^-}\ \text{Eq. (3·85)}\ \Big|_0 :$$

$$\tilde{\Gamma}^{+-}_{\mu\nu}\cdot \mathrm{FT}\left.\frac{\delta^2\tilde{\Gamma}}{\delta K_\nu^+\delta c^-}\right|_0 + \tilde{\Gamma}^{+-}_{W_\mu\chi}\cdot \mathrm{FT}\left.\frac{\delta^2\tilde{\Gamma}}{\delta K_\chi^+\delta c^-}\right|_0 =0\,, \tag{3·96}$$

$$\frac{\delta^2}{\delta \chi^+\delta c^-}\ \text{Eq. (3·85)}\ \Big|_0 :$$

$$\tilde{\Gamma}^{-+}_{W_\mu\chi}\cdot \mathrm{FT}\left.\frac{\delta^2\tilde{\Gamma}}{\delta K_\nu^+\delta c^-}\right|_0 + \tilde{\Gamma}^{+-}_{\chi\chi}\cdot \mathrm{FT}\left.\frac{\delta^2\tilde{\Gamma}}{\delta K_\chi^+\delta c^-}\right|_0 =0. \tag{3·97}$$

Here use has been made of the ghost number conservation. With the definition of invariant functions:

$$p^\nu J(p^2)\equiv \mathrm{FT}\left.\frac{\delta^2\tilde{\Gamma}}{\delta K_\nu^+\delta c^-}\right|_0 , \tag{3·98a}$$

$$I(p^2)\equiv \mathrm{FT}\left.\frac{\delta^2\tilde{\Gamma}}{\delta K_\chi^+\delta c^-}\right|_0 , \tag{3·98b}$$

Eqs. (3·96) and (3·97) are rewritten as

$$p_\mu B(p^2)J(p^2)+ip_\mu C^{+-}(p^2)I(p^2)=0\,, \tag{3·99a}$$

$$-ip^2C^{-+}(p^2)J(p^2)+p^2F(p^2)I(p^2)=0\,. \tag{3·99b}$$

Equation (3·99b) assures the factorizability in Eq. (3·89). The p^2 factor comes from the fact that the χ field represents the massless NG mode. From the above WT identities an important relation of $\tilde{\Gamma}^{(2)}$ components is derived:

$$B(p^2)F(p^2)=|C^{+-}(p^2)|^2. \tag{3·100}$$

The inverse propagator $\Gamma^{(2)}$ is obtained by adding the contribution of a gauge fixing Lagrangian to $\tilde{\Gamma}^{(2)}$. For the gauge fixing term we take the bilinear form without using auxiliary B fields,

$$\mathcal{L}_{\mathrm{GF}}=-\frac{1}{\alpha}(\partial^{\mu}W_{\mu}{}^{+}-\alpha X^{+}\cdot\chi^{+})(\partial^{\mu}W_{\mu}{}^{-}-\alpha X^{-}\cdot\chi^{-})+\cdots, \tag{3·101}$$

where only the charged gauge boson part is explicitly written and $X^{\pm}\cdot\chi^{\pm}$ denotes the convolution integral in the coordinate space as follows:

$$X^{\pm}\cdot\chi^{\pm}\equiv\int d^4yX^{\pm}(x-y)\chi^{\pm}(y), \tag{3·102}$$

which is a local product in the momentum space. The case where $X^{\pm}$ function is taken to be

$$X^{\pm}(x)=\delta^4(x)M_W, \tag{3·103}$$

is called the 't Hooft gauge.

The contribution of the gauge fixing part of the effective action,

$$\Gamma_{\mathrm{GF}}=\int d^4x\mathcal{L}_{\mathrm{GF}}, \tag{3·104}$$

to the two-point functions is calculated as follows:

$$\Gamma^{+-}_{\mathrm{GF}\mu\nu}\equiv\mathrm{FT}\frac{\delta^2\Gamma_{\mathrm{GF}}}{\delta W^{+\mu}(x)\delta W^{-\nu}(y)}$$

$$=-\frac{1}{\alpha}p_{\mu}p_{\nu}, \tag{3·105}$$

$$\Gamma^{+-}_{\mathrm{GF}W_{\mu}\chi}=ip_{\mu}\tilde{X}^{-}(p^2), \tag{3·106}$$

$$\Gamma^{-+}_{\mathrm{GF}W_{\mu}\chi}=ip_{\mu}\tilde{X}^{+}(p^2), \tag{3·107}$$

$$\Gamma^{+-}_{\mathrm{GF}\chi\chi}=-\alpha\tilde{X}^{-}(p^2)\tilde{X}^{+}(p^2), \tag{3·108}$$

where $\tilde{X}^{\pm}(p^2)$ is the Fourier transform of $X^{\pm}(x)$. For simplicity of the argument we take $\tilde{X}^{\pm}(p^2)$ so as to cancel the $W\chi$ mixing part of the inverse propagator:

$$\tilde{X}^{\pm}(p^2)=-C^{\mp\pm}(p^2). \tag{3·109}$$

Finally the full inverse propagator in this gauge fixing condition (3·109) is

$$
\Gamma^{(2)} = \begin{array}{c||c|c}
 & W_\nu^- & \chi^- \\ \hline\hline
W_\mu^+ & A(p^2)T_{\mu\nu} - \dfrac{1}{\alpha}\{p^2-\alpha B(p^2)\}L_{\mu\nu} & 0 \\ \hline
\chi^+ & 0 & F(p^2)\{p^2-\alpha B(p^2)\}
\end{array}, \tag{3·110}
$$

where use has been made of Eq. (3·100). By adopting this type of non-local gauge fixing condition we could make the structure of the two-point functions very simple, although in non-local gauges we have various complications in the procedure of quantization.

How about the ghost propagator in this gauge condition? First go back to the second type of the WT identity (2·119):

$$
F_i{}^a \frac{\delta\tilde{\Gamma}}{\delta K_i} + \frac{\delta\tilde{\Gamma}}{\delta\bar{c}^a} = 0, \tag{3·111}
$$

which is deduced from the ghost equation of motion. In our gauge it is written as

$$
\partial_\mu \frac{\delta\tilde{\Gamma}}{\delta K^{+\mu}} - \alpha X^- \cdot \frac{\delta\tilde{\Gamma}}{\delta K_\chi^+} + \frac{\delta\tilde{\Gamma}}{\delta\bar{c}^+} = 0, \tag{3·112}
$$

where

$$
K^{\pm\mu} \equiv (K_1^\mu \mp iK_2{}^\mu)/\sqrt{2}\,.
$$

With the operation of $\delta/\delta c^-|_0$ we obtain

$$
-ip^2J(p^2) + \alpha\tilde{X}^-(p^2)I(p^2) = \gamma^{-+}(p^2), \tag{3·113}
$$

and we have

$$
\gamma^{-+}(p^2) = -iJ(p^2)\{p^2-\alpha B(p^2)\}, \tag{3·114}
$$

where we have used Eqs. (3·99), (3·100) and (3·109). Thus the ghost inverse propagator is

$$
\Gamma_c^{(2)} = \begin{array}{c||c|c}
 & c^- & \bar{c}^- \\ \hline\hline
c^+ & 0 & -J^*(p^2)\{p^2-\alpha B(p^2)\} \\ \hline
\bar{c}^+ & -J(p^2)\{p^2-\alpha B(p^2)\} & 0
\end{array} \tag{3·115}
$$

We easily see the evidence of the quartet mechanism which guarantees the unitarity of physical S-matrix explained in § 2. 2. The propagators of the four particles [the scalar component of the gauge boson ($L_{\mu\nu}$ part), the NG boson (χ), the ghost (c) and the antighost ($\bar{c}$)] have a pole in a common position which is the solution of

$$p^2-\alpha B(p^2)=0\,. \tag{3·116}$$

These quartet particles can appear in the physical subspace of the total Hilbert space only in a zero-norm combination and hence cannot be observed. The only one physical mode of this sector is the massive vector boson which is represented by the $T_{\mu\nu}$ mode in the inverse propagator.

Let us proceed to the renormalization procedure. We introduce renormalization constants:

$$W_\mu{}^\pm(x)\equiv Z_3^{1/2}W^\pm_{\mu\mathrm{R}}(x)\,, \tag{3·117}$$

$$\chi^\pm(x)\equiv Z_\chi^{1/2}\chi_\mathrm{R}{}^\pm(x)\,, \tag{3·118}$$

$$M_{W0}^2\equiv M_W^2+\delta M_W^2\,, \tag{3·119}$$

$$\delta\overline{M}_W^2\equiv\delta M_W^2 Z_3\,. \tag{3·120}$$

From the bilinear terms of the bare Lagrangian we separate out the counter terms for $\Gamma^{(2)}$, which will be shown systematically in § 4. 3. The $\widetilde{\Gamma}$ parts of renormalized inverse propagators are written by the use of renormalization constants:

$$\begin{aligned}\widetilde{\Gamma}^\mathrm{R}_{\mu\nu}&\equiv Z_3\widetilde{\Gamma}_{\mu\nu}=Z_3A(p^2)T_{\mu\nu}+Z_3B(p^2)L_{\mu\nu}\\&\equiv A_\mathrm{R}(p^2)T_{\mu\nu}+B_\mathrm{R}(p^2)L_{\mu\nu}\,,\end{aligned} \tag{3·121}$$

$$\widetilde{\Gamma}^\mathrm{R}_{W_\mu\chi}\equiv Z_3^{1/2}Z_\chi^{1/2}\widetilde{\Gamma}_{W_\mu\chi}\equiv ip_\mu C_\mathrm{R}(p^2)\,, \tag{3·122}$$

$$\widetilde{\Gamma}^\mathrm{R}_{\chi\chi}\equiv Z_\chi\widetilde{\Gamma}_{\chi\chi}\equiv p^2F_\mathrm{R}(p^2)\,. \tag{3·123}$$

They are also obtained directly by differentiating $\widetilde{\Gamma}$ with respect to the renormalized fields.

Suppose that the renormalization procedure up to $(n-1)$-loop is completed. We calculate n-loop contribution which is written as

$$A_\mathrm{R}^{(n)}(p^2)=Z_3^{(n)}(M_W^2-p^2)-\delta\overline{M}_W^{2(n)}+\overline{A}^{(n)}(p^2)\,, \tag{3·124}$$

$$B_\mathrm{R}^{(n)}(p^2)=Z_3^{(n)}M_W^2-\delta\overline{M}_W^{2(n)}+\overline{B}^{(n)}(p^2)\,, \tag{3·125}$$

where the suffix (n) denotes the n-loop contribution and the first two terms of each equation are n-loop counter terms not yet determined. The n-loop momentum integration part is involved in $\overline{A}^{(n)}$ and $\overline{B}^{(n)}$, and all their inner divergences are eliminated by the renormalization up to $(n-1)$-loop and there remain only overall divergences. The on-shell renormalization conditions derived in Eq. (3·83),

$$A_R^{(n)}(M_W^2)=0\,,\quad A_R^{(n)\prime}(M_W^2)=0\,,\tag{3·126}$$

determine the counter terms:

$$\delta\overline{M}_W^{2(n)}=\bar{A}^{(n)}(M_W^2),\quad Z_3^{(n)}=\bar{A}^{(n)\prime}(M_W^2).\tag{3·127}$$

The resultant invariant functions $A_R^{(n)}(p^2)$ become a finite function including the $(p^2-M_W{}^2)^2$ factor.

To prove the finiteness of $B_R^{(n)}(p^2)$ whose counter terms are already fixed by Eq. (3·127), we should note an important relation,

$$A(0)=B(0).\tag{3·128}$$

This relation is obtained from the condition for non-existence of any pole in one-particle irreducible Green functions, that is, inverse propagators. This condition is always satisfied in perturbation theory. The above relation is also required to give rise to the Higgs mechanism: The unphysical modes (B^{as} and χ^{as}) in the vector boson field (W_μ^{as}) are eliminated and the massive vector boson asymptotic field is constructed. The relation (3·128) holds in any order of loop expansion:

$$A_R^{(n)}(0)=B_R^{(n)}(0).\tag{3·129}$$

As is seen in Eq. (3·125), the renormalization of $B(p^2)$ is only to subtract p^2-independent constant terms. It can be expressed as

$$B_R^{(n)}(p^2)=\overline{B}^{(n)}(p^2)-\overline{B}^{(n)}(0)+A_R^{(n)}(0),\tag{3·130}$$

where we take account of Eq. (3·129). Here another point should be noted that $\overline{B}^{(n)}(p^2)$ includes the factor $M_W{}^2$ because $\overline{B}(p^2)$ is equal to zero in the case of unbroken gauge symmetry ($M_W=0$). This indicates that $\overline{B}^{(n)}(p^2)$ has only logarithmic divergence and we obtain finite $B_R^{(n)}(p^2)$ from Eq. (3·130).

The renormalized χ propagator in n-loop is expressed as

$$p^2F_R^{(n)}(p^2)=p^2Z_\chi^{(n)}+p^2\overline{F}^{(n)}(p^2),\tag{3·131}$$

where the first term on the right-hand side is the counter term. The n-loop term $\overline{F}^{(n)}(p^2)$ has only logarithmic divergence. Therefore $F_R^{(n)}(p^2)$ is made finite by adjusting Z_χ as, for example,

$$Z_\chi^{(n)}=-\overline{F}^{(n)}(\mu^2),\tag{3·132}$$

where the corresponding renormalization condition is

$$F_R^{(n)}(\mu^2)=0\,.\tag{3·133}$$

The renormalized mixing part $C_R(p^2)$ is related to $B_R(p^2)$ and $F_R(p^2)$ by the identity (3·100):

$$|C_{\rm R}^{+-}(p^2)|^2 = B_{\rm R}(p^2) F_{\rm R}(p^2). \tag{3·134}$$

The mixing part is automatically finite after the renormalization for B and F. We have made all $W\chi$ components of $\widetilde{\Gamma}^{(2)}$ finite.

Next we discuss the contribution of gauge fixing term. We define $X_{\rm R}^{\pm}$ and $\alpha_{\rm R}$ as

$$X_{\rm R}^{\pm}(x) \equiv Z_\chi^{1/2} Z_3^{1/2} X^{\pm}(x), \tag{3·135}$$

$$\alpha_{\rm R} \equiv Z_3^{-1}\alpha, \tag{3·136}$$

where $\alpha_{\rm R}$ is supposed to be finite. The gauge fixing condition (3·109) leads

$$\widetilde{X}_{\rm R}^{\pm}(p^2) = -C_{\rm R}^{\pm\mp}(p^2). \tag{3·137}$$

The gauge fixing part of the effective action,

$$\begin{aligned}\Gamma_{\rm GF} &= \int \mathcal{L}_{\rm GF} d^4x \\ &= \int \frac{-1}{\alpha_{\rm R}} (\partial^\mu W_{\mu\rm R}^{+} - \alpha_{\rm R} X_{\rm R}^{+} \cdot \chi_{\rm R}^{+})(\partial^\mu W_{\mu\rm R}^{-} - \alpha_{\rm R} X_{\rm R}^{-} \cdot \chi_{\rm R}^{-}) d^4x\end{aligned} \tag{3·138}$$

becomes finite functional of renormalized quantities. Thus two-point functions of $\Gamma_{\rm GF}$ are renormalized to be finite.

Finally, the ghost inverse propagator is written with the renormalized quantities $\alpha_{\rm R}$ and $B_{\rm R}$:

$$\gamma^{+-}(p^2) = -J^*(p^2)\{p^2 - \alpha_{\rm R} B_{\rm R}(p^2)\}. \tag{3·139}$$

This includes only logarithmic divergence because of the factorization of the finite term, $\{p^2 - \alpha_{\rm R} B_{\rm R}(p^2)\}$. Therefore we can make it finite by adjusting Z_c factor defined by

$$c(x) \equiv Z_c c_{\rm R}(x), \tag{3·140}$$

appropriately.

We have made the pole structure of two-point functions clear and proved explicitly the renormalizability in the charged gauge boson sector.

3.3.2 *Neutral gauge boson sector*

In this subsection we study the neutral gauge boson sector which includes two gauge bosons (Z_μ and A_μ, or, W_μ^3 and W_μ^0), two Higgs fields (ϕ and χ_3), two ghosts (c^3 and c^0) and two antighosts ($\bar{c}^3$ and $\bar{c}^0$). We assume CP invariance of Lagrangian. In this case the physical Higgs field ϕ decouples completely from other particles. We adopt the following gauge fixing Lagrangian,

$$\mathcal{L}_{\mathrm{GF}}^{\mathrm{Neutral}}=\frac{-1}{2\alpha_3}(\partial^\mu W_\mu^3-\alpha_3\omega\cdot\partial^\mu W_\mu^0-\alpha_3 X_3\cdot\chi_3)^2-\frac{1}{2\alpha_0}(\partial^\mu W_\mu^0)^2, \qquad (3\cdot141)$$

where $\omega\cdot\partial^\mu W_\mu{}^0$ and $X_3\cdot\chi_3$ represent convolution integrals similar to Eq. (3·102). Similar to charged gauge boson case two-point functions obtained from $\widetilde{\Gamma}$ are defined as

$\widetilde{\Gamma}^{(2)}=$

	W_ν^3	W_ν^0	χ_3
W_μ^3	$A_3(p^2)T_{\mu\nu}+B_3(p^2)L_{\mu\nu}$	$A_{30}(p^2)T_{\mu\nu}+B_{30}(p^2)L_{\mu\nu}$	$ip_\mu C_{3\chi}(p^2)$
W_μ^0	$A_{30}(p^2)T_{\mu\nu}+B_{30}(p^2)L_{\mu\nu}$	$A_0(p^2)T_{\mu\nu}+B_0(p^2)L_{\mu\nu}$	$ip_\mu C_{0\chi}(p^2)$
χ_3	$-ip_\nu C_{3\chi}(p^2)$	$-ip_\nu C_{0\chi}(p^2)$	$p^2F_3(p^2)$

, (3·142)

$$\frac{\delta^2\widetilde{\Gamma}}{\delta c^i\delta\bar{c}^j}=-\frac{\delta^2\widetilde{\Gamma}}{\delta\bar{c}^j\delta c^i}=$$

	$\bar{c}^3$	$\bar{c}^0$
c^3	$\gamma_{33}(p^2)$	$\gamma_{30}(p^2)$
c^0	$\gamma_{03}(p^2)$	$\gamma_{00}(p^2)$

. (3·143)

Note that γ_{30} has nothing to do with γ_{03}. Gauge fixing part contribution is obtained as follows:

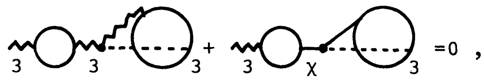
$\Gamma_{\mathrm{GF}}^{(2)}=$

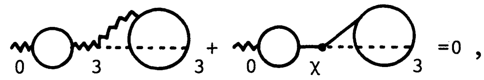
	W_ν^3	W_ν^0	χ_3
W_μ^3	$-\dfrac{p^2}{\alpha_3}L_{\mu\nu}$	$p^2\widetilde{\omega}_3(p^2)L_{\mu\nu}$	$ip_\mu\widetilde{X}_3(p^2)$
W_μ^0	$p^2\widetilde{\omega}_3(p^2)L_{\mu\nu}$	$-\left(\alpha_3\widetilde{\omega}_3{}^2(p^2)+\dfrac{1}{\alpha_0}\right)p^2L_{\mu\nu}$	$-ip_\mu\alpha_3\widetilde{\omega}_3(p^2)\widetilde{X}_3(p^2)$
χ_3	$-ip_\nu\widetilde{X}_3(p^2)$	$ip_\mu\alpha_3\widetilde{\omega}_3(p^2)\widetilde{X}_3(p^2)$	$-\alpha_3\widetilde{X}_3{}^2(p^2)$

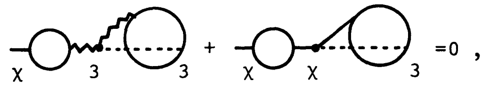
, (3·144)

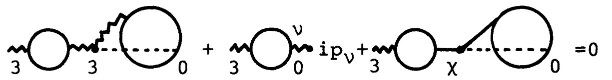
which is to do with $L_{\mu\nu}$ part in the vector sector.

By differentiating the WT identity (3·85) with respect to various combinations of fields, we obtain the following relations among invariant functions already defined. Here we represent these identities in the momentum representation in a graphical way explained in §§ 2.4 and 2.5.

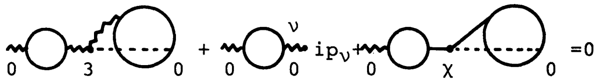
$\dfrac{\delta^2}{\delta W_\mu^3 \delta c^3}$ Eq. (3·85)$\Big|_0$:  (3·145a)

$\dfrac{\delta^2}{\delta W_\mu^0 \delta c^3}$ Eq. (3·85)$\Big|_0$:  (3·145b)

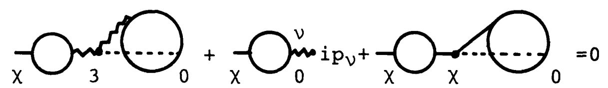
$\dfrac{\delta^2}{\delta \chi^3 \delta c^3}$ Eq. (3·85)$\Big|_0$:  (3·145c)

$\dfrac{\delta^2}{\delta W_\mu^3 \delta c^0}$ Eq. (3·85)$\Big|_0$: 

(3·145d)

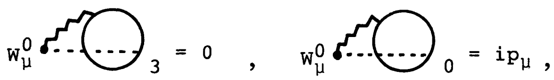
$\dfrac{\delta^2}{\delta W_\mu^0 \delta c^0}$ Eq. (3·85)$\Big|_0$: 

(3·145e)

$\dfrac{\delta^2}{\delta \chi^3 \delta c^0}$ Eq. (3·85)$\Big|_0$: 

(3·145f)

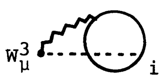
where we have used the explicit form of Green functions,

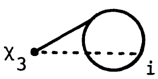
 (3·146)

derived from the BRS transform of $W_\mu{}^0$,

$$\delta^{\mathrm{BRS}} W_\mu^0 = \partial_\mu c^0 \,. \tag{3·147}$$

With the definitions of new invariant functions,

 $\equiv p_\mu G_{3i}(p^2)$, (3·148a)

 $\equiv G_{\chi i}(p^2)$, (3·148b)

Eqs. (3·145) are rewritten as follows:

$$B_3 G_{33} + iC_{3\chi} G_{\chi 3} = 0 \,, \tag{3·149a}$$

$$B_{30} G_{33} + iC_{0\chi} G_{\chi 3} = 0 \,, \tag{3·149b}$$

$$C_{3\chi}G_{33}+iF_3G_{\chi 3}=0\,, \tag{3·149c}$$

$$B_3G_{30}+iC_{3\chi}G_{\chi 3}+iB_{30}=0\,, \tag{3·149d}$$

$$B_{30}G_{30}+iC_{0\chi}G_{\chi 0}+iB_0=0\,, \tag{3·149e}$$

$$C_{3\chi}G_{30}+iF_3G_{\chi 0}+iC_{0\chi}=0\,. \tag{3·149f}$$

In these identities five equations are independent of each other:

$$B_3F_3=(C_{3\chi})^2\,, \tag{3·150a}$$

$$B_3B_0=(B_{30})^2\,, \tag{3·150b}$$

$$B_3C_{0\chi}=B_{30}C_{3\chi}\,, \tag{3·150c}$$

$$G_{\chi 3}=iG_{33}B_3/C_{3\chi}\,, \tag{3·150d}$$

$$G_{\chi 0}=(iG_{30}B_3-B_{30})/C_{3\chi}\,. \tag{3·150e}$$

From the second type of the WT identities (2·119),

$$\partial_\mu\frac{\delta\tilde{\Gamma}}{\delta K_\mu{}^3}-\alpha_3\omega\cdot\partial_\mu\frac{\delta\tilde{\Gamma}}{\delta K_\mu{}^0}-\alpha_3X_3\cdot\frac{\delta\tilde{\Gamma}}{\delta K_{\chi_3}}+\frac{\delta\tilde{\Gamma}}{\delta\bar{c}^3}=0\,, \tag{3·151a}$$

$$\partial_\mu\frac{\delta\tilde{\Gamma}}{\delta K_\mu{}^0}+\frac{\delta\tilde{\Gamma}}{\delta\bar{c}^0}=0\,, \tag{3·151b}$$

we have the following identities of the ghost inverse propagator with suitable operations:

$$\frac{\delta}{\delta c^3}\,\mathrm{Eq.}(3\cdot 151\mathrm{a})\Big|_0:\quad ip^2G_{33}+\alpha_3\tilde{X}_3G_{\chi 3}=\gamma_{33}\,, \tag{3·152a}$$

$$\frac{\delta}{\delta c^0}\,\mathrm{Eq.}(3\cdot 151\mathrm{a})\Big|_0:\quad ip^2G_{30}+\alpha_\chi\tilde{\omega}_3p^2+\alpha_3\tilde{X}_3G_{\chi 0}=\gamma_{03}\,, \tag{3·152b}$$

$$\frac{\delta}{\delta c^3}\,\mathrm{Eq.}(3\cdot 151\mathrm{b})\Big|_0:\quad 0=\gamma_{30}\,, \tag{3·152c}$$

$$\frac{\delta}{\delta c^0}\,\mathrm{Eq.}(3\cdot 151\mathrm{b})\Big|_0:\quad -p^2=\gamma_{00}\,. \tag{3·152d}$$

The ghost inverse propagator is expressed as

$$\Gamma_c^{(2)}=\begin{array}{c||c|c} & \bar{c}^3 & \bar{c}^0 \\ \hline\hline c^3 & i(p^2+\alpha_3\tilde{X}_3B_3/C_{3\chi})G_{33} & 0 \\ \hline c^0 & \begin{array}{l} i(p^2+\alpha_3\tilde{X}_3B_3/C_{3\chi})(G_{30}+iB_{30}/B_3) \\ +ip^2(B_{30}/B_3-\alpha_3\tilde{\omega})\end{array} & -p^2 \end{array}\,, \tag{3·153}$$

where we have replaced $G_{\chi 3}$ and $G_{\chi 0}$ by the right-hand side of Eqs. (3·150d, e). This expression will play an important role in the proof of charge universality (§ 3. 4).

We clarify the pole structure of unphysical mode in this sector. In order to cancel the mixing terms between vector and scalar in $\Gamma^{(2)}$,

$$\Gamma^{(2)}=\widetilde{\Gamma}^{(2)}+\Gamma^{(2)}_{\mathrm{GF}}\,, \tag{3·154}$$

we choose the following gauge fixing conditions,

$$\widetilde{X}_3(p^2)=-C_{3\chi}(p^2)\,, \tag{3·155a}$$

$$\widetilde{\omega}(p^2)=-C_{0\chi}(p^2)/\alpha_3 C_{3\chi}(p^2)\,. \tag{3·155b}$$

In this gauge $\Gamma^{(2)}$ and $\Gamma_c{}^{(2)}$ become

$\Gamma^{(2)}=$

	W_ν^3	W_ν^0	χ_3
W_μ^3	$A_3T_{\mu\nu}+\left(B_3-\dfrac{p^2}{\alpha_3}\right)L_{\mu\nu}$	$A_{30}T_{\mu\nu}+\left(B_{30}-\dfrac{p^2B_{30}}{\alpha_3B_3}\right)L_{\mu\nu}$	0
W_μ^0	$A_{30}T_{\mu\nu}+\left(B_{30}-\dfrac{p^2B_{30}}{\alpha_3B_3}\right)L_{\mu\nu}$	$A_0T_{\mu\nu}+\left(B_0-\dfrac{p^2B_0}{\alpha_3B_3}-\dfrac{p^2}{\alpha_0}\right)L_{\mu\nu}$	0
χ_3	0	0	$(p^2-\alpha_3B_3)F_3$

, (3·156)

$\Gamma_c{}^{(2)}=$

	$\bar{c}^3$	$\bar{c}^0$
c^3	$i(p^2-\alpha_3B_3)G_{33}$	0
c^0	$(p^2-\alpha_3B_3)\left(G_{30}+i\dfrac{B_{30}}{B_3}\right)$	$-p^2$

, (3·157)

where use has been made of Eq. (3·150). The determinant of the invariant functions of $L_{\mu\nu}$ part is

$$\det L_{\mu\nu}=\frac{1}{\alpha_3\alpha_0}p^2(p^2-\alpha_3B_3)\,, \tag{3·158}$$

and that of the ghost part is

$$\det \Gamma_c{}^{(2)}=p^2(p^2-\alpha_3B_3)G_{33}\,. \tag{3·159}$$

We have obtained two zeros in these unphysical modes, that is, one at $p^2=0$ and the other at the root of $p^2-\alpha_3B_3(p^2)=0$, which we call m^2. There are two quartets in this neutral sector: One quartet at $p^2=0$ consists of c_A, $\bar{c}_A$,

$L^A_{\mu\nu}$ and the longitudinal polarization in $T^A_{\mu\nu}$, and the other one at $p^2=m^2$ consists of c_Z, $\bar{c}_Z$, $L^Z_{\mu\nu}$ and χ_3.

Next we investigate the on-shell renormalization conditions for A (photon)-Z boson system. The renormalized inverse propagator of this sector is defined as follows,

$$\begin{aligned}\Gamma^{(2)\mathrm{R}}_{ij} &\equiv \mathrm{FT}\frac{\delta^2\Gamma}{\delta R_i \delta R_j} \\ &= (Z^{1/2}_{im})^t\,\mathrm{FT}\frac{\delta^2\Gamma}{\delta B_m \delta B_n} Z^{1/2}_{nj} \\ &= \{(Z^{1/2})^t \Gamma^{(2)} Z^{1/2}\}_{ij},\end{aligned} \tag{3·160}$$

where R_i and B_i represent renormalized and bare fields respectively:

$$R_i:\ R_Z = Z, \quad R_A = A\,,$$

$$B_i:\ B_3 = W_3, \quad B_0 = W_0\,.$$

The matrix $Z^{1/2}_{ij}$ in Eq. (3·160) connects R_i and B_i:

$$B_i = Z^{1/2}_{ij} R_j\,. \tag{3·161}$$

In the tree approximation the $Z^{1/2}_{ij}$ matrix is the well-known mixing matrix,

$$Z^{1/2} = \begin{pmatrix} \cos\theta_\mathrm{W} & \sin\theta_\mathrm{W} \\ -\sin\theta_\mathrm{W} & \cos\theta_\mathrm{W} \end{pmatrix}. \tag{3·162}$$

The on-shell renormalization conditions are imposed as follows (see Eq. (3·83)),

$$A^A_\mathrm{R}(0) = 0\,, \qquad A^{ZA}_\mathrm{R}(0) = A^{AZ}_\mathrm{R}(0) = 0\,, \tag{3·163a}$$

$$A^{ZA}_\mathrm{R}(M_Z^2) = A^{AZ}_\mathrm{R}(M_Z^2) = 0\,, \qquad A^Z_\mathrm{R}(M_Z^2) = 0\,, \tag{3·163b}$$

$$A^{A\prime}_\mathrm{R}(0) = -1\,, \qquad A^{Z\prime}_\mathrm{R}(M_Z^2) = -1\,, \tag{3·164}$$

where $A^{ij}_\mathrm{R}(p^2)$ is invariant functions of $T_{\mu\nu}$ part of $\Gamma^{(2)\mathrm{R}}_{ij}$ parametrized similarly in Eq. (3·142). There are six conditions while the number of adjustable renormalization constants are four of $Z^{1/2}_{ij}$ factor and one of δM_Z^2. Therefore in order to satisfy six conditions there must be an identity between invariant functions. We first recognize the non-pole conditions in the inverse propagator,

$$\begin{aligned} A_3(0) &= B_3(0)\,, \\ A_{30}(0) &= B_{30}(0)\,, \\ A_0(0) &= B_0(0)\,, \end{aligned} \tag{3·165}$$

which are similar to Eq. (3·128). The WT identities (3·150b) shows

$$\det \begin{vmatrix} B_3(0) & B_{30}(0) \\ B_{30}(0) & B_0(0) \end{vmatrix} = 0\,, \tag{3·166}$$

and hence,

$$\det \begin{vmatrix} A_3(0) & A_{30}(0) \\ A_{30}(0) & A_0(0) \end{vmatrix} = 0\,. \tag{3·167}$$

This relation guarantees the existence of a massless pole in this sector. For the renormalized amplitudes the same type of identity is deduced as

$$\det \begin{vmatrix} A_{\mathrm{R}}^{Z}(0) & A_{\mathrm{R}}^{ZA}(0) \\ A_{\mathrm{R}}^{AZ}(0) & A_{\mathrm{R}}^{A}(0) \end{vmatrix} = 0\,, \tag{3·168}$$

where we have assumed

$$\det Z_{ij}^{1/2} \neq 0\,. \tag{3·169}$$

Due to the identity (3·168) two conditions (3·163a) are not independent. Consequently the number of independent renormalization conditions is five and is equal to the degrees of freedom of counter terms. The existence of massless pole is a direct result of gauge invariance; an unbroken gauge invariance guarantees the existence of a massless gauge boson.

Let us comment on the structure of the $Z_{ij}^{1/2}$ matrix. This matrix was once defined as an orthogonal type of matrix in a previous literature [Ross and Taylor 73]. As a result it was claimed that the on-shell renormalization conditions could not be satisfied for Z-A mixing sector. We understand that

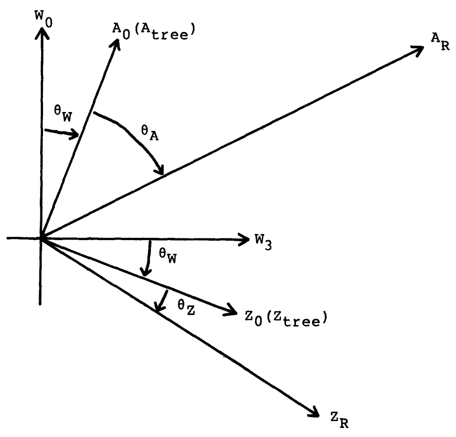


Fig. 3. 1. Schematical diagram of the relation between renormalized on-shell fields and bare fields.

the degrees of freedom of this $Z^{1/2}$ factor were not enough. We allow four degrees of freedom to $Z_{ij}^{1/2}$ and we can satisfy the complete on-shell renormalization conditions. The circumstance that renormalized on-shell fields are not obtained by rotationlike transformation of bare fields is schematically shown in Fig. 3.1. It should be mentioned that angles θ_A and θ_Z in Fig. 3.1 are not physical quantities which could be calculated perturbatively.

Let us investigate details of our renormalization scheme from another point of view which we have mentioned in § 3.2.3. We denote the matrices of the invariant functions of $T_{\mu\nu}$ part in $\Gamma_{ij}^{(2)}$ and $\Gamma_{ij}^{(2)\mathrm{R}}$ by $T_{ij}(p^2)$ and $T_{ij}^{\mathrm{R}}(p^2)$ respectively,

$$T_{ij}(p^2)=\begin{pmatrix} A_3(p^2) & A_{30}(p^2) \\ A_{30}(p^2) & A_0(p^2) \end{pmatrix},$$

$$T_{ij}^{\mathrm{R}}(p^2)=\begin{pmatrix} A_{\mathrm{R}}^{Z}(p^2) & A_{\mathrm{R}}^{ZA}(p^2) \\ A_{\mathrm{R}}^{ZA}(p^2) & A_{\mathrm{R}}^{A}(p^2) \end{pmatrix}. \tag{3·170}$$

The on-shell renormalization conditions (3·163) are expressed in terms of $T_{ij}^{\mathrm{R}}(p^2)$,

$$T_{ij}^{\mathrm{R}}(0)\begin{pmatrix} 0 \\ 1 \end{pmatrix}_j=0\,, \tag{3·171a}$$

$$T_{ij}^{\mathrm{R}}(M_Z^2)\begin{pmatrix} 1 \\ 0 \end{pmatrix}_j=0\,. \tag{3·171b}$$

These equations are rewritten as

$$T_{ij}(0)\,Z_{jA}^{1/2}=0\,, \tag{3·172a}$$

$$T_{ij}(M_Z^2)\,Z_{iZ}^{1/2}=0\,. \tag{3·172b}$$

We find that the first type of the on-shell renormalization conditions (3·163) is nothing but the eigenvalue equations of T_{ij} with zero eigenvalue. Their eigenvectors correspond to $Z_{ij}^{1/2}$ matrix. With this interpretation the second type of conditions (3·164) may be regarded as normalization conditions for eigenvectors.

The eigenvalue equations are formally solved:

$$Z_{jA}^{1/2}=f_A\begin{pmatrix} A_0(0) \\ -A_{30}(0) \end{pmatrix}=f_A\begin{pmatrix} B_0(0) \\ -B_{30}(0) \end{pmatrix}, \tag{3·173a}$$

$$Z_{jZ}^{1/2}=f_Z\begin{pmatrix} A_0(M_Z^2) \\ -A_{30}(M_Z^2) \end{pmatrix}. \tag{3·173b}$$

We determine the normalization factors f_A and f_Z. From the renormalization conditions (3·164) written in the forms,

$$\frac{\partial}{\partial p^2}\{(Z^{1/2})^t T Z^{1/2}\}_{ZZ}\Big|_{p^2=M_Z^2} = -1\,, \tag{3·174a}$$

$$\frac{\partial}{\partial p^2}\{(Z^{1/2})^t T Z^{1/2}\}_{AA}\Big|_{p^2=0} = -1, \tag{3·174b}$$

we obtain

$$f_Z = \{2A_0 A_{30} A_{30}' - (A_0)^2 A_3' - (A_{30})^2 A_0'\}^{-1/2}|_{p^2=M_Z^2}\,, \tag{3·175a}$$

$$f_A = \{2A_0 A_{30} A_{30}' - (A_0)^2 A_3' - (A_{30})^2 A_0'\}^{-1/2}|_{p^2=0}\quad . \tag{3·175b}$$

Equations (3·175) correspond to the following relation in QED:

$$Z_3^{1/2} = \{-A_0'(0)\}^{-1/2}\,. \tag{3·176}$$

In this section we have investigated the structure of two-point functions and explicitly proved that the on-shell renormalization conditions defined in § 3. 2 are consistently imposed even in the *Z-A* mixing sector with the aid of the WT identities.

§ 3. 4 Charge universality

In this section the electric charge universality is investigated. We mean by the electric charge universality that the on-shell coupling constant of the photon and a particle is proportional to the bare coupling constant in a matter-independent way. In a practical case, for example, the absolute value of the bare coupling constant of the proton is equal to that of the electron. Hence in this case the charge universality means that the proton and the electron have the same physical charge which is the on-shell coupling strength to the photon. This universality should hold in any model because the agreement between the charge of the proton and that of the electron is verified experimentally with quite good accuracy.

3. 4. 1 *What is the problem?*

In QED the charge universality is obtained as follows. The Lagrangian including gauge fixing terms,

$$\mathcal{L} = -\frac{1}{4}F^0_{\mu\nu}F^{0\mu\nu} + B^0\partial_\mu A^{0\mu} + \frac{\alpha_0}{2}B^0B^0$$
$$+\sum_i \bar{\psi}_i^{\,0}(i\partial_\mu\gamma^\mu + q_i e^0 A_\mu^{\,0}\gamma^\mu - m_i)\psi_i^{\,0}\,, \tag{3·177}$$

has a global $U(1)$ invariance and leads to the conserved current:

$$j_\mu = \sum q_i e_0 \overline{\psi}_i^0 \gamma_\mu \psi_i^0 , \tag{3·178}$$

$$\partial^\mu j_\mu = 0 . \tag{3·179}$$

The charge defined by

$$Q \equiv \int j_0(x) d^3x , \tag{3·180}$$

generates a corresponding $U(1)$ transformation of fields ψ_i^0 such that

$$[\psi_i^0, Q] = q_i e_0 \psi_i^0 . \tag{3·181}$$

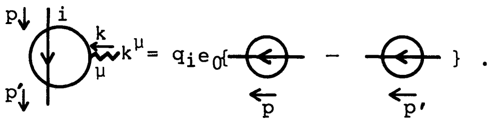
From the conservation law (3·179) a trivial identity holds:

$$\langle 0 | \mathrm{T} \partial^\mu j_\mu(x) \psi_i^0(y) \overline{\psi}_i^0(z) | 0 \rangle = 0 . \tag{3·182}$$

With the aid of the equations of motion and the equal time commutators, we obtain a Ward-Takahashi (WT) identity in the momentum representation [Takahashi 57],

$$\Gamma_i^{0\nu} k_\nu = q_i e_0 \{ S_i^{0-1}(p) - S_i^{0-1}(p') \} , \tag{3·183}$$

where Γ^0 and S^{-1} represent the photon proper vertex and the inverse propagator, respectively. This relation is graphically expressed as follows (see § 2.5):



(3·184)

For renormalized amplitudes the WT identity (3·183) takes the form,

$$\Gamma_i^\nu k_\nu = Z_3^{1/2} q_i e_0 \{ S_i^{-1}(p) - S_i^{-1}(p') \} . \tag{3·185}$$

With the operation $\partial / \partial k_\mu |_{k=0, p^2=m_i^2}$, we obtain the on-shell photon coupling as follows [Ward 50]:

$$\Gamma_i^\mu = \gamma^\mu Z_3^{1/2} q_i e_0 , \tag{3·186}$$

where use has been made of the on-shell renormalization condition for $S^{-1}(p)$:

$$\frac{\partial}{\partial \not{p}} S^{-1}(p) \bigg|_{\text{on-shell}} = -1 . \tag{3·187}$$

The expression (3·186) assures the charge universality because the proportionality factor $Z_3^{1/2}$ is independent of i. Thus in QED the charge universality holds in a trivial way according to the property that the photon couples to the conserved current j^μ which generates the $U(1)$ gauge transformation.

In the Weinberg-Salam theory the situation is different, that is, the current to which the photon couples is not conserved,

$$\partial^{\mu} j_{\mu}{}^{\rm WS} = O_1 \neq 0\,, \tag{3·188}$$

and it is not equal to the $U(1)_{\rm QED}$ generator current J:

$$j_{\mu}{}^{\rm WS} - J_{\mu} = O_2 \neq 0\,. \tag{3·189}$$

Therefore the proof of the charge universality based on the WT identities is non-trivial. Here it should be noted that Eqs. (3·188) and (3·189) do not bring about physical difficulties. Operators $O_{1,2}$ are expressed in any gauge as

$$O_{1,2} = \{Q_{\rm B}, \bar{O}_{1,2}\}\,, \tag{3·190}$$

that is, the BRS transform of appropriate operators $\bar{O}_{1,2}$. Such a type of operator does not develop the expectation value in a physical state because of the physical state condition,

$$Q_{\rm B}|{\rm phys}\rangle = 0\,. \tag{2·47}$$

In other words, $O_{1,2}$ are practically equivalent to null operators.

We comment on the unitary gauge, which is given by the limit in which the gauge parameter α goes to infinity in the 't Hooft gauge (3·103). In this gauge the mass of unphysical quartet $(B, \chi, c, \bar{c})$ is αM and diverges in this limit. Following the decoupling argument, the Lagrangian in the unitary gauge is simply obtained by neglecting all unphysical fields relevant to the broken symmetries. In this gauge O_1 and O_2 equal zero due to the local $U(1)_{\rm QED}$ invariance of the Lagrangian except for the gauge fixing part for the photon. This local invariance does not exist in a usual covariant gauge because the gauge fixing terms for $W^{\pm}$ bosons break it. The proof of the charge universality in this gauge is trivial as in QED apart from a little modification as follows. With the use of the relation

$$\partial^{\mu} j_{\mu}^{\rm WS}[{\rm unitary}] = 0\,, \tag{3·191a}$$

$$j_{\mu}^{\rm WS}[{\rm unitary}] = J_{\mu}\,, \tag{3·191b}$$

we have the same WT identity as Eq. (3·183),

$$\Gamma^{0\nu}_{Ai} k_{\nu} = q_i e_0 \{S_i^{0-1}(p) - S_i^{0-1}(p')\}\,. \tag{3·192}$$

The renormalized photon vertex is obtained as follows,

$$\begin{aligned}\Gamma^{\nu}_{Ai} k_{\nu} = {} & Z^{1/2}_{AA} q_i e_0 \{S_i^{-1}(p) - S_i^{-1}(p')\} \\ & + Z^{1/2}_{ZA} \times [\text{bare } Z \text{ boson vertex}]\,,\end{aligned} \tag{3·193}$$

where $Z^{1/2}_{AA}$ and $Z^{1/2}_{ZA}$ are components of the matrix renormalization constant defined in Eq. (4·27). Because of the Z-A mixing the second term in Eq.

(3·193) appears. The conservation law (3·191a) guarantees the following identities of two-point functions defined in § 4. 3. 2,

$$A_{AA}(0)=0, \quad A_{ZA}(0)=0\,, \tag{3·194}$$

which imply (see § 3. 3. 2)

$$Z_{ZA}^{1/2}=0\,. \tag{3·195}$$

Using this property, we have the charge universality from Eq. (3·193):

$$\Gamma_{Ai}^{\mu}=\gamma^{\mu}Z_{AA}^{1/2}q_i e_0\,. \tag{3·196}$$

As is seen above, the WS theory in the unitary gauge can be regarded as the simplest extension of QED to include charged vector bosons $W_\mu{}^{\pm}$ and has properties similar to QED as far as the photon sector is concerned, although there may exist some complications in the renormalizability in this gauge.

Here we comment on the relation between the "charge universality"*) and the renormalizability. In § 2. 5 we have demonstrated the renormalizability with the use of the "charge universality" which is obtained from the WT identities. The renormalizability is related only to the divergent part of amplitudes. On the other hand, the "charge universality" is concerned with the relations among full amplitudes including all finite terms as well as infinite terms. Therefore the "charge universality" is sufficient but not necessary condition for the renormalizability. Even if the symmetry is spontaneously broken, the renormalizability holds with the same counter terms as those in symmetric theory (see § 2. 5), but the "charge universality" is broken in this case; for instance, the $W^{\pm}$ boson coupling constants are not universal, once higher order corrections are taken into account.

There are the following three cases for remaining symmetry in spontaneously broken gauge theory:

a) non-Abelian type,

b) pure $U(1)$ type,

c) $U(1)$ which includes non-Abelian components of the original theory.

In Case a) the proof of the "charge universality" holds as shown in § 2.5. Case b) essentially corresponds to pure QED and the proof of the "charge universality" is trivial as in QED. The actual QED in broken WS theory corresponds to Case c), that is, the photon field is the linear combination of $W_\mu{}^0(U(1))$ and $W_\mu{}^3(SU(2))$. In this case there are some difficulties for the proof of the charge universality. The inclusion of non-Abelian component in the photon causes $O_{1,2}\neq 0$ (see Eqs. (3·188) and (3·189)). Furthermore invariant tensors do not exist because the remaining symmetry is $U(1)$ instead of non-Abelian symmetry. Thus the charge universality in the WS theory

*) We denote the charge universality in a general case which is not restricted to the photon coupling by the "charge universality" (with quotation mark).

becomes non-trivial. We comment on another type of the breakdown of the "charge universality". Consider the case where an original symmetry G is broken to leave a symmetry $G_1 \times G_2$ (non-simple). The universal charge g_1 of G_1 gauge symmetry does not necessarily equal the universal charge g_2 of G_2. For example, the separating behavior of running coupling constants in grand unified theory corresponds to this type of the breakdown of the "charge universality".

In the following, first, we discuss the "charge universality" in the framework of S-matrix theory. Second, we give a concise proof of the charge universality with the use of the WT identities. Finally, we give an alternative straightforward proof with the aid of equations of motion and the physical state condition [Aoki 79].

3.4.2 *S-matrix theoretical approach*

In this subsection we give an S-matrix theoretical proof of the charge universality without referring to an explicit form of the Lagrangian. An original argument was given by Weinberg where the photon and the graviton were treated [Weinberg 64]. Here we generalize it for non-Abelian gauge bosons [Aoki 79]. A general theorem treating any massless particle with spin j (≥ 1) is given by Kugo-Uehara, which will be stated later [Kugo and Uehara 81].

Consider a G invariant theory including massless vector particles which belong to the adjoint representation of G, where G is a simple Lie group. The S-matrix element with emission of a massless vector boson is written as

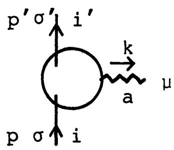
$$S = \varepsilon^{*\mu} M_\mu , \tag{3·197}$$

where $\varepsilon^{*\mu}$ is a polarization vector of the massless vector particle. The Lorentz invariance of S requires

$$k^\mu M_\mu = 0 , \tag{3·198}$$

where k^μ is the momentum of the emitted vector boson. This is because in a covariant field theory a Lorentz transformation of $\varepsilon^{*\mu}$ must be accompanied by the inhomogeneous term proportional to k^μ. This inhomogeneous term is the very result coming from masslessness of the particle. The "gauge invariance" of the S-matrix, Eq. (3·198), is required by the Lorentz invariance [Kugo and Uehara 81].

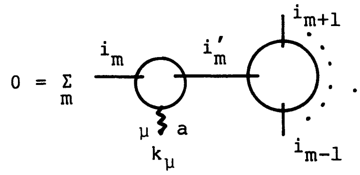
We define the on-shell charge g_m of a particle whose representation of G is $R(m)$, by referring to the on-shell S-matrix element as follows:



$$\varepsilon^*_\mu \xrightarrow{k^\mu \to 0} g_m (T^a_{R(m)})_{i'i}\, 2p^\mu \varepsilon^*_\mu \delta_{\sigma\sigma'} , \tag{3·199}$$

where $p^{(\prime)}$, $\sigma^{(\prime)}$ and $i^{(\prime)}$ represent the momentum, the spin and the component of the representation of G respectively, $T_R{}^a$ is the generator matrix of the representation R and a is the component of the emitted vector boson. In Eq. (3·199) we have omitted a trivial kinematical factor. Invariant amplitudes other than (3·199) must vanish in the limit $k \to 0$, which is shown later.

Consider M_μ including i_m ($m=1\sim N$) particle of $R(m)$ representation. In the limit $k\to 0$ in Eq. (3·198), similarly in the usual soft pion technique, non-vanishing contributions come from pole terms in M:



(3·200)

The intermediate propagator in each graph in Eq. (3·200) has a pole as

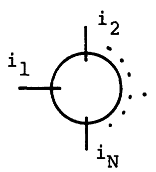
$$\frac{1}{(p_m+\eta_m k)^2-M_m^2} \sim \frac{\eta_m}{2p_m k},$$

$$\eta_m = \begin{cases} +1: \text{ for outgoing particle,} \\ -1: \text{ for incoming particle,} \end{cases} \tag{3·201}$$

where M_m is the mass of particle m. This pole cancels the external factor k_μ and we obtain the identity,

$$\sum_{m=1}^{N} \eta_m g_m (T^a_{R(m)})_{i_m i_m'} \Gamma_{i_1\cdots i_m'\cdots i_N} = 0\,, \tag{3·202}$$

where $\Gamma_{i_1\cdots i_N}$ is the S-matrix element without vector emission:

$\Gamma_{i_1\cdots i_N} \equiv$ 

(3·203)

We note here that $\Gamma_{i_1\cdots i_N}$ is proportional to an invariant G tensor and is invariant under an infinitesimal G transformation:

$$\sum_{m=1}^{N} \eta_m (T^a_{R(m)})_{i_m i_m'} \Gamma_{i_1\cdots i_m'\cdots i_N} = 0\,. \tag{3·204}$$

By comparing Eq. (3·202) with Eq. (3·204), we conclude the m independence of g_m:

$$g_m = g \qquad \text{for all } m\,. \tag{3·205}$$

This indicates the "charge universality". One can also understand that the on-shell coupling has the unique form (3·199) because other forms of amplitudes must break the gauge invariance of the S-matrix (3·200). This type of argument, however, does not give any information about quartic couplings which is also indispensable for the proof of the renormalizability (see Eq. (2·199)). Of course the above S-matrix theoretical approach is not applied to unphysical particles.

If the symmetry group G is not simple but $U(1)$, we obtain

$$\sum_{m=1}^{N} \eta_m g_m = 0\,, \tag{3·206}$$

$$\sum_{m=1}^{N} \eta_m g_m^0 = 0\,, \tag{3·207}$$

instead of Eqs. (3·202) and (3·204), where g_m^0 is the bare coupling constant of particle m. Since these equations hold for any S-matrix element, we may conclude

$$g_m \propto g_m^0 \tag{3·208}$$

also in this $U(1)$ case. In the case where there is some other symmetry independent of G, however, we cannot completely deduce the charge universality (3·208). For instance, if the electron number and the muon number conserve independently, we cannot conclude the equality of the electron charge and muon charge. Furthermore even if there is $\mu \to e\gamma$ process, we cannot deduce the vanishing charge of neutrinos in the case where the fermion number conservation holds.

By treating an emission of a massless particle with spin $j \geq 1$, the general gauge invariance of the S-matrix is obtained and the existence of the corresponding conserved charge with tensorial rank $j-1$ is concluded [Kugo and Uehara 81]. For example, massless particles with spin 1, 3/2 and 2 (graviton) imply scalar charge Q, super charge Q_α and vector charge Q_μ (energy-momentum P_μ) respectively. These conserved charges are related to the corresponding symmetries in the Lagrangian formalism. The proportionality between the charges and the quantum numbers is implied in every case; especially in the case of graviton, it means the equality between the gravitational and the inertial mass.

To sum up the S-matrix theoretical argument, the Lorentz invariance of the S-matrix element including massless vector is guaranteed by the cancellation among poles existing in the contribution of one particle intermediate state, and this cancellation implies the on-shell charge conservation and further, the "charge universality".

3. 4. 3 *Proof by the use of the WT identities*

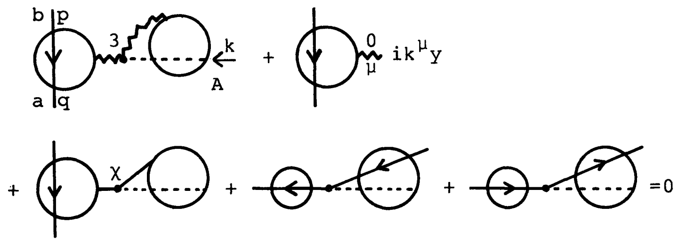
We give a proof of the charge universality in the WS theory with the use of the WT identities [Aoki 79]. As the proof is non-trivial and very lengthy as mentioned in § 3. 4. 1, we present it in a concise form. We start with the differentiation of the WT identity as follows:

$$\frac{\delta^3}{\delta\overline{\psi}_a\delta\psi_b\delta c_A}\ \text{Eq.}(2\cdot120)\bigg|_0\,, \tag{3·209}$$

where ψ and $\overline{\psi}$ are renormalized fields and $\delta/\delta c_A$ is defined by

$$\frac{\delta}{\delta c_A}\equiv x\frac{\delta}{\delta c_3}+y\frac{\delta}{\delta c_0}\,. \tag{3·210}$$

The coefficients x and y in Eq. (3·210) are determined later. Equation (3·209) is expressed graphically as



, (3·211)

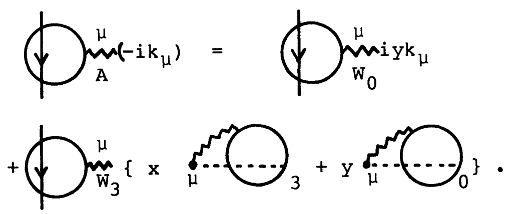
where we have assumed for simplicity that ϕ does not mix with the photon sector. According to the renormalization of ψ and $\overline{\psi}$, we introduce the renormalized BRS source currents K_ψ and $K_{\overline{\psi}}$ satisfying

$$\frac{\delta}{\delta K^0_{\psi_a}}\cdot\frac{\delta}{\delta{\psi_a}^0}=\frac{\delta}{\delta K_{\psi_a}}\cdot\frac{\delta}{\delta\psi_a}\,,$$

$$\frac{\delta}{\delta K^0_{\overline{\psi}_a}}\cdot\frac{\delta}{\delta{\overline{\psi}_a}^0}=\frac{\delta}{\delta K_{\overline{\psi}_a}}\cdot\frac{\delta}{\delta\overline{\psi}_a}\,. \tag{3·212}$$

All graphs in (3·211) represent those obtained by the differentiation with respect to renormalized quantities $(\psi, \overline{\psi}, K_\psi, K_{\overline{\psi}})$.

First, we determine the coefficients x and y so that the sum of the first two terms in Eq. (3·211) becomes the renormalized physical photon vertex in the limit $k^2\to 0$:



(3·213)

By the use of invariant functions defined in § 3. 3. 2, we obtain

$$x=-f_A\{iB_0(0)+B_{30}(0)G_{30}(0)\}/G_{33}(0)\,, \tag{3·214a}$$

$$y=f_AB_{30}(0)\,, \tag{3·214b}$$

where the formal solution of the renormalized photon channel (3·173a) is used. With the aid of the WT identities (3·150), these equations are re-written as

$$x = if_A C_{0\chi}(0) G_{\chi 0}(0)/G_{33}(0), \qquad (3\cdot 215\text{a})$$

$$y = -if_A C_{0\chi}(0) G_{\chi 3}(0)/G_{33}(0). \qquad (3\cdot 215\text{b})$$

From Eqs. (3·215a) and (3·215b) we have

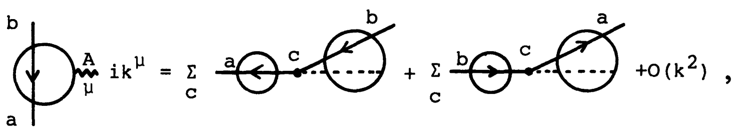
$$xG_{\chi 3}(0) + yG_{\chi 0}(0) = 0, \qquad (3\cdot 216)$$

which represents that the scalar mixing contribution $G_{\chi A}$ (the third term in Eq. (3·211)) vanishes in the limit $k^2 \to 0$. With the use of Eq. (3·153), we have also

$$x\gamma_{33}(0) + y\gamma_{03}(0) = 0, \qquad (3\cdot 217)$$

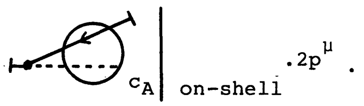
which means that c_A corresponds to the massless channel of ghosts. In short, the renormalized photon channel is obtained by the differentiation with respect to the massless channel of ghosts c_A.

The resultant WT identity is



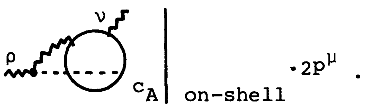
(3·218)

which resembles that of a symmetric theory (2·181). There is, however, a difference between them regarding the existence of particle mixing. This is the critical point to make the proof very lengthy. With the extensive analysis of the structure of on-shell renormalized inverse propagators and other invariant functions, one can derive the following relation,

$\Gamma^{\mu}_{\text{on-shell}} =$ 

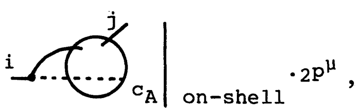
(3·219)

For other particles than fermions, we obtain the on-shell charge in a similar way. For example, for the W^+ boson we have

$\Gamma^{\mu,\rho\nu}_{W^+\text{on-shell}} =$ 

(3·220)

We easily generalize the above results as

$\Gamma^{\mu,ij}_{\text{on-shell}} =$ 

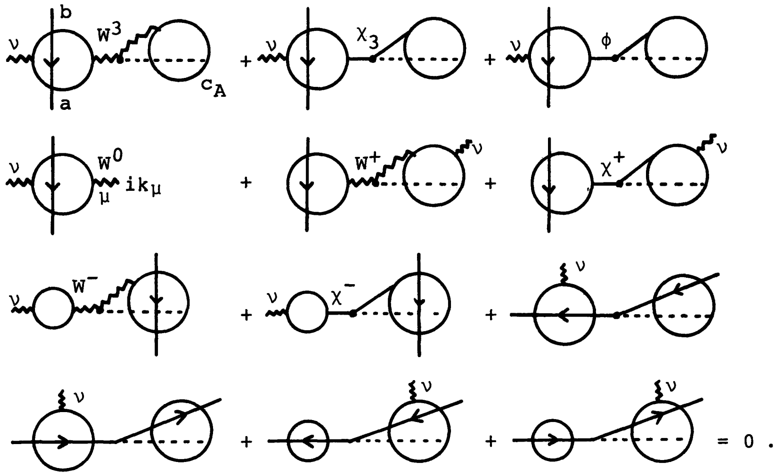
(3·221)

which we call the (renormalized) charge graph.

In order to evaluate the charge graph we proceed to a fourth derivative of the WT identity (2·120). For fermions, for instance, applying the operation,

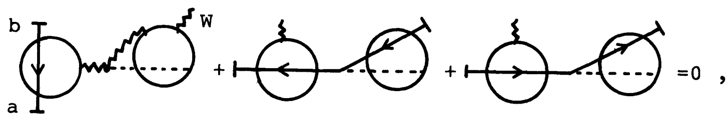
$$\frac{\delta^4}{\delta\bar{\psi}_a\delta\psi_b\delta W_\nu^+\delta c_A}\cdot\Big|_0 , \tag{3·222}$$

we have the following identity,



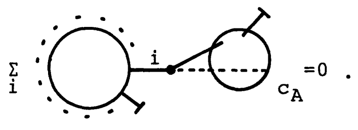
(3·223)

By putting all external lines of the above amplitudes on mass-shell, one finds Eq. (3·223) reduces to



(3·224)

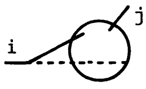
which is similar to Eq. (2·190). We can generalize Eq. (3·224) for the case of arbitrary number of external lines to obtain,



(3·225)

This is nothing but the on-shell charge conservation law (3·206).

To prove the charge universality, we must extract the $U(1)_{\mathrm{QED}}$ quantum number q_i from the charge graph leaving a matter independent factor. In the symmetric theory with simple group, we have obtained the "charge universality" easily because there exists invariant tensors in that case (see Eqs. (2·191) and (3·204)). In the case of pure $U(1)$ symmetry, we can evaluate the charge graph directly as follows. The ghost c_A corresponds to the $U(1)$ channel and hence it is a free particle, which is related to the well-known fact that QED Lagrangian does not need a ghost field. Thus the charge graph is just a constant given by the coefficient of c_A term in the BRS transform of the relevant field:

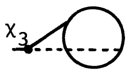


$$c_A = $$  $$= \delta_{ij} q_i e_0 \quad . \tag{3·226}$$

The photon in the WS theory corresponds to the $U(1)$ symmetry which is the combination of the original $U(1)$ and non-Abelian component. The above fact obstructs the gauge independent proof of the charge universality. We adopt the Landau gauge such as

$$\mathcal{L}_{\mathrm{GF}} = \sum_{a=0}^{3} B^a \partial^\mu W_\mu^a \, . \tag{3·227}$$

In this gauge some amplitudes are evaluated very easily because of the transversal structure of gauge boson propagators as follows. Consider, for example, the Green function $G_{\chi 3}$ (bare) which is graphically expressed as



$$c_3 \; , \tag{3·228}$$

where the dot represents the BRS transform of χ_3. The BRS transform of Higgs scalar ϕ is obtained by the following replacement in its gauge transform: gauge parameters $\to$ corresponding ghost fields (see (2·39)),

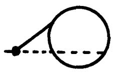
$$\delta^{\mathrm{BRS}} \Phi(x) = i\lambda g^a c^a(x) T^a \Phi(x) \, . \tag{3·229}$$

In terms of each component of Higgs field defined in Eq. (2·65), the BRS transform is given by

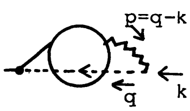
$$\delta\phi = -\lambda \frac{g}{2} \chi_a c^a + \lambda \frac{g'}{2} \chi_3 c^0 \, ,$$

$$\delta\chi_a = \lambda \frac{g}{2} [(v+\phi) c^a + f^{abc} \chi_b c^c] + \begin{cases} \lambda \dfrac{g'}{2} \chi_2 c^0 \, , & (a=1) \\ -\lambda \dfrac{g'}{2} \chi_1 c^0 \, , & (a=2) \\ -\lambda \dfrac{g'}{2} (v+\phi) c^0 \, , & (a=3) \end{cases} \tag{3·230}$$

where all quantities are bare ones. With the use of Eq. (3·230), $G_{\chi 3}$ is separated into two parts:



$$= \frac{1}{2} g_0 v_0 \; + $$



$$. \tag{3·231}$$

In the limit $k \to 0$ the second term including the gauge coupling with ghost fields vanishes. The reason for this is that ghost-gauge coupling constant has a factor q^ν (the ghost momentum), which is contracted with the gauge boson propagator in the Landau gauge,

$$\frac{g_{\mu\nu} - p_\mu p_\nu / p^2}{p^2}, \tag{3·232}$$

and vanish in the limit $k \to 0$. Thus we obtain

$$G_{\chi 3}(0) = \frac{1}{2} g_0 v_0, \tag{3·233}$$

and in a similar way we have

$$G_{\chi 0}(0) = -\frac{1}{2} g_0' v_0. \tag{3·234}$$

The differentiation with respect to c_A is, in this gauge, given by

$$\frac{\delta}{\delta c_A} = -i f_A v_0 \frac{C_{0\chi}(0)}{2G_{33}(0)} \left(g_0' \frac{\delta}{\delta c_3} + g_0 \frac{\delta}{\delta c_0} \right), \tag{3·235}$$

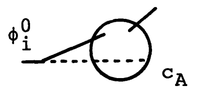
where use has been made of Eq. (3·215). The BRS transform of a field ϕ_i^0 is expressed as

$$\delta^{\mathrm{BRS}} \phi_i^0 = i \left(-i f_A v_0 \frac{C_{3\chi}(0)}{2G_{33}(0)} \right) g_0 g_0' q_i c_A \phi_i^0 + \cdots, \tag{3·236}$$

where q_i is the $U(1)_{\mathrm{QED}}$ generator:

$$q_i = \left(I_3 + \frac{Y}{2} \right)_i. \tag{3·237}$$

The bare charge graph of ϕ_i^0 is easily evaluated in a way similar to the evaluation of $G_{\chi 3}$ and we obtain



$$= -i f_A v_0 \frac{C_{0\chi}(0)}{2G_{33}(0)} g_0 g_0' q_1 \cdot \mathbf{1}, \tag{3·238}$$

where $\mathbf{1}$ is the unit matrix in all spin and flavour indices. The renormalized charge graph (3·221) equals the bare one (3·238) because of index-independence ($\mathbf{1}$) of Eq. (3·238). The proof is now completed with this equation. Other expressions of the on-shell charge are obtained,

$$\begin{aligned} e^i_{\text{on-shell}} &= Z_{0A}^{1/2} g_0' q_i \\ &= \frac{i}{G_{33}(0)} Z_{3A}^{1/2} g_0 q_i. \end{aligned} \tag{3·239}$$

They may be interesting in comparison with that of QED,

$$e^i_{\text{on-shell}}(\mathrm{QED}) = Z_3^{1/2} e_0 q_i. \tag{3·240}$$

3.4.4 *"Maxwell" equation and charge universality*

We investigate here the "Maxwell" equations, the equations of motion of gauge fields, to give another proof for the charge universality. For simplicity, gauge fixing terms are taken as

$$\mathcal{L}_{\mathrm{GF}} = -\partial_\mu B^a W_\mu^a + \frac{\alpha}{2} B^a B^a, \tag{3·241}$$

which preserve the global $SU(2)\times U(1)$ symmetry. In this gauge the equations of motion of $W_\mu{}^a$ are

$$\partial^\nu F_{\mu\nu}^a = gJ_\mu^a - \{Q_B, D_\mu \bar{c}^a\} \qquad \text{for } a=1, 2, 3, \tag{3·242}$$

$$\partial^\nu F_{\mu\nu}^0 = \frac{1}{2} g' J_\mu^0 - \{Q_B, \partial_\mu \bar{c}^0\}, \tag{3·243}$$

where $J_\mu{}^a$ $(a=0\sim 3)$ are the corresponding generator currents of the $SU(2)\times U(1)$ global symmetry. On the true vacuum only one unbroken symmetry is $U(1)_{\mathrm{QED}}$ whose charge is formally written as

$$Q = I_3 + \frac{1}{2} Y = Q^3 + \frac{1}{2} Q^0. \tag{3·244}$$

With this combination for the charge, "Maxwell" equations (3·242) can be rewritten as

$$\partial^\nu F_{\mu\nu} = e^0 \left(J_\mu^3 + \frac{1}{2} J_\mu^0\right) - N_\mu, \tag{3·245}$$

where we define

$$e^0 \equiv \frac{gg'}{\sqrt{g^2+g'^2}}, \tag{3·246}$$

$$F_{\mu\nu} \equiv \frac{1}{\sqrt{g^2+g'^2}} (g' F_{\mu\nu}^3 + g F_{\mu\nu}^0), \tag{3·247}$$

$$N_\mu \equiv \left\{ Q_{\mathrm{B}}, \frac{1}{\sqrt{g^2+g'^2}} [g'(D_\mu \bar{c})^3 + g\partial_\mu \bar{c}^0] \right\}. \tag{3·248}$$

Here we should notice both of the formal charges,

$$\tilde{Q} \equiv \int d^3x \left(J_0{}^3 + \frac{1}{2} J_0{}^0\right) \tag{3·249}$$

and

$$N \equiv \frac{1}{e^0} \int d^3x N_0, \tag{3·250}$$

are conserved and equally generate the $U(1)_{\mathrm{QED}}$ transformation, since the difference between them is a space integration of total divergence and is replaced by a surface term which does not contribute to commutators with local fields. In fact, neither of them is well-defined because of the massless one-particle contributions. The unbroken symmetry assures only one well-defined charge which is a certain combination of $\tilde{Q}$ and N,

$$Q \equiv \int d^3x\left[\left(J_0^3+\frac{1}{2}J_0^0\right)-\frac{1}{e^0}\zeta N_0\right]\frac{1}{1-\zeta}$$
$$\equiv \int d^3x J_0^Q\,, \tag{3·251}$$

where ζ is a parameter to be determined dynamically so that the massless pole cancels out. For instance, in QED this ζ parameter is easily evaluated to be $(1-Z_3^{1/2})$ because the massless pole in this case is B field which is free. We obtain from Eqs. (3·245) and (3·251),

$$\frac{1}{1-\zeta}\partial^\nu F_{\mu\nu}=e^0 J_\mu^Q - N_\mu\,. \tag{3·252}$$

This $1/(1-\zeta)$ factor is important to obtain the correct result in the following, although in the usual proof with differentiation of J_μ (see § 3. 4. 1), this factor is irrelevant.

Sandwiching Eq. (3·252) by physical one-particle states $|i\rangle$ and $|f\rangle$, we have

$$(1-\zeta)^{-1}\langle f|\partial^\mu F_{\mu\nu}|i\rangle = e^0\langle f|J_\mu^Q|i\rangle\,. \tag{3·253}$$

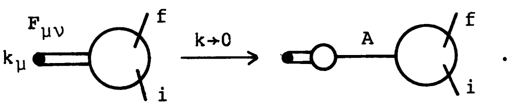
Note that N_μ does not contribute,

$$\langle f|N_\mu|i\rangle=\langle f|\{Q_{\rm B}, *\}|i\rangle=0\,, \tag{3·254}$$

due to the physical state condition:

$$Q_{\rm B}|i\rangle=0,\quad Q_{\rm B}|f\rangle=0\,. \tag{3·255}$$

We apply $\int d^4x e^{ikx}$ to both sides of Eq. (3·253) and take the limit $k\to 0$. On the left-hand side of Eq. (3·253), only the contribution coming from a massless particle in the $F_{\mu\nu}$ channel, namely, the photon A_μ, survives:



. (3·256)

It should be noted that a massless scalar particle such as B or χ cannot contribute to the $F_{\mu\nu}$ channel because $F_{\mu\nu}$ is antisymmetric in μ and ν. Thus the left-hand side reduces to the on-shell photon coupling up to a constant independent of matters i and f. The right-hand side ($\mu=0$) in this limit is nothing but the matrix element of the well-defined electromagnetic charge operator Q, that is, $q_i\delta_{fi}$, where q_i is the eigen value of the charge eigen state $|i\rangle$. Thus we have obtained the charge universality in a fairly simple way.

This simplification of the proof is due to the physical state condition (3·255). Furthermore the proof here does not limit the states $|i\rangle$ and $|f\rangle$ to those made out of elementary fields and can be applied to any physical particles including bound states. Thus we have got a proof for the equality of the electric charges of the proton and the electron, assuming that the proton is a bound state of the three quark channel, *uud*. Here we have seen an example to suggest that the canonical operator formalism with equations of motion is very useful and powerful to reveal essential structures of gauge theories.

To confirm that the above result agrees with the former results (3·238) and (3·239) obtained only by using the WT identities, we evaluate the proportionality constant of the on-shell charge to the bare charge. The one-photon contribution to the $F_{\mu\nu}$, C is estimated by the use of the asymptotic form of $F_{\mu\nu}$,

$$(F_{\mu\nu})^{\mathrm{as}}=C(\partial_\mu A_\nu{}^{\mathrm{as}}-\partial_\nu A_\mu{}^{\mathrm{as}})+\cdots, \tag{3·257}$$

where $A_\mu{}^{\mathrm{as}}$ is the asymptotic field of the physical photon and dots stand for other particles such as Z boson. Noting that Eq. (3·257) leads to

$$\mathrm{FT}\langle 0|\mathrm{T}F_{\mu\nu}(x)A_\rho(y)|0\rangle=iC(p_\mu g_{\nu\rho}-p_\nu g_{\mu\rho})\frac{1}{p^2}, \tag{3·258}$$

we have

$$\lim_{k\to 0}\int d^4x e^{ikx}\langle f|\partial^\nu F_{\nu\mu}(x)|i\rangle(1-\zeta)^{-1}$$
$$=-C(1-\zeta)^{-1}\lim_{p^f\to p^i}(2\pi)^4\delta^4(p^f-p^i)\cdot e_{fi}(p^f+p^i)_\mu, \tag{3·259}$$

where the state normalization convention is

$$\langle f|i\rangle=(2\pi)^3 2p_0{}^i\delta_{fi}\delta^3(p^f-f^i). \tag{3·260}$$

The kinematical form $(p^f+p^i)_\mu$ is implied by the following expression of the right-hand side ($\mu=0$) in Eq. (3·253),

$$\lim_{k\to 0}\int d^4x e^{ikx}\langle f|e^0 J_0{}^Q(x)|i\rangle$$
$$=e^0 q_i\delta_{fi}\lim_{p^f\to p^i}(2\pi)^4\delta^4(p^f-p^i)\cdot 2p_0{}^i. \tag{3·261}$$

By comparing Eq. (3·259) with (3·261), the on-shell charge e_{fi} is represented by

$$e_{fi}=-C^{-1}(1-\zeta)\delta_{fi}e^0 q_i. \tag{3·262}$$

The factor $C^{-1}(1-\zeta)$ is evaluated as follows. From the equation of motion (3·252), this factor is given by the residue of the massless pole of the following Green function,

$$-\mathrm{FT}\langle 0|TA_\rho N_\mu|0\rangle\,, \tag{3·263}$$

where we have taken into account that $J_\mu{}^Q$ has no massless one-particle contribution. The Green function is easily estimated in the Landau gauge ($\alpha=0$ in $\mathcal{L}_{\rm GF}$ (3·241)) and the result is

$$-C^{-1}(1-\zeta)\,e^0=Z_{0A}^{1/2}g_0'\,. \tag{3·264}$$

This relation agrees with Eq. (3·239) obtained by the use of the WT identities.

Chapter 4

On-Shell Renormalization in Electroweak Theory. II

In this chapter we summarize the necessary machinery to carry out higher-order calculations practically in the framework of the on-shell renormalization in the Weinberg-Salam theory.

§ 4. 1 Structure of the full Lagrangian

The basic Lagrangian to start with is given by Eq. (2·86) together with Eqs. (2·18), (2·36a), (2·36b), (2·66), (2·87) and (2·88). For convenience we here present the full form of the Lagrangian once again:

$$\mathcal{L}=\mathcal{L}_{\mathrm{G}}+\mathcal{L}_{\mathrm{F}}+\mathcal{L}_{\mathrm{GF}}+\mathcal{L}_{\mathrm{FP}}+\mathcal{L}_{\mathrm{H}}+\mathcal{L}_{\mathrm{M}}\,, \tag{4·1}$$

$$\mathcal{L}_{\mathrm{G}}=-\frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu}_{a}\,, \tag{4·2}$$

$$\mathcal{L}_{\mathrm{F}}=i\sum_{i}\bar{L}_{i}\not{D}_{\mathrm{L}}L_{i}+i\sum_{n=i,I}\bar{R}_{n}\not{D}_{\mathrm{R}}R_{n}\,, \tag{4·3}$$

$$\mathcal{L}_{\mathrm{GF}}=B^{a}F_{i}{}^{a}\phi^{i}+\frac{\alpha}{2}(B^{a})^{2}\,, \tag{4·4}$$

$$\mathcal{L}_{\mathrm{FP}}=-\bar{c}^{a}\delta^{a}(F_{i}{}^{b}\phi^{i})\,c^{b}\,, \tag{4·5}$$

$$\mathcal{L}_{\mathrm{H}}=(D_{\mathrm{L}}{}^{\mu}\Phi)^{\dagger}(D_{\mathrm{L}\mu}\Phi)+\mu^{2}\Phi^{\dagger}\Phi-\lambda(\Phi^{\dagger}\Phi)^{2}\,, \tag{4·6}$$

$$\mathcal{L}_{\mathrm{M}}=-\sum_{i}f_{i}\bar{L}_{i}\Phi R_{i}-\sum_{I}f_{I}\bar{L}_{I}(i\tau_{2}\Phi^{*})R_{I}+\text{h.c.}\,, \tag{4·7}$$

where the definitions of $F^{a}_{\mu\nu}$, $D^{\mu}_{\mathrm{R,L}}$, $D_{\mu}{}^{ab}$, Φ, $L_{i,I}$ and $R_{i,I}$ are given in Eqs. (2·20), (2·21), (2·22), (2·37a), (2·65), (2·81) and (2·84) respectively. It is understood that all the fields and parameters in this Lagrangian are bare quantities. Following the procedure described in § 2. 3, we reexpress the fields in the basic Lagrangian in terms of bare physical fields (see Eq. (2·69)) and give a nonvanishing vacuum expectation value to the Higgs field. Then we choose the most convenient set of independent bare parameters. Introducing renormalization constants, we define renormalized fields and parameters and split the Lagrangian into tree parts and counter terms. At this stage we fix the gauge fixing term $\mathcal{L}_{\mathrm{GF}}$ so as to be convenient for the calculation of higher order corrections. Finally we determine the FP ghost term $\mathcal{L}_{\mathrm{FP}}$ by using the BRS transformation. The Feynman rules will be derived in § 4.2 from the tree Lagrangian and the counter terms will be tabulated in § 4.3.

The basic Lagrangian (4·1) reexpressed in terms of physical fields takes a lengthy expression which will be given in the following. Here the vacuum expectation value v of the Higgs field is introduced as a parameter. We shall discuss, in a moment, how v is determined in terms of the other parameters. The relations between the original bare fields $(W_\mu{}^a, \chi_1, \chi_2)$ and the bare physical fields $(W_\mu{}^\pm, Z_\mu, A_\mu, \chi^\pm)$ are given as in § 2.3 by

$$W_\mu{}^\pm = (W_\mu{}^1 \mp i W_\mu{}^2)/\sqrt{2}\,, \tag{4·8}$$

$$Z_\mu = (g W_\mu{}^3 - g' W_\mu{}^0)/\sqrt{g^2+g'^2}\,, \tag{4·9}$$

$$A_\mu = (g' W_\mu{}^3 + g W_\mu{}^0)/\sqrt{g^2+g'^2}\,, \tag{4·10}$$

$$\chi^\pm = (\chi_1 \mp i\chi_2)/\sqrt{2}\,. \tag{4·11}$$

The Lagrangian in terms of the bare physical fields takes the following expression:

$$\mathcal{L} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{M}} + \mathcal{L}_{\mathrm{GF}} + \tilde{\mathcal{L}}_{\mathrm{FP}}\,, \tag{4·12}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{G}} = &-\frac{1}{2}F^+_{\mu\nu}F^{-\mu\nu} - \frac{1}{4}F^Z_{\mu\nu}F^{Z\mu\nu} - \frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} \\
&+ \frac{ig}{\sqrt{g^2+g'^2}}(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma})\,[g\{(\partial_\alpha W_\beta{}^+)\,W_\gamma{}^- Z_\delta + (\partial_\alpha W_\beta{}^-)\,Z_\gamma W_\delta{}^+ \\
&\quad + (\partial_\alpha Z_\beta)\,W_\gamma{}^+ W_\delta{}^-\} \\
&\quad + g'\{(\partial_\alpha W_\beta{}^+)\,W_\gamma{}^- A_\delta + (\partial_\alpha W_\beta{}^-)\,A_\gamma W_\delta{}^+ + (\partial_\alpha A_\beta)\,W_\gamma{}^+ W_\delta{}^-\}] \\
&- \frac{g^2}{g^2+g'^2}[(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta})\,W_\alpha{}^+ W_\beta{}^- (g^2 Z_\gamma Z_\delta + g'^2 A_\gamma A_\delta) \\
&\qquad + gg'(2g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma})\,W_\alpha{}^+ W_\beta{}^- A_\gamma Z_\delta] \\
&+ \frac{g^2}{2}(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta})\,W_\alpha{}^+ W_\beta{}^+ W_\gamma{}^- W_\delta{}^-\,,
\end{aligned} \tag{4·13}$$

where $F^\pm_{\mu\nu}$, $F^Z_{\mu\nu}$ and $F^A_{\mu\nu}$ are given by

$$F^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu\,, \text{ etc.,} \tag{4·14}$$

$$\mathcal{L}_{\mathrm{F}} = \mathcal{L}^{\mathrm{Kin}}_{\mathrm{F}} + \mathcal{L}^{\mathrm{Int}}_{\mathrm{F}}\,, \tag{4·15}$$

$$\mathcal{L}^{\mathrm{Kin}}_{\mathrm{F}} = i\bar{\psi}_n \partial\!\!\!/ \psi_n\,, \quad (n = i, I) \tag{4·16}$$

$$\begin{aligned}
\mathcal{L}^{\mathrm{Int}}_{\mathrm{F}} = &\frac{g}{\sqrt{2}}\left(\bar{\psi}_I U^\dagger_{Ii}\partial^\mu \frac{1-\gamma_5}{5}\psi_i W_\mu{}^+ + \bar{\psi}_i U_{iI}\partial^\mu \frac{1-\gamma_5}{2}\psi_I W_\mu{}^-\right) \\
&+ \frac{gg'}{\sqrt{g^2+g'^2}} Q_n \bar{\psi}_n \gamma^\mu \psi_n A_\mu
\end{aligned}$$

$$+\frac{\sqrt{g^2+g'^2}}{2}\left\{\bar{\psi}_I\gamma^\mu\left(\frac{1-\gamma_5}{2}-\frac{2Q_I g'^2}{g^2+g'^2}\right)\psi_I-\bar{\psi}_i\gamma^\mu\left(\frac{1-\gamma_5}{2}+\frac{2Q_i g'^2}{g^2+g'^2}\right)\psi_i\right\}Z_\mu, \tag{4·17}$$

$$\mathcal{L}_{\rm H}=\mathcal{L}_{\rm H}^{(2)}+\mathcal{L}_{\rm H}^{(3)}+\mathcal{L}_{\rm H}^{(4)}+\mathcal{L}_{\rm V}, \tag{4·18}$$

$$\begin{aligned}\mathcal{L}_{\rm H}^{(2)}=&-\frac{1}{2}\phi\Box\phi-\frac{1}{2}\chi_3\Box\chi_3-\chi^+\Box\chi^-\\&+\frac{1}{4}g^2v^2W_\mu^+W^{-\mu}+\frac{1}{2}\frac{1}{4}(g^2+g'^2)v^2Z_\mu Z^\mu\\&-\frac{1}{2}gv(W_\mu^+\partial^\mu\chi^-+W_\mu^-\partial^\mu\chi^+)-\frac{1}{2}\sqrt{g^2+g'^2}\,vZ_\mu\partial^\mu\chi_3,\end{aligned} \tag{4·19}$$

$$\begin{aligned}\mathcal{L}_{\rm H}^{(3)}=&\frac{1}{2}g\,W_\mu^+(\chi^-\overleftrightarrow{\partial^\mu}\phi)+\frac{1}{2}g\,W_\mu^-(\chi^+\overleftrightarrow{\partial^\mu}\phi)\\&+\frac{i}{2}g\,W_\mu^+(\chi_3\overleftrightarrow{\partial^\mu}\chi^-)-\frac{i}{2}g\,W_\mu^-(\chi_3\overleftrightarrow{\partial^\mu}\chi^+)+\frac{1}{2}\sqrt{g^2+g'^2}\,Z_\mu(\chi_3\overleftrightarrow{\partial^\mu}\phi)\\&+\frac{i}{2}\frac{g^2-g'^2}{\sqrt{g^2+g'^2}}Z_\mu(\chi^-\overleftrightarrow{\partial^\mu}\chi^+)+\frac{gg'}{\sqrt{g^2+g'^2}}A_\mu(\chi^-\overleftrightarrow{\partial^\mu}\chi^+),\\&\qquad(A\overleftrightarrow{\partial^\mu}B\equiv A\cdot\partial^\mu B-\partial^\mu A\cdot B)\end{aligned} \tag{4·20}$$

$$\begin{aligned}\mathcal{L}_{\rm H}^{(4)}=&\frac{1}{4}g^2W_\mu^+W^{-\mu}(2v\phi+\phi\phi+2\chi^+\chi^-+\chi_3\chi_3)\\&+\frac{1}{4}\frac{(g^2-g'^2)^2}{g^2+g'^2}Z_\mu Z^\mu\chi^+\chi^-+\frac{1}{8}(g^2+g'^2)Z_\mu Z^\mu(2v\phi+\phi\phi+\chi_3\chi_3)\\&+\frac{g^2g'^2}{g^2+g'^2}A_\mu A^\mu\chi^+\chi^-+\frac{1}{2}\frac{gg'^2}{\sqrt{g^2+g'^2}}Z_\mu W^{+\mu}\chi^-(\chi_3+iv+i\phi)\\&+\frac{1}{2}\frac{gg'^2}{\sqrt{g^2+g'^2}}Z_\mu W^{-\mu}\chi^+(\chi_3-iv-i\phi)\\&-\frac{1}{2}\frac{g^2g'}{\sqrt{g^2+g'^2}}A_\mu W^{+\mu}\chi^-(\chi_3+iv+i\phi)\\&-\frac{1}{2}\frac{g^2g'}{\sqrt{g^2+g'^2}}A_\mu W^{-\mu}\chi^+(\chi_3-iv-i\phi)\\&+\frac{gg'(g^2-g'^2)}{g^2+g'^2}Z_\mu A^\mu\chi^+\chi^-,\end{aligned} \tag{4·21}$$

$$\begin{aligned}\mathcal{L}_{\rm V}=&\mu^2\Phi^\dagger\Phi-\lambda(\Phi^\dagger\Phi)^2\\=&v(\mu^2-\lambda v^2)\phi+(\mu^2-\lambda v^2)\chi^-\chi^+\end{aligned}$$

$$+\frac{1}{2}(\mu^2-\lambda v^2)\chi_3\chi_3+\frac{1}{2}(\mu^2-3\lambda v^2)\phi\phi$$
$$-2v\lambda(\phi\chi^-\chi^+)-v\lambda(\phi\chi_3\chi_3)-v\lambda(\phi\phi\phi)$$
$$-\lambda(\chi^-\chi^-\chi^+\chi^+)-\lambda(\chi^-\chi^+\chi_3\chi_3)-\lambda(\phi\phi\chi^-\chi^+)$$
$$-\frac{1}{4}\lambda(\chi_3\chi_3\chi_3\chi_3)-\frac{1}{4}\lambda(\phi\phi\phi\phi)-\frac{1}{2}\lambda(\phi\phi\chi_3\chi_3), \tag{4·22}$$

$$\mathcal{L}_{\mathrm{M}}=-\frac{f_n}{\sqrt{2}}v\bar{\psi}_n\psi_n-\frac{i}{2}\bar{\psi}_I U^\dagger_{Ii}\left\{(f_i-f_I)+(f_i+f_I)\gamma_5\right\}\psi_i\cdot\chi^+$$
$$-\frac{i}{2}\bar{\psi}_i U_{iI}\left\{(f_I-f_i)+(f_I+f_i)\gamma_5\right\}\psi_I\cdot\chi^-$$
$$-\frac{f_n}{\sqrt{2}}\bar{\psi}_n\psi_n\phi-\frac{if_n}{\sqrt{2}}\bar{\psi}_n\gamma_5\psi_n\chi_3\,. \tag{4·23}$$

The independent bare parameters present in the Lagrangian (4·1) are g, g', λ, μ^2, f_i and f_I. These are free parameters and are fixed only by confronting the theory with experiments. For this purpose it is convenient to choose, instead of the original parameters, another set of bare parameters corresponding to the following physical quantities: the unit of charge (e) and the masses of the W boson, Z boson, Higgs boson and fermions (M_W, M_Z, m_ϕ, m_i and m_I). The new bare parameters corresponding to these physical quantities will be called the bare physical parameters and may be related to the original bare parameters ($g, g', \lambda, \mu^2, f_i, f_I$) through the use of the vacuum expectation value v of the Higgs boson.

The vacuum expectation value v is not an independent parameter but should be determined in the theory as a function of g, g', λ, μ^2, f_i and f_I. In practice, v is determined to be equal to the minimum point of the effective potential of ϕ to each order of perturbative expansion. The parameter v is introduced as if it were an independent parameter. The term linear in ϕ in the Lagrangian $\mathcal{L}_{\mathrm{V}}$ (Eq. (4·22)) represents the counter term for the tadpole. To each order of perturbation we require that the tadpole term in the effective action disappears, i.e., the coefficient of the term linear in ϕ vanishes. For later convenience we write

$$T=v(\mu^2-\lambda v^2). \tag{4·24}$$

Summing up, we introduce the redundant parameter v (or T) and add the condition that "tadpole $=0$". The counter term corresponding to this condition is nothing but T. Allowing v to be an independent parameter, we have seven kinds of parameters g, g', v, λ, μ^2, f_i and f_I, which corresponds six kinds of bare physical parameters e, M_W, M_Z, m_ϕ, m_i, m_I, and the tadpole counter term T. The relation between these two sets of parameters is readily given by Eqs. (2·68), (2·70), (2·74), (2·89) and (4·24). Inverted relations for these parameters read

$$
\begin{aligned}
&g = eM_Z/\sqrt{M_Z^2 - M_W^2}\,,\\
&g' = eM_Z/M_W\,,\\
&v = 2M_W\sqrt{M_Z^2 - M_W^2}/(eM_Z)\,,\\
&\lambda = \frac{e^2 M_Z^2}{8M_W^2(M_Z^2 - M_W^2)}\left(m_\phi^2 - \frac{eTM_Z}{M_W\sqrt{M_Z^2 - M_W^2}}\right),\\
&\mu^2 = m_\phi^2/2\,,\\
&f_{i,I} = \sqrt{2}\, m_{i,I}/v\,,
\end{aligned}
\tag{4·25}
$$

where we have taken the sign convention $\mathrm{sgn}\, e = \mathrm{sgn}\, g = \mathrm{sgn}\, g'$.

Using Eq. (4·25), we can replace all the bare parameters g, g', v, λ, μ^2, f_i and f_I in the Lagrangian by the bare physical parameters. For the later use in § 4. 2 where the Feynman rule is developed, we tabulate useful relations in the following:

$$
\begin{aligned}
&\frac{g}{\sqrt{g^2+g'^2}} = \frac{M_W}{M_Z}\,, \quad \sqrt{g^2+g'^2} = \frac{eM_Z^2}{M_W\sqrt{M_Z^2 - M_W^2}}\,,\\
&\frac{g^2 - g'^2}{\sqrt{g^2+g'^2}} = \frac{e(2M_W^2 - M_Z^2)}{M_Z\sqrt{M_Z^2 - M_W^2}}\,,\\
&\mu^2 - \lambda v^2 = \frac{eTM_Z}{2M_W\sqrt{M_Z^2 - M_W^2}}\,,\\
&\mu^2 - 3\lambda v^2 = -m_\phi^2 + \frac{3eTM_Z}{2M_W\sqrt{M_Z^2 - M_W^2}}\,,\\
&v\lambda = \frac{eM_Z}{4M_W\sqrt{M_Z^2 - M_W^2}}\left(m_\phi^2 - \frac{eTM_Z}{M_W\sqrt{M_Z^2 - M_W^2}}\right).
\end{aligned}
\tag{4·26}
$$

We now define renormalization constants and renormalized quantities in such a way that

$$
\begin{aligned}
&W_0^{\pm} = Z_W^{1/2} W^{\pm}\,,\\
&\begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = (Z_{ij}^{1/2}) \begin{pmatrix} Z \\ A \end{pmatrix},\\
&\psi^n_{0\mathrm{R,L}} = (Z^{1/2}_{\mathrm{R,L}})^{nm} \psi^m_{\mathrm{R,L}}\,, \quad (Q_n = Q_m)\\
&\chi_0^{\pm} = Z_\chi^{1/2} \chi^{\pm}\,,\\
&\chi_{30} = Z_{\chi 3}^{1/2} \chi_3\,,\\
&\phi_0 = Z_\phi^{1/2} \phi\,,\\
&M_{W0}^2 = M_W^2 + \delta M_W^2\,,
\end{aligned}
$$

$$
\begin{aligned}
&M^2_{Z0}=M_Z^2+\delta M_Z^2\,,\\
&m^2_{\phi 0}=m_\phi^2+\delta m_\phi^2\,,\\
&m_{n0}=m_n+\delta m_n\,,\\
&e_0=Ye\,,
\end{aligned}
\tag{4·27}
$$

where quantities with (without) suffix 0 denote bare (renormalized) quantities (so far the quantities without suffix 0 have been used to denote the bare one, but we follow the above notation henceforth). As already mentioned in §§ 3.2 and 3.3, (Z_{ij}) is a real matrix with independent four components and $Z_{\mathrm{R,L}}$ are complex matrices. We need not deal with mixing between ψ_{L} and $\psi_{\mathrm{R}}{}^*$ since we assume the fermion number conservation.

We make loop expansions following our Lagrangian. In zeroth order of the loop expansion $Z=1$ and $\delta m=0$. We call the corresponding effective Lagrangian the tree Lagrangian. We split the Lagrangian into two parts: a part consisting of the zeroth order terms, i.e., the tree part, and a part consisting of higher-loop terms, i.e., the counter-term part. We choose the gauge fixing term $\mathcal{L}_{\mathrm{GF}}$ so that bilinear terms in the tree part, i.e., inverse propagator terms, take the simplest form; in fact, we adopt the following form to cancel the mixing terms among W and χ (Z and χ_3):

$$
\begin{aligned}
\mathcal{L}_{\mathrm{GF}}=&-\frac{1}{\alpha}(\partial^\mu W_\mu{}^+ +\alpha M_W\chi^+)(\partial^\mu W_\mu{}^- +\alpha M_W\chi^-)\\
&-\frac{1}{2\alpha_Z}(\partial^\mu Z_\mu+\alpha_Z M_Z\chi_3)^2-\frac{1}{2\alpha_A}(\partial^\mu A_\mu)^2,
\end{aligned}
\tag{4·28}
$$

where α, α_Z and α_A are the gauge parameters and, in particular, the case with $\alpha=\alpha_Z=\alpha_A=1$ is called the 't Hooft-Feynman gauge.

Introducing auxiliary fields B, the Lagrangian $\mathcal{L}_{\mathrm{GF}}$ of Eq. (4·28) may be written as

$$
\begin{aligned}
\mathcal{L}_{\mathrm{GF}}=&B^+(\partial^\mu W_\mu{}^- +\alpha M_W\chi^-)+B^-(\partial^\mu W_\mu{}^+ +\alpha M_W\chi^+)\\
&+B^Z(\partial^\mu Z_\mu+\alpha_Z M_Z\chi_3)+B^A\partial^\mu A_\mu+\alpha B^+B^-\\
&+\frac{\alpha_Z}{2}B^ZB^Z+\frac{\alpha_A}{2}B^AB^A\,.
\end{aligned}
\tag{4·29}
$$

It is convenient to replace the renormalized quantities in Eq. (4·29) by corresponding bare ones for the purpose of finding the form of $\mathcal{L}_{\mathrm{FP}}$:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{GF}}=&B_0{}^+(\partial^\mu W^-_{0\mu}+\alpha_0 M_{W0}\chi_0{}^-)+B^-(\partial^\mu W^+_{0\mu}+\alpha_0 M_{W0}\chi_0{}^+)\\
&+B_0{}^Z(\partial^\mu Z_{0\mu}+\alpha_{Z0}M_{Z0}\chi_{30})+B_0{}^A(\partial^\mu A_{0\mu}+\beta M_{Z0}\chi_{30})\\
&+\text{terms quadratic in the } B_0\text{'s}\,,
\end{aligned}
\tag{4·30}
$$

where the bare quantities B_0, α_0, α_{Z0} and β are defined by

$$\begin{aligned}
&B_0{}^{\pm} = Z_W^{-1/2} B^{\pm} ,\\
&\alpha_0 = \alpha Z_W^{1/2} Z_\chi^{-1/2} M_W / M_{W0} ,\\
&\alpha_{Z0} = \alpha_Z Z_{ZZ}^{1/2} Z_{\chi 3}^{-1/2} M_Z / M_{Z0} ,\\
&\beta = \alpha_Z Z_{AZ}^{1/2} Z_{\chi 3}^{-1/2} M_Z / M_{Z0} ,
\end{aligned} \tag{4·31}$$

and the last terms in Eq. (4·30) are not written explicitly since they do not affect the FP-ghost terms according to the fact that the BRS transforms of the B's vanish. We can now easily find the corresponding FP-ghost Lagrangian $\mathcal{L}_{\mathrm{FP}}$ by using Eq. (2·41) resulting in

$$\begin{aligned}
\mathcal{L}_{\mathrm{FP}} = &-\bar{c}^{+}{}_0 \delta^{\mathrm{BRS}} (\partial^\mu W_{0\mu}^{-} + \alpha_0 M_{W0} \chi_0{}^{-}) - \bar{c}_0{}^{-} \delta^{\mathrm{BRS}} (\partial^\mu W_{0\mu}^{+} + \alpha_0 M_{W0} \chi_0{}^{+})\\
&- \bar{c}_0{}^{Z} \delta^{\mathrm{BRS}} (\partial^\mu Z_{0\mu} + \alpha_{Z0} M_{Z0} \chi_{30}) - \bar{c}_0{}^{A} \delta^{\mathrm{BRS}} (\partial^\mu A_{0\mu} + \beta M_{Z0} \chi_{30}) ,
\end{aligned} \tag{4·32}$$

where the rule of the BRS transformation is given in Eqs. (2·39) and (3·229). To find the explicit form of $\mathcal{L}_{\mathrm{FP}}$, it is more convenient to reexpress Eqs. (2·39) and (3·229) in terms of the physical fields:

$$\begin{aligned}
&\delta W_\mu{}^{\pm} = \partial_\mu c^{\pm} \pm \frac{ig}{\sqrt{g^2+g'^2}} [W_\mu{}^{\pm} (g c^Z + g' c^A) - (g Z_\mu + g' A_\mu) c^{\pm}],\\
&\delta Z_\mu = -\frac{ig^2}{\sqrt{g^2+g'^2}} (W_\mu{}^{+} c^{-} - W_\mu{}^{-} c^{+}) + \partial_\mu c^Z ,\\
&\delta A_\mu = -\frac{igg'}{\sqrt{g^2+g'^2}} (W_\mu{}^{+} c^{-} - W_\mu{}^{-} c^{+}) + \partial_\mu c^A ,\\
&\delta\phi = -\frac{g}{2} (\chi^+ c^- + \chi^- c^+) - \frac{\sqrt{g^2+g'^2}}{2} \chi_3 c^Z ,\\
&\delta\chi^{\pm} = \frac{g}{2} [(v+\phi) c^{\pm} \mp \chi_3 c^{\pm}] \pm \frac{i}{2\sqrt{g^2+g'^2}} \chi^{\pm} [(g^2 - g'^2) c^Z + 2gg' c^A],\\
&\delta\chi_3 = \frac{\sqrt{g^2+g'^2}}{2} (v+\phi) c^Z - \frac{ig}{2} (\chi^+ c^- - \chi^- c^+),
\end{aligned} \tag{4·33}$$

where the superfix BRS is neglected for δ^{BRS} and we have omitted the suffix 0 for the bare quantities and the fields c^Z and c^A are constructed from c^3 and c^0 just like the fields Z and A:

$$\begin{aligned}
&c^Z = (g c^3 - g' c^0) / \sqrt{g^2+g'^2} ,\\
&c^A = (g' c^3 + g c^0) / \sqrt{g^2+g'^2} .
\end{aligned} \tag{4·34}$$

We finally find

$$\mathcal{L}_{\mathrm{FP}} = \mathcal{L}_{\mathrm{FP}}^{(2)} + \mathcal{L}_{\mathrm{FP}}^{(3)} , \tag{4·35}$$

where

$$\mathcal{L}_{\rm FP}^{(2)}=-\bar{c}^{+}(\Box+\alpha M_W^2)c^{-}-\bar{c}^{-}(\Box+\alpha M_W^2)c^{+}$$
$$-\bar{c}^{Z}(\Box+\alpha_Z M_Z^2)c^{Z}-\bar{c}^{A}\Box c^{A}-\bar{c}^{A}(\beta M_Z^2)c^{Z}, \tag{4·36}$$

$$\mathcal{L}_{\rm FP}^{(3)}=\frac{ig^2}{\sqrt{g^2+g'^2}}W_\mu^{+}[\partial^\mu\bar{c}^{-}\cdot c^{Z}-\partial^\mu\bar{c}^{Z}\cdot c^{-}]+ieW_\mu^{+}[\partial^\mu\bar{c}^{-}\cdot c^{A}-\partial^\mu\bar{c}^{A}\cdot c^{-}]$$
$$-\frac{ig^2}{\sqrt{g^2+g'^2}}W_\mu^{-}[\partial^\mu\bar{c}^{+}\cdot c^{Z}-\partial^\mu\bar{c}^{Z}\cdot c^{+}]-ieW_\mu^{-}[\partial^\mu\bar{c}^{+}\cdot c^{A}-\partial^\mu\bar{c}^{A}\cdot c^{+}]$$
$$+\frac{ig^2}{\sqrt{g^2+g'^2}}Z_\mu[\partial^\mu\bar{c}^{+}\cdot c^{-}-\partial^\mu\bar{c}^{-}\cdot c^{+}]+ieA_\mu[\partial^\mu\bar{c}^{+}\cdot c^{-}-\partial^\mu\bar{c}^{-}\cdot c^{+}]$$
$$+i\chi^{+}\left[\frac{-\alpha M_W(-g'^2+g^2)}{2\sqrt{g^2+g'^2}}\bar{c}^{-}c^{Z}-\alpha M_W e\bar{c}^{-}c^{A}\right.$$
$$\left.+\frac{\beta}{2}M_Z g\bar{c}^{A}c^{-}+\frac{\alpha_Z}{2}M_Z g\bar{c}^{Z}c^{-}\right]$$
$$+i\chi^{-}\left[\frac{\alpha M_W(-g'^2+g^2)}{2\sqrt{g^2+g'^2}}\bar{c}^{+}c^{Z}+\alpha M_W e\bar{c}^{+}c^{A}\right.$$
$$\left.-\frac{\beta}{2}M_Z g\bar{c}^{A}c^{+}-\frac{\alpha_Z}{2}M_Z g\bar{c}^{Z}c^{+}\right]$$
$$+\frac{i\alpha}{2}M_W g\chi_3[-\bar{c}^{+}c^{-}+\bar{c}^{-}c^{+}]-\frac{\alpha}{2}M_W g\phi[\bar{c}^{+}c^{-}+\bar{c}^{-}c^{+}]$$
$$-\frac{\alpha^Z}{2}M_Z\sqrt{g^2+g'^2}\phi\bar{c}^{Z}c^{Z}-\frac{\beta}{2}M_Z\sqrt{g^2+g'^2}\phi\bar{c}^{A}c^{Z}. \tag{4·37}$$

The renormalization constants and renormalized fields for the FP ghosts are defined by

$$c_0^{\pm}=\tilde{Z}_3 c^{\pm},\quad \begin{pmatrix} c_0^{Z} \\ c_0^{A}\end{pmatrix}=(\tilde{Z}_{ij})\begin{pmatrix} c^{Z} \\ c^{A}\end{pmatrix},$$
$$\bar{c}_0^{\pm}=\bar{c}^{\pm},\quad \bar{c}_0^{Z}=\bar{c}^{Z},\quad \bar{c}_0^{A}=\bar{c}^{A}. \tag{4·38}$$

In Eq. (4·38) we have introduced the renormalization constants only for the c's and kept the $\bar{c}$'s unmodified. This is legitimate because the ghost fields appear only in internal loops in Feynman diagrams. The decomposition of the Lagrangian $\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP}$ into the tree part and the counter terms may be made in the same way as before.

Summing up, in this section we have rewritten the original Lagrangian in terms of the bare physical quantities and then have decomposed the resulting Lagrangian into the tree and counter terms. The procedure may be summarized as follows,

$$\begin{aligned}
&\mathcal{L}(g, g', \lambda, \mu^2, f_n, \psi, \alpha_0) \\
&\quad = \tilde{\mathcal{L}}(e_0, M_{W0}, M_{Z0}, m_{\phi 0}, m_{n0}, \psi_0, \alpha_0, [T]) \\
&\quad = \mathcal{L}_{\text{tree}}(e, M_W, M_Z, m_\phi, m_n, \psi, \alpha) \\
&\quad + \mathcal{L}_{\text{counter}}(e, M_W, M_Z, m_\phi, m_n, \psi, \alpha, Y, \delta M_W^2, \delta M_Z^2, \delta m_\phi^2, \delta m_n, Z, [T]),
\end{aligned} \tag{4·39}$$

where ψ, α and Z represent generically fields, gauge parameters and field renormalization constants respectively.

For the use in the next section, we finally present the bilinear terms $\mathcal{L}_{\text{tree}}^{(2)}$ in $\mathcal{L}_{\text{tree}}$, i.e., the inverse propagator part,

$$\begin{aligned}
\mathcal{L}_{\text{tree}}^{(2)} = {} & W_\mu^+ \left[(g^{\mu\nu}\Box - \partial^\mu\partial^\nu) + \frac{1}{\alpha}\partial^\mu\partial^\nu + M_W^2 g^{\mu\nu} \right] W_\nu^- \\
& + \frac{1}{2} Z_\mu \left[(g^{\mu\nu}\Box - \partial^\mu\partial^\nu) + \frac{1}{\alpha_Z}\partial^\mu\partial^\nu + M_Z^2 g^{\mu\nu} \right] Z_\nu \\
& + \frac{1}{2} A_\mu \left[(g^{\mu\nu}\Box - \partial^\mu\partial^\nu) + \frac{1}{\alpha_A}\partial^\mu\partial^\nu \right] A_\nu \\
& + \chi^+(-\Box - \alpha M_W^2)\chi^- + \frac{1}{2}\phi(-\Box - m_\phi^2)\phi + \frac{1}{2}\chi_3(-\Box - \alpha_Z M_Z^2)\chi_3 \\
& + \bar{\psi}_i(i\gamma^\mu\partial_\mu - m_i)\psi_i + \bar{\psi}_I(i\gamma^\mu\partial_\mu - m_I)\psi_I \\
& - \bar{c}^+(\Box + \alpha M_W^2)c^- - \bar{c}^-(\Box + \alpha M_W^2)c^+ \\
& - \bar{c}^Z(\Box + \alpha_Z M_Z^2)c^Z - \bar{c}^A \Box c^A .
\end{aligned} \tag{4·40}$$

§ 4.2 Feynman rules

In this section we present a full list of Feynman rules corresponding to the Lagrangian (4·12). The S-matrix is given in terms of the interaction Lagrangian $\mathcal{L}_{\text{I}}(x)$ by

$$S = \text{T} \exp\left[i \int d^4x \mathcal{L}_{\text{I}}(x) \right], \tag{4·41}$$

where T represents an ordinary time ordering. The S-matrix element S_{fi} for specific initial (i) and final (f) states is related to the transition amplitude T_{fi} through the following equation:

$$S_{fi} = 1 + i(2\pi)^4 \delta^4(\sum_j p_j) T_{fi} . \tag{4·42}$$

By using our Feynman rule which will be given in this section, one directly obtains T_{fi} defined in Eq. (4·42) for the process $i \to f$.

We define the propagators by

$$\Delta_{ij}(k)=i\int d^4x\, e^{-ik\cdot x}\langle 0|\mathrm{T}[\phi_i(x)\phi_j(0)]|0\rangle\,, \tag{4·43}$$

where ϕ_i generically represents all the relevant fields in the following. Propagators in the Feynman diagram will be denoted by wavy, dashed, full and dotted lines for gauge bosons, Higgs particles, fermions and ghosts respectively (see § 4. 2. 1). In some cases we put an arrow on the propagator line in order to specify the direction of the flow of a certain quantum number: for the W^+ boson and χ^+ particle the arrow represents the flow of the positive charge, for the fermion that of the fermion number and for the ghost that of the ghost number.

The vertex functions for the interactions of gauge-bosons, fermions, Higgs fields and FP ghosts are derived from $\mathcal{L}_{\mathrm{I}}$ (note that we do not use $i\mathcal{L}_{\mathrm{I}}$). All the momenta of particles associated with vertices are taken to flow in. (The case of the FP-ghost is exceptional. The momentum of the FP ghost is taken to flow along the direction of the ghost number.)

The loop integration is performed by the rule

$$\int\frac{d^Dk}{(2\pi)^Di}\,, \tag{4·44}$$

where we have in mind the dimensional regularization with D the space-time dimension. To the fermion and FP-ghost loops we associate the factor (-1) to account for the anticommutation of fermion and ghost fields.

It should be stressed here that our Feynman rule is designed to minimize the number of times that imaginary unit i appears. Thus an extra factor i has been introduced in the definition of Δ_{ij} in Eq. (4·43) and also the vertices are defined through $\mathcal{L}_{\mathrm{I}}$ instead of $i\mathcal{L}_{\mathrm{I}}$. For example, the fermion tree propagator is $1/(m-\not{p})$ with fermion mass m and the photon-fermion vertex is $e\gamma_\mu$.

Let us explain the notations used in our Feynman rules by taking some typical examples. The propagator of the W^+ field is defined by

$$\Delta^+_{\mu\nu}(q)\equiv i\int d^4x\, e^{-iq\cdot x}\langle 0|\mathrm{T}[W_\mu{}^+(x)\,W_\nu{}^-(0)]|0\rangle\,. \tag{4·45}$$

The corresponding diagram together with the tree propagator is given at the top of the list in § 4. 2. 1. There the arrow on the wavy line represents the flow of a positive charge so that W^+ is created at the end point denoted by $W_\nu{}^-$ and is annihilated at the end point denoted by $W_\mu{}^+$. The W-W-Z proper vertex function is depicted in the first diagram of § 4. 2. 2 (1). It should be noted that the vertex function is defined as an amputated 3- or 4-point Green function and has no propagator as external lines. The letter W^+ (or W^-) attached to the vertex diagram shows that the W^+ (or W^-) particle comes in the vertex. The same remark also applies to the gauge 4-point functions as depicted in § 4. 2. 2 (2).

To see the virtue of our Feynman rule we present two examples of the calculation. The first example deals with the W-boson contribution to the one-loop self-energy part of Z-bosons, $\Pi^Z_{\mu\nu}(q)$. The diagram to be considered is shown in Fig. 4.1.

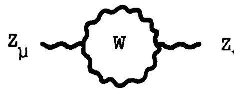


Fig. 4.1. W-boson contribution to the one-loop self-energy part of Z boson.

Following our Feynman rule, we have

$$\Pi^Z_{\mu\nu}(q)=\int\frac{d^Dk}{(2\pi)^Di}\frac{eM_W}{\sqrt{M_Z^2-M_W^2}}\left\{(k-2q)^\alpha g_\mu{}^\beta+(q-2k)_\mu g^{\beta\alpha}+(k+q)^\beta g_\mu{}^\alpha\right\}$$
$$\times\frac{1}{(k^2-M_W^2)((k-q)^2-M_W^2)}$$
$$\times\frac{eM_W}{\sqrt{M_Z^2-M_W^2}}\left\{(k+q)_\beta g_{\nu\alpha}+(q-2k)_\nu g_{\alpha\beta}+(k-2q)_\alpha g_{\beta\nu}\right\},\quad(4\cdot46)$$

where we have adopted the Feynman gauge. Obviously we have the least number of the i's in Eq. (4·46) compared with the expressions for $\Pi^Z_{\mu\nu}(q)$ in ordinary Feynman rules. Our $\Pi^Z_{\mu\nu}(q)$ is normalized by the following perturbative relation,

$$\Delta^Z_{\mu\nu}(q)=\Delta^{Z0}_{\mu\nu}(q)+\Delta^{Z0}_{\mu\alpha}(q)\Pi^{Z,\alpha\beta}(q)\Delta^{Z0}_{\beta\nu}(q)+\cdots,\quad(4\cdot47)$$

where $\Delta^Z_{\mu\nu}$ is an unrenormalized full propagator of the Z-boson and $\Delta^{Z0}_{\mu\nu}$ is the tree propagator appearing in the Feynman rule § 4.2.1. Equation (4·46) can be calculated explicitly in a straightforward manner. The result may be found in Appendix B.

The next example is one of the correction diagrams in the one-loop order for ν_μ-e scattering. The diagram we consider is shown in Fig. 4.2. By the use of our Feynman rule, we obtain its contribution to the scattering amplitude $A(p_e',p_\nu';p_e,p_\nu)$,

$$A(p_e',p_\nu';p_e,p_\nu)$$
$$=\int\frac{d^Dk}{(2\pi)^Di}\bar{u}_\nu(p_\nu')\left(\frac{eM_Z^2}{2M_W\sqrt{M_Z^2-M_W^2}}\right)^2\gamma_\alpha\frac{1-\gamma_5}{2}\frac{1}{\not{k}-\not{p}_\nu}\gamma_\beta\frac{1-\gamma_5}{2}u_\nu(p_\nu)$$
$$\times\bar{u}_e(p_e')\left(\frac{eM_Z^2}{2M_W\sqrt{M_Z^2-M_W^2}}\right)^2\gamma^\alpha\left(\frac{1-\gamma_5}{2}-2\frac{M_Z^2-M_W^2}{M_Z^2}\right)\frac{1}{m_e-\not{p}_e-\not{k}}$$
$$\times\gamma^\beta\left(\frac{1-\gamma_5}{2}-2\frac{M_Z^2-M_W^2}{M_Z^2}\right)u_e(p_e)\frac{1}{k^2-M_Z^2}\frac{1}{(k-q)^2-M_Z^2},\quad(4\cdot48)$$

where we have adopted the Feynman gauge, and p_ν and p_e (p_ν' and p_e') are

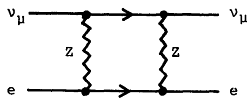


Fig. 4.2. One of the diagrams of the one-loop corrections for the process $\nu_\mu e\to\nu_\mu e$.

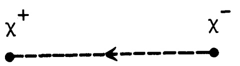
initial (final) neutrino and electron momenta respectively with $q=p_e'-p_e$. Here again we observe that the imaginary unit i appears only once in Eq. (4·48). Note that the amplitude A coincides exactly with T_{fi} defined in Eq. (4·42). The loop integral in Eq. (4·48) can be performed by applying the Feynman parametrization. The resulting expression may be found in Appendix B. If one takes into account only a few leading terms in powers of $1/M_Z$, the expression for the amplitude A is further simplified and the Feynman-parameter integrals are trivially done. The final form for A may be easily obtained by using the formula given in Appendix C.

4.2.1 *Propagators*

$$\frac{1}{k^2+i\varepsilon-M_W^2}\left[g^{\mu\nu}-(1-\alpha)\frac{k^\mu k^\nu}{k^2+i\varepsilon-\alpha M_W^2}\right]$$

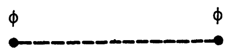
$$\frac{1}{k^2+i\varepsilon-M_Z^2}\left[g^{\mu\nu}-(1-\alpha_Z)\frac{k^\mu k^\nu}{k^2+i\varepsilon-\alpha_Z M_Z^2}\right]$$

$$\frac{g^{\mu\nu}}{k^2+i\varepsilon}-(1-\alpha_A)\frac{k^\mu k^\nu}{(k^2+i\varepsilon)^2}$$

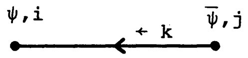


$$\frac{1}{\alpha M_W^2-k^2-i\varepsilon}$$

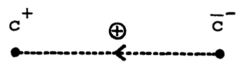
$$\frac{1}{\alpha_Z M_Z^2-k^2-i\varepsilon}$$



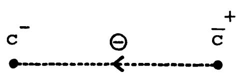
$$\frac{1}{m_\phi^2-k^2-i\varepsilon}$$



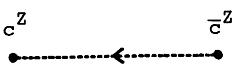
$$\left.\frac{1}{m-\not k-i\varepsilon}\right|_{ij}=\left.\frac{m+\not k}{m^2-k^2-i\varepsilon}\right|_{ij}$$



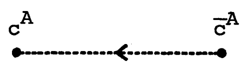
$$\frac{1}{\alpha M_W^2-k^2-i\varepsilon}$$



$$\frac{1}{\alpha M_W^2-k^2-i\varepsilon}$$



$$\frac{1}{\alpha_Z M_Z^2-k^2-i\varepsilon}$$

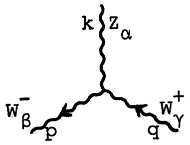


$$\frac{1}{-k^2-i\varepsilon}$$

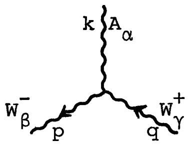
The arrow on the propagator line represents the direction of the flow of a certain quantum number; the charge for $W^\pm$, $\chi^\pm$, the fermion number for ψ, the ghost number for c, $\bar{c}$. The symbol $\oplus$ or $\ominus$ beside charged ghost propagators represents the sign of the charge carried by the arrow.

4.2.2 *Vertices in* $\mathcal{L}_G$

(1) Gauge-boson three-vertices

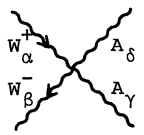


$$\frac{eM_W}{\sqrt{M_Z^2-M_W^2}}\{(k-p)^\gamma g^{\alpha\beta}+(p-q)^\alpha g^{\beta\gamma}+(q-k)^\beta g^{\gamma\alpha}\}$$

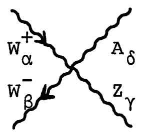


$$e\{(k-p)^\gamma g^{\alpha\beta}+(p-q)^\alpha g^{\beta\gamma}+(q-k)^\beta g^{\gamma\alpha}\}$$

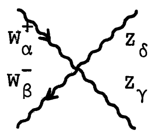
(2) Gauge-boson four-vertices



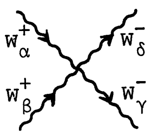
$$e^2(g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}-2g^{\alpha\beta}g^{\gamma\delta})$$



$$\frac{e^2M_W}{\sqrt{M_Z^2-M_W^2}}(g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}-2g^{\alpha\beta}g^{\gamma\delta})$$



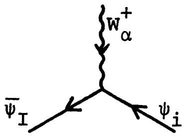
$$\frac{e^2M_W^2}{M_Z^2-M_W^2}(g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}-2g^{\alpha\beta}g^{\gamma\delta})$$



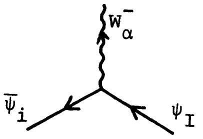
$$\frac{-e^2M_Z^2}{M_Z^2-M_W^2}(g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}-2g^{\alpha\beta}g^{\gamma\delta})$$

All the momenta of particles associated with vertices are taken to flow in.

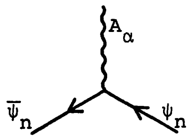
4.2.3 *Vertices in $\mathcal{L}_F$*



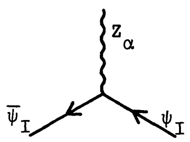
$$\frac{eM_Z}{2\sqrt{2(M_Z^2-M_W^2)}}U^\dagger_{Ii}\gamma_\alpha(1-\gamma_5)$$



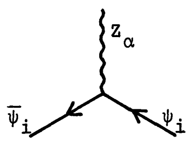
$$\frac{eM_Z}{2\sqrt{2(M_Z^2-M_W^2)}}U_{iI}\gamma_\alpha(1-\gamma_5)$$



$$eQ_n\gamma_\alpha \quad (n=i, I)$$



$$\frac{eM_Z^2}{2M_W\sqrt{M_Z^2-M_W^2}}\gamma_\alpha\left(\frac{1}{2}-2Q_I\frac{M_Z^2-M_W^2}{M_Z^2}-\frac{1}{2}\gamma_5\right)$$

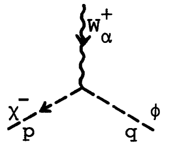


$$\frac{eM_Z^2}{2M_W\sqrt{M_Z^2-M_W^2}}\gamma_\alpha\left(-\frac{1}{2}-2Q_i\frac{M_Z^2-M_W^2}{M_Z^2}+\frac{1}{2}\gamma_5\right)$$

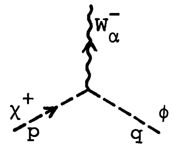
$Q_{i,I}$ represent electromagnetic charges of fields $\psi_{i,I}$ in units of e.

4.2.4 *Vertices in* $\mathcal{L}_{\rm H}$

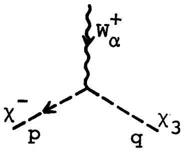
(1) Gauge-boson-Higgs three-vertices



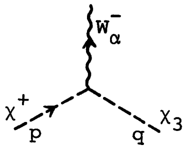
$$\frac{ieM_Z}{2\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



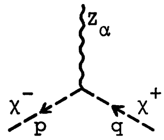
$$\frac{ieM_Z}{2\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



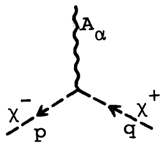
$$\frac{eM_Z}{2\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



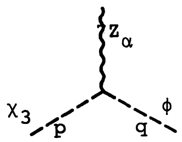
$$\frac{-eM_Z}{2\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



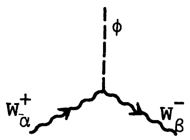
$$\frac{e(M_Z^2-2M_W^2)}{2M_W\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



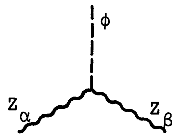
$$-e(p-q)_\alpha$$



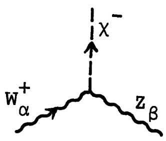
$$\frac{ieM_Z^2}{2M_W\sqrt{M_Z^2-M_W^2}}(p-q)_\alpha$$



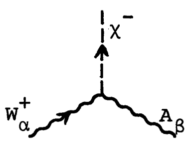
$$\frac{eM_WM_Z}{\sqrt{M_Z^2-M_W^2}}g_{\alpha\beta}$$



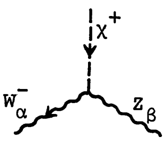
$$\frac{eM_Z^3}{M_W\sqrt{M_Z^2-M_W^2}}g_{\alpha\beta}$$



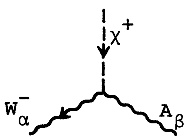
$$ie\sqrt{M_Z^2-M_W^2}\,g_{\alpha\beta}$$



$$-ieM_Wg_{\alpha\beta}$$

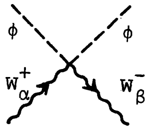


$$-ie\sqrt{M_Z^2-M_W^2}\,g_{\alpha\beta}$$

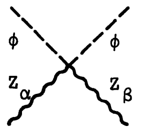


$$ieM_Wg_{\alpha\beta}$$

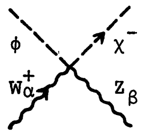
(2) Gauge-boson-Higgs four-vertices



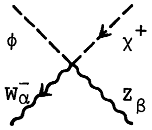
$$\frac{e^2M_Z^2}{2(M_Z^2-M_W^2)}g_{\alpha\beta}$$



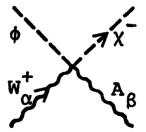
$$\frac{e^2M_Z^4}{2M_W^2(M_Z^2-M_W^2)}g_{\alpha\beta}$$



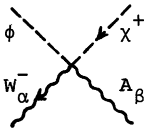
$$\frac{ie^2M_Z}{2M_W}g_{\alpha\beta}$$



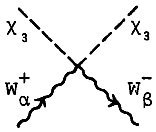
$$\frac{-ie^2M_Z}{2M_W}g_{\alpha\beta}$$



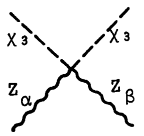
$$\frac{-ie^2M_Z}{2\sqrt{M_Z^2-M_W^2}}g_{\alpha\beta}$$



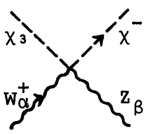
$$\frac{ie^2M_Z}{2\sqrt{M_Z^2-M_W^2}}g_{\alpha\beta}$$



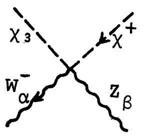
$$\frac{e^2M_Z^2}{2(M_Z^2-M_W^2)}g_{\alpha\beta}$$



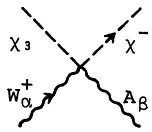
$$\frac{e^2M_Z^4}{2M_W^2(M_Z^2-M_W^2)}g_{\alpha\beta}$$



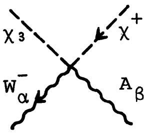
$$\frac{e^2M_Z}{2M_W}g_{\alpha\beta}$$



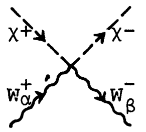
$$\frac{e^2M_Z}{2M_W}g_{\alpha\beta}$$



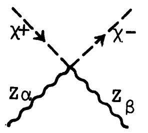
$$\frac{-e^2M_Z}{2\sqrt{M_Z^2-M_W^2}}g_{\alpha\beta}$$



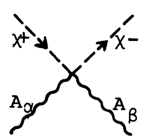
$$\frac{-e^2M_Z}{2\sqrt{M_Z^2-M_W^2}}g_{\alpha\beta}$$



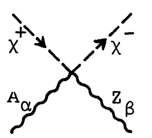
$$\frac{e^2M_Z^2}{2(M_Z^2-M_W^2)}g_{\alpha\beta}$$



$$\frac{e^2(2M_W^2-M_Z^2)^2}{2M_W^2(M_Z^2-M_W^2)}g_{\alpha\beta}$$

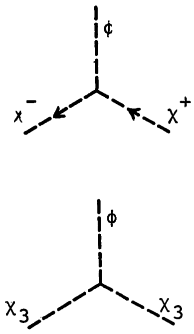


$$2e^2g_{\alpha\beta}$$

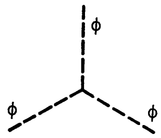


$$\frac{e^2(2M_W^2-M_Z^2)}{M_W\sqrt{M_Z^2-M_W^2}}g_{\alpha\beta}$$

(3) Higgs three-vertices

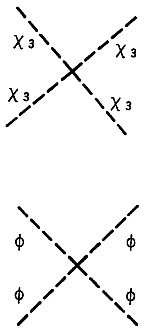


$$\frac{-em_\phi{}^2M_Z}{2M_W\sqrt{M_Z^2-M_W^2}}$$

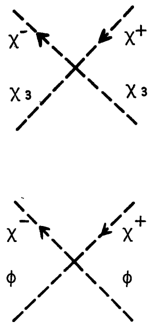


$$\frac{-3em_\phi{}^2M_Z}{2M_W\sqrt{M_Z^2-M_W^2}}$$

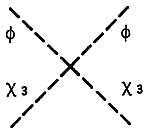
(4) Higgs four-vertices



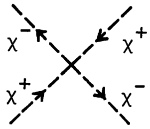
$$\frac{-3e^2m_\phi{}^2M_Z^2}{4M_W^2(M_Z^2-M_W^2)}$$



$$\frac{-e^2m_\phi{}^2M_Z^2}{4M_W^2(M_Z^2-M_W^2)}$$

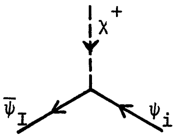


$$\frac{-e^2m_\phi{}^2M_Z^2}{4M_W^2(M_Z^2-M_W^2)}$$

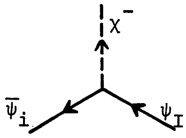


$$\frac{-e^2m_\phi{}^2M_Z^2}{2M_W^2(M_Z^2-M_W^2)}$$

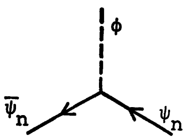
4.2.5 *Vertices in* $\mathcal{L}_M$



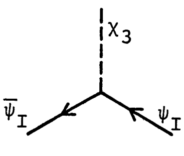
$$\frac{-ieM_Z}{2M_W\sqrt{2(M_Z^2-M_W^2)}}U^\dagger_{Ii}\{(m_i-m_I)+(m_i+m_I)\gamma_5\}$$



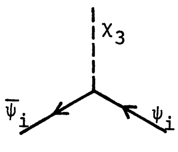
$$\frac{-ieM_Z}{2M_W\sqrt{2(M_Z^2-M_W^2)}}U_{iI}\{(m_I-m_i)+(m_I+m_i)\gamma_5\}$$



$$\frac{-em_nM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}$$



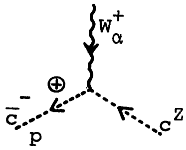
$$\frac{-iem_IM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}\gamma_5$$



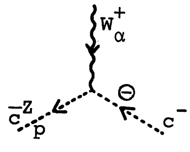
$$\frac{iem_iM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}\gamma_5$$

4.2.6 *Vertices in* $\mathcal{L}_{\text{FP}}$

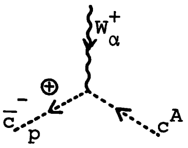
(1) Gauge-boson-ghost three-vertices



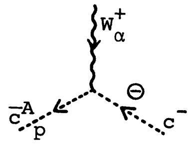
$$\frac{-eM_W}{\sqrt{M_Z^2-M_W^2}}p_\alpha$$



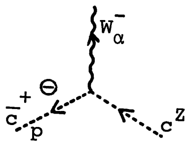
$$\frac{eM_W}{\sqrt{M_Z^2-M_W^2}}p_\alpha$$



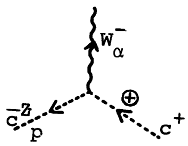
$$-ep_\alpha$$



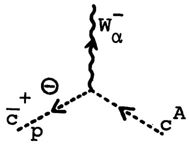
$$ep_\alpha$$



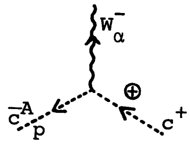
$$\frac{eM_W}{\sqrt{M_Z^2-M_W^2}}p_\alpha$$



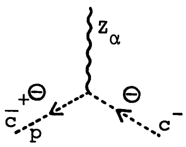
$$\frac{-eM_W}{\sqrt{M_Z^2-M_W^2}}p_\alpha$$



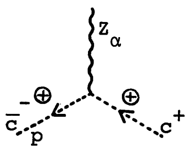
$$ep_\alpha$$



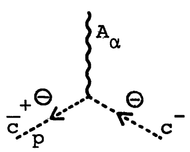
$$-ep_\alpha$$



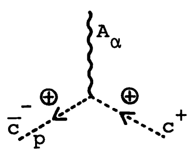
$$\frac{-eM_W}{\sqrt{M_Z^2-M_W^2}}p_\alpha$$



$$\frac{eM_W}{\sqrt{M_Z^2-M_W^2}}p_\alpha$$

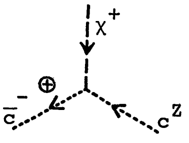


$$-ep_\alpha$$

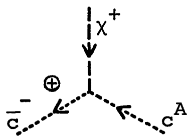


$$ep_\alpha$$

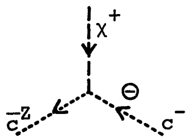
(2) Higgs-ghost three-vertices



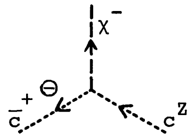
$$\frac{-i\alpha e(2M_W^2-M_Z^2)}{2\sqrt{M_Z^2-M_W^2}}$$



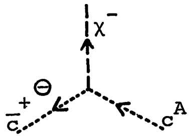
$$-i\alpha e M_W$$



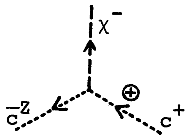
$$\frac{i\alpha_Z e M_Z^2}{2\sqrt{M_Z^2-M_W^2}}$$



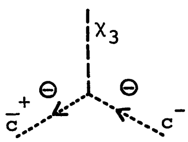
$$\frac{i\alpha e\,(2M_W^2-M_Z^2)}{2\sqrt{M_Z^2-M_W^2}}$$



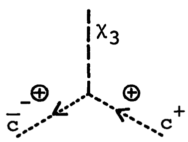
$$i\alpha e M_W$$



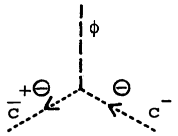
$$\frac{-i\alpha_Z e M_Z^2}{2\sqrt{M_Z^2-M_W^2}}$$

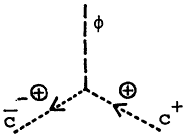


$$\frac{-i\alpha e M_W M_Z}{2\sqrt{M_Z^2-M_W^2}}$$

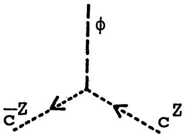


$$\frac{i\alpha e M_W M_Z}{2\sqrt{M_Z^2-M_W^2}}$$





$$\frac{-\alpha e M_W M_Z}{2\sqrt{M_Z^2-M_W^2}}$$



$$\frac{-\alpha_Z e M_Z^3}{2M_W\sqrt{M_Z^2-M_W^2}}$$

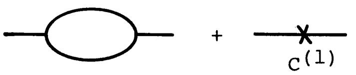
§ 4.3 Renormalization conditions and counter terms

In this section we tabulate the on-shell renormalization conditions and counter terms for the tadpole and for all two-, three- and four-point functions. Every counter term is presented at the place where the related renormalization condition appears.

Renormalized n-point functions are coefficients of the renormalized fields defined in Eq. (4·27) in the effective action. They are given by the sum of one-particle irreducible Feynman diagrams including the contribution of counter terms. In the case of two-point functions they are nothing but the inverse propagators.

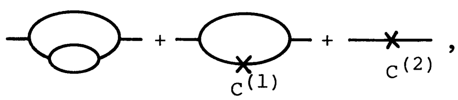
As explained in § 4.1, we split the bare Lagrangian into two parts: a part consisting of the zeroth order terms of the loop expansion (the tree part) and the remaining part consisting of higher-loop terms (the counter-term part). According to this separation, we generate the loop expansion of the effective action systematically. At each order of the loop expansion we determine the counter terms so that the amplitudes satisfy the renormalization conditions.

This process is schematically shown as follows. Consider a renormalized two-point function Γ_i. In the one-loop approximation we have two kinds of diagrams for $\Gamma_i^{(1)}$,

$\Gamma_i^{(1)} =$ 

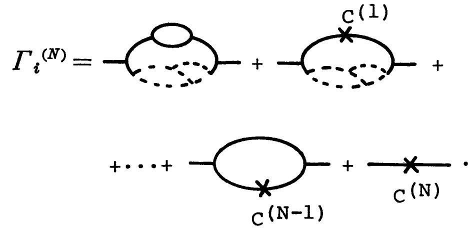
, (4·49)

where the second term $C^{(1)}$ on the right-hand side represents the one-loop part of the counter terms. We determine $C^{(1)}$ by the renormalization condition for Γ_i. With $C^{(1)}$ thus determined the contribution from counter terms cancels out the divergence included in the first term in Eq. (4·49) and we obtain the finite $\Gamma_i^{(1)}$. In the two-loop approximation there are three kinds of diagrams,

$\Gamma_i^{(2)} =$ 

, (4·50)

where $C^{(2)}$ is the two-loop part of the counter terms. The inner divergences of the first term are cancelled out by the second term and there remains the overall divergence. We determine $C^{(2)}$ to satisfy the renormalization condition for Γ_i and the third term in Eq. (4·50) cancels out the above-mentioned overall divergence to give finite $\Gamma_i^{(2)}$. We proceed to the N-loop approximation.



(4·51)

The N-loop part of the counter terms $C^{(N)}$ is determined by the renormalization condition and it cancels out the overall divergence in the N-loop diagrams. In this way we obtain the loop expansion of Γ_t in which every term is finite.

In the following tables we first present the renormalized n-point function and write down the corresponding counter terms explicitly, where the N-loop parts of these counter terms should be regarded as the last term in Eq. (4·51); then we give the on-shell renormalization conditions and the renormalization constants which are determined by these conditions. All the renormalization constants except Y are determined by the renormalization conditions for the tadpole and for the two-point functions. The constant Y is determined by the renormalization condition for the photon vertex of any of the charged particles. For all other vertices (three- and four-point functions) no more independent renormalization conditions and counter terms appear. They are already fixed by the foregoing renormalization conditions. The explicit expressions for the renormalization constants are given in Appendix E.

For mixing channels the renormalization conditions are imposed following the formulas discussed in detail in §§ 3. 2 and 3. 3. The renormalization conditions for diagonal elements are the same as the usual ones and those for off-diagonal elements require that amplitudes vanish at the on-shell momentum of each particle (§ 4. 3. 2(1) (2)).

It should be mentioned that no counter terms come out from the gauge fixing Lagrangian $\mathcal{L}_{\mathrm{GF}}$ as discussed in § 4. 1.

In what follows we arrange the tables of renormalization conditions and counter terms for amplitudes, from which their tree parts are removed, following the order of presentation of Feynman rules in the last section. On the next page we presnt abbreviated notations used in the following tables.

Notations used in tables of counter terms:

$$G_W \equiv \sqrt{1+\frac{\delta M_W^2}{M_W^2}}\,,$$

$$G_Z \equiv \sqrt{1+\frac{\delta M_Z^2}{M_Z^2}}\,,$$

$$G_\phi \equiv 1+\frac{\delta m_\phi^2}{m_\phi^2}\,,$$

$$G_{mj} \equiv 1+\frac{\delta m_j^2}{m_j^2}\,,$$

$$H \equiv \sqrt{1+\frac{\delta M_Z^2-\delta M_W^2}{M_Z^2-M_W^2}}\,;$$

$$G_1 \equiv G_W/H\,,$$

$$G_2 \equiv G_Z/H\,,$$

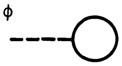
$$G_3 \equiv G_Z/G_W\,,$$

$$G_4 \equiv \left\{1+\frac{2\delta M_W^2-\delta M_Z^2}{2M_W^2-M_Z^2}\right\}\Big/(G_W H)\,;$$

$$Z_\psi^{1/2} \equiv \frac{Z_L^{1/2}+Z_R^{1/2}}{2}+\frac{Z_R^{1/2}-Z_L^{1/2}}{2}\gamma^5\,,$$

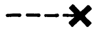
$$Z_{\bar\psi}^{1/2} \equiv \frac{Z_L^{1/2\dagger}+Z_R^{1/2\dagger}}{2}+\frac{Z_L^{1/2\dagger}-Z_R^{1/2\dagger}}{2}\gamma^5\,.$$

4.3.1 *Renormalization condition and counter term for tadpole*



$= \text{tadpole},$

Counter term

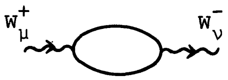

$= T\,.$

Renormalization condition — Renormalization constant

$$\text{Tadpole} = 0 \quad \Rightarrow \quad T$$

4.3.2 *Renormalization conditions and counter terms for two-point functions (proper self-energies) of physical particles*

(1) Gauge-bosons



$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) A^W(k^2) + \frac{k_\mu k_\nu}{k^2} B^W(k^2)\,,$$

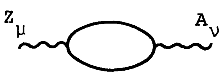
$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) [\delta M_W^2 \cdot Z_W + (M_W^2 - k^2) Z_W]$$
$$+ \frac{k_\mu k_\nu}{k^2} [\delta M_W^2 \cdot Z_W + M_W^2 \cdot Z_W]\,.$$

Renormalization conditions — Renormalization constants

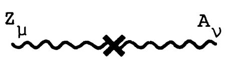
$$\left.\begin{aligned} A^W(M_W^2) &= 0 \\ A^{W\prime}(M_W^2) &= 0 \end{aligned}\right\} \quad \Rightarrow \quad Z_W,\ \delta M_W^2\,.$$

$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) A^Z(k^2) + \frac{k_\mu k_\nu}{k^2} B^Z(k^2)\,,$$

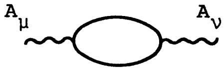
$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) [(M_Z^2 + \delta M_Z^2)(Z_{ZZ}^{1/2})^2$$
$$- k^2 \{(Z_{ZZ}^{1/2})^2 + (Z_{AZ}^{1/2})^2\}]$$
$$+ \frac{k_\mu k_\nu}{k^2} [(M_Z^2 + \delta M_Z^2)(Z_{ZZ}^{1/2})^2]\,.$$



$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) A^{ZA}(k^2) + \frac{k_\mu k_\nu}{k^2} B^{ZA}(k^2)\,,$$



$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) [(M_Z^2 + \delta M_Z^2)(Z_{ZZ}^{1/2} Z_{ZA}^{1/2})$$
$$- k^2 \{(Z_{ZZ}^{1/2} Z_{ZA}^{1/2}) + (Z_{AZ}^{1/2})(Z_{AA}^{1/2})\}]$$
$$+ \frac{k_\mu k_\nu}{k^2} [(M_Z^2 + \delta M_Z^2)(Z_{ZZ}^{1/2} Z_{ZA}^{1/2})]\,.$$



$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) A^A(k^2) + \frac{k_\mu k_\nu}{k^2} B^A(k^2)\,,$$

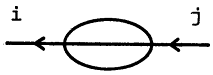
$$= \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) [(M_Z^2 + \delta M_Z^2)(Z_{ZA}^{1/2})^2$$
$$- k^2 \{(Z_{ZA}^{1/2})^2 + (Z_{AA}^{1/2})^2\}]$$
$$+ \frac{k_\mu k_\nu}{k^2} [(M_Z^2 + \delta M_Z^2)(Z_{ZA}^{1/2})^2].$$

Renormalization conditions — Renormalization constants

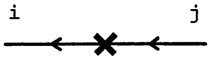
$$\left.\begin{array}{ll} A^Z(M_Z^2) = 0\,, & A^{Z\prime}(M_Z^2) = 0\,; \\ A^A(0) = 0\,, & A^{A\prime}(0) = 0\,; \\ A^{ZA}(M_Z^2) = 0\,, & A^{ZA}(0) = 0\,. \end{array}\right\} \Rightarrow Z_{ZZ}^{1/2},\ Z_{ZA}^{1/2},\ Z_{AZ}^{1/2},\ Z_{AA}^{1/2},\ \delta M_Z^2\,.$$

In the A-Z mixing channel there are six renormalization conditions for five constants. In fact, as discussed in § 3. 3, six conditions are not independent. The Ward-Takahashi identity representing the unbroken $U(1)$ symmetry guarantees the existence of the massless pole in the $A-Z$ channel. Two equations $A^A(0) = 0$ and $A^{ZA}(0) = 0$ gives an identical condition. This situation corresponds to that in the QED case: $\Pi^{\mu\nu}(k^2) = (g^{\mu\nu}k^2 - k^\mu k^\nu) \times \Pi(k^2)$.

(2) Fermions



$$= K_1^{ij}(k^2)\mathbf{1} + K_5^{ij}(k^2)\gamma^5 + K_\gamma^{ij}(k^2)\not{k} + K_{5\gamma}^{ij}(k^2)\not{k}\gamma^5\,,$$



$$= \mathbf{1}\left[-\frac{1}{2}(Z_L^{1/2\dagger})_{iI}(m_I + \delta m_I)(Z_R^{1/2})_{Ij} - \frac{1}{2}(Z_R^{1/2\dagger})_{iI}(m_I + \delta m_I)(Z_L^{1/2})_{Ij}\right]$$
$$+ \gamma^5\left[-\frac{1}{2}(Z_L^{1/2\dagger})_{iI}(m_I + \delta m_I)(Z_R^{1/2})_{Ij} + \frac{1}{2}(Z_R^{1/2\dagger})_{iI}(m_I + \delta m_I)(Z_L^{1/2})_{Ij}\right]$$
$$+ \not{k}\left[\frac{1}{2}(Z_L^{1/2\dagger})_{iI}(Z_L^{1/2})_{Ij} + \frac{1}{2}(Z_R^{1/2\dagger})_{iI}(Z_R^{1/2})_{Ij}\right]$$
$$+ \not{k}\gamma^5\left[-\frac{1}{2}(Z_L^{1/2\dagger})_{iI}(Z_L^{1/2})_{Ij} + \frac{1}{2}(Z_R^{1/2\dagger})_{iI}(Z_R^{1/2})_{Ij}\right].$$

Renormalization conditions

$i=j$:

$$\begin{cases} K^{ii}(m_i^2)u(m_i)=\bar{u}(m_i)K^{ii}(m_i^2)=0\,, \\ \dfrac{1}{\not{k}-m_i}K^{ii}(m_i^2)u(m_i)=\bar{u}(m_i)K^{ii}(m_i^2)\dfrac{1}{\not{k}-m_i}=0; \end{cases}$$

i.e.,

$$\begin{cases} K_1^{ii}(m_i^2)+m_iK_r^{ii}(m_i^2)=0\,, \\ K_5^{ii}(m_i^2)=0\,, \\ K_{5r}^{ii}(m_i^2)=0\,, \\ \left[\dfrac{\partial}{\partial \not{k}}(K_1^{ii}(k^2)+\not{k}K_r^{ii}(k^2))\right]_{\not{k}=m_i}=0\,. \end{cases}$$

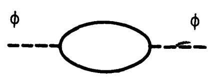
$i>j$:

$$\begin{cases} K^{ij}(m_j^2)u(m_j)=0\,, \\ \bar{u}(m_i)\,K^{ij}(m_i^2)=0\,; \end{cases} \qquad \text{i.e.,} \qquad \begin{cases} K_1^{ij}(m_j^2)+m_jK_r^{ij}(m_j^2)=0\,, \\ K_5^{ij}(m_j^2)-m_jK_{5r}^{ij}(m_j^2)=0\,, \\ K_1^{ij}(m_i^2)+m_iK_r^{ij}(m_i^2)=0\,, \\ K_5^{ij}(m_i^2)+m_iK_{5r}^{ij}(m_i^2)=0\,. \end{cases}$$

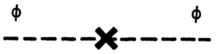
By these conditions are determined the renormalization constants $(Z_L^{1/2})_{ij}$, $(Z_R^{1/2})_{ij}$ (complex non-unitary matrices) and δm_i. We need not impose the conditions for $i<j$ since the renormalization conditions with $i<j$ are not independent of those with $i>j$. This is because the effective action Γ is hermite due to the hermiticity of the Lagrangian as explained in § 3. 2 explicitly.

In the case where the CP-invariance is imposed, the condition $K_5^{ii}(m_i^2)=0$ holds automatically and no counter terms are required.

(3) Higgs particle



$$=F_\phi(k^2)\,,$$



$$=-\delta m_\phi^2 Z_\phi+(k^2-m_\phi^2)Z_\phi+3TY\frac{eM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}G_3H^{-1}Z_\phi\,.$$

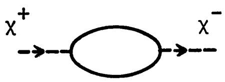
Renormalization conditions $\qquad\qquad$ Renormalization constants

$$\left.\begin{matrix} F_\phi(m_\phi^2)=0 \\ F_\phi'(m_\phi^2)=0 \end{matrix}\right\} \quad \Rightarrow \quad Z_\phi,\,\delta m_\phi^2\,.$$

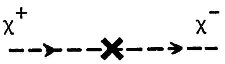
4.3.3 *Renormalization conditions and counter terms for two-point functions of unphysical particles*

Since the unphysical particles do not appear in external lines, the finite parts of their renormalization constants are irrelevant. Hence we may impose any renormalization condition for these particles. It is only our task to eliminate divergences caused by the presence of these particles. Practically, it is most convenient to adopt the minimal subtraction procedure.

(1) Nambu-Goldstone particles

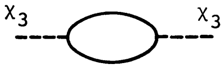


$$=k^2F(k^2),$$

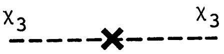


$$=k^2Z_\chi+TY\frac{eM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}G_3H^{-1}Z_\chi\,.$$

Relevant renormalization condition ⇨ Renormalization constant Z_χ.



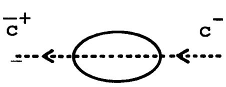
$$=k^2F_3(k^2),$$

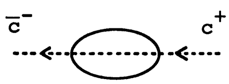


$$=k^2Z_{\chi 3}+TY\frac{eM_Z}{2M_W\sqrt{M_Z^2-M_W^2}}G_3H^{-1}Z_{\chi 3}\,.$$

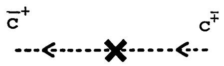
Relevant renormalization condition ⇨ Renormalization constant $Z_{\chi 3}$.

(2) Faddeev-Popov ghosts



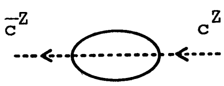


$$\left.\begin{array}{l}=\gamma_{+-}(k^2),\\[2em] =\gamma_{-+}(k^2),\end{array}\right\}$$



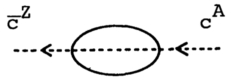
$$=k^2\tilde{Z}_3-\alpha M_W^2G_W\tilde{Z}_3Z_W{}^{1/2}Z_\chi{}^{-1/2}\,.$$

Relevant renormalization condition ⇨ Renormalization constant $\tilde{Z}_3$.



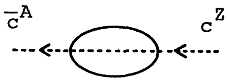
$$=\gamma_{ZZ}(k^2),$$

$$=k^2\tilde{Z}_{ZZ}-\alpha_Z M_Z^2 G_Z \tilde{Z}_{ZZ} Z_{ZZ}^{1/2}(Z_{\chi 3})^{-1/2}.$$



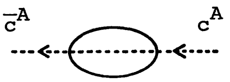
$$=\gamma_{ZA}(k^2),$$

$$=k^2\tilde{Z}_{ZA}-\alpha_Z M_Z^2 G_Z \tilde{Z}_{ZA} Z_{ZZ}^{1/2}(Z_{\chi 3})^{-1/2}.$$

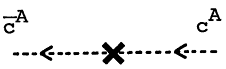


$$=\gamma_{AZ}(k^2),$$

$$=k^2\tilde{Z}_{AZ}-\alpha_Z M_Z^2 G_Z \tilde{Z}_{ZZ} Z_{AZ}^{1/2}(Z_{\chi 3})^{-1/2}.$$



$$=\gamma_{AA}(k^2),$$

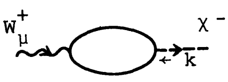


$$=k^2\tilde{Z}_{AA}-\alpha_Z M_Z^2 G_Z \tilde{Z}_{ZA} Z_{AZ}^{1/2}(Z_{\chi 3})^{-1/2}.$$

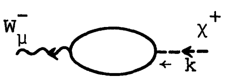
Relevant renormalization conditions	⇨	Renormalization constants $\tilde{Z}_{ZZ}$, $\tilde{Z}_{ZA}$, $\tilde{Z}_{AZ}$, $\tilde{Z}_{AA}$.

(3) Two-point functions between physical and unphysical particles

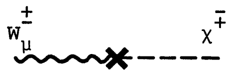
We have determined all the renormalization constants except the constant Y. In terms of these renormalization constants determined already the counter terms are specified for the following two-point functions:



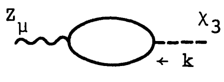
$$=ik_\mu c^{+-}(k^2),$$



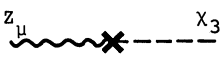
$$=ik_\mu c^{-+}(k^2),$$



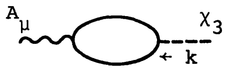
$$=ik_\mu M_W G_W Z_W^{1/2} Z_\chi^{1/2}\ .$$



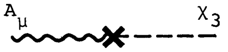
$$=ik_\mu c_{Z\chi}(k^2),$$



$$=ik_\mu M_Z G_Z Z_{ZZ}^{1/2} Z_{\chi 3}^{1/2}\ .$$



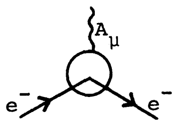
$$=ik_\mu c_{A\chi}(k^2).$$



$$=ik_\mu M_Z G_Z Z_{ZA}^{1/2} Z_{\chi 3}^{1/2}.$$

4.3.4 *Charge renormalization constant Y*

We have exhausted all the renormalization constants and the counter terms for the two-point functions. Only one remaining renormalization constant Y can be determined as follows:



$$=\Gamma^\mu(p, k).$$

See § 4.3.6 on the counter term.

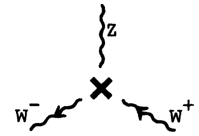
Renormalization condition		Renormalization constant
$\bar{u}(m_e)\Gamma^\mu u(m_e)\|_{k^\mu=0}=0$	⇨	Y

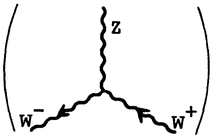
We may, in fact, impose the above renormalization condition on any charged particle and obtain the same value for the constant Y, since the universality of the on-shell charge (coupling constant with the photon) has been proved in § 3.4.

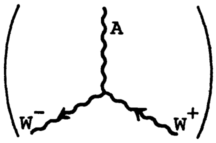
In the rest of tables we are concerned with the counter terms for the three- and four-point functions at least with one loop. In these expressions for counter terms, it should be understood that the diagrams in parentheses indicate the tree vertices given by the relevant Feynman rules and that their tree parts are omitted.

4.3.5 *Counter terms for three- and four-point functions in* $\mathcal{L}_G$

(1) Gauge-boson three-point functions



$=YG_1Z_WZ_{ZZ}^{1/2}$


$+YZ_WZ_{AZ}^{1/2}$


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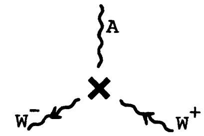
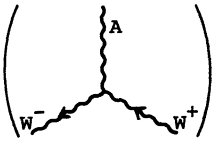
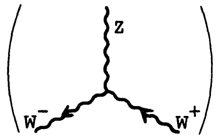


$=YZ_WZ_{AA}^{1/2}$


$+YG_1Z_WZ_{ZA}^{1/2}$


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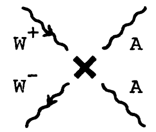
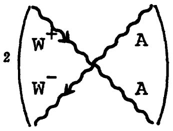
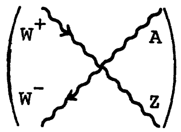
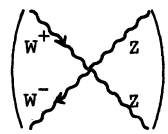
(2) Gauge-boson four-point functions



$=Y^2Z_W(Z_{AA}^{1/2})^2$


$+Y^2G_1Z_W(Z_{AA}^{1/2})(Z_{ZA}^{1/2})$


$+Y^2(G_1)^2Z_W(Z_{ZA}^{1/2})^2$


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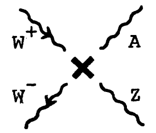
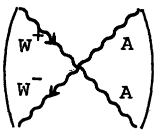
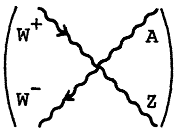
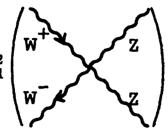


$=Y^2Z_W(Z_{AA}^{1/2})(Z_{AZ}^{1/2})$


$+Y^2G_1Z_W\{(Z_{AA}^{1/2})(Z_{ZZ}^{1/2})$

$+(Z_{AZ}^{1/2})(Z_{ZA}^{1/2})\}$


$+Y^2(G_1)^2Z_WZ_{ZZ}^{1/2}Z_{ZA}^{1/2}$


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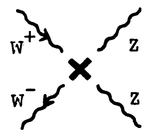
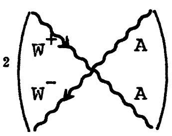
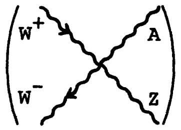
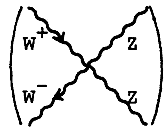


$=Y^2Z_W(Z_{AZ}^{1/2})^2$


$+Y^2G_1Z_W(Z_{AZ}^{1/2})(Z_{ZZ}^{1/2})$


$+Y^2(G_1)^2Z_W(Z_{ZZ}^{1/2})^2$


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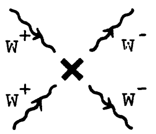
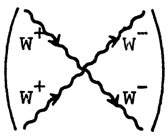


$=Y^2(G_2)^2Z_W^2$


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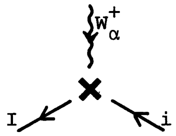
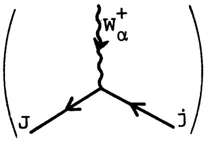
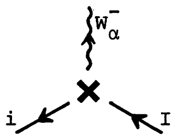
4.3.6 *Counter terms for three-point functions in* $\mathcal{L}_F$



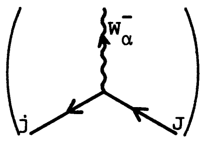
$= YG_2(Z_L{}^{1/2\dagger})_{IJ}$



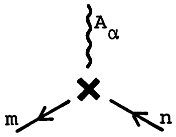
$(Z_L{}^{1/2\dagger})_{ji} Z_W{}^{1/2}$.



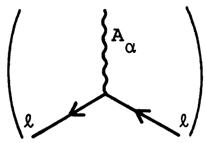
$= YG_2(Z_L{}^{1/2\dagger})_{ij}$



$(Z_L{}^{1/2})_{JI} Z_W{}^{1/2}$.

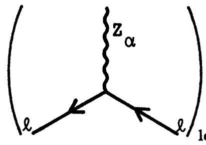


$= YZ_{AA}^{1/2}(Z_{\bar{\phi}}^{-1/2})_{ml}$



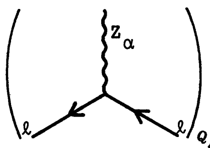
$(Z_\phi{}^{1/2})_{ln}$

$+ YG_2G_3Z_{ZA}^{1/2}(Z_L{}^{1/2})_{ml}$

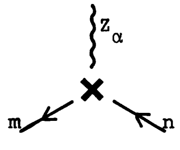


left part $(Z_L{}^{1/2})_{ln}$

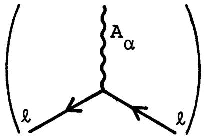
$+ YG_2{}^{-1}G_3Z_{ZA}^{1/2}(Z_{\bar{\phi}}^{-1/2})_{ml}$



Q_ℓ part $(Z_\phi{}^{1/2})_{ln}$.

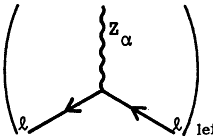


$= YZ_{AZ}^{1/2}(Z_{\bar{\phi}}^{-1/2})_{ml}$



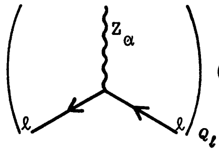
$(Z_\phi{}^{1/2})_{ln}$

$+ YG_2G_3Z_{ZZ}^{1/2}(Z_L{}^{1/2})_{ml}$



left part $(Z_L{}^{1/2})_{ln}$

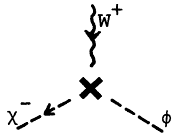
$+ YG_2{}^{-1}G_3Z_{ZZ}^{1/2}(Z_{\bar{\phi}}^{-1/2})_{ml}$

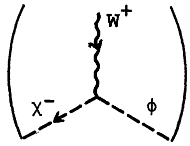


Q_ℓ part $(Z_\phi{}^{1/2})_{ln}$.

4.3.7 *Counter terms for three- and four-point functions in* $\mathcal{L}_H$

(1) Gauge-boson—Higgs three-point functions



$=YG_2Z_W^{1/2}Z_\chi^{1/2}Z_\phi^{1/2}$


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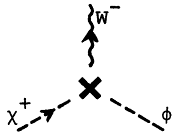
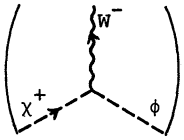


$=YG_2Z_W^{1/2}Z_\chi^{1/2}Z_\phi^{1/2}$


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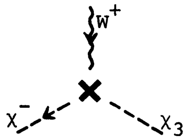
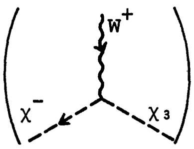


$=YG_2Z_W^{1/2}Z_\chi^{1/2}Z_{\chi 3}^{1/2}$


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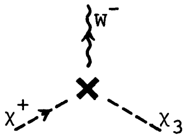
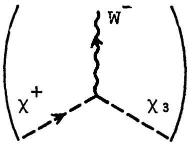


$=YG_2Z_W^{1/2}Z_\chi^{1/2}Z_{\chi 3}^{1/2}$


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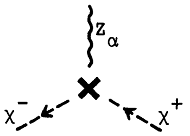
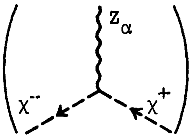
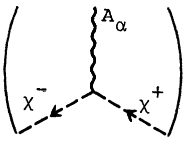


$=YG_4Z_\chi Z_{ZZ}^{1/2}$


$+YZ_\chi Z_{AZ}^{1/2}$


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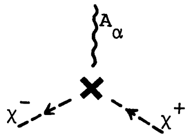
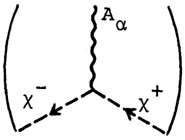
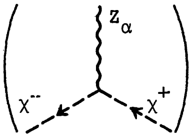


$=YZ_\chi Z_{AA}^{1/2}$


$+YG_4Z_\chi Z_{ZA}^{1/2}$


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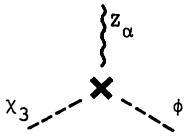
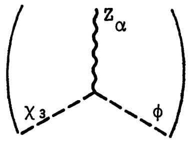


$=YG_2G_3Z_{\chi 3}^{1/2}Z_\phi^{1/2}Z_{ZZ}^{1/2}$


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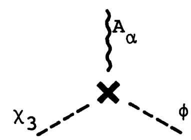
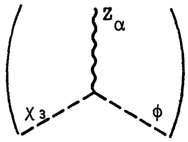


$=YG_2G_3Z_{\chi 3}^{1/2}Z_\phi^{1/2}Z_{ZA}^{1/2}$


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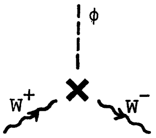
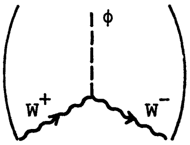


$$=YG_2G_WZ_WZ_\phi^{1/2}$$



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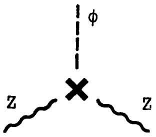
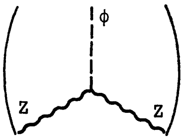


$$=YG_2G_3G_ZZ_\phi^{1/2}(Z_{ZZ}^{1/2})^2$$



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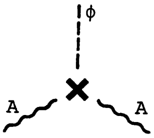
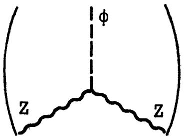


$$=YG_2G_3G_ZZ_\phi^{1/2}(Z_{ZA}^{1/2})$$



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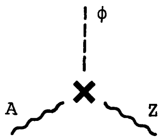
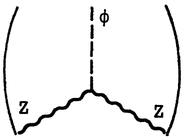


$$=YG_2G_3G_ZZ_\phi^{1/2}Z_{ZA}^{1/2}Z_{ZZ}^{1/2}$$



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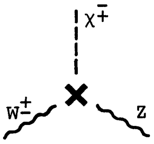
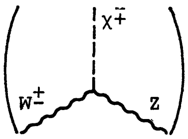
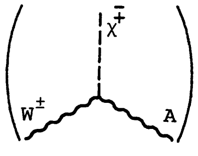


$$=YHZ_\chi^{1/2}Z_W^{1/2}Z_{ZZ}^{1/2}$$



$$+YG_WZ_\chi^{1/2}Z_W^{1/2}Z_{AZ}^{1/2}$$



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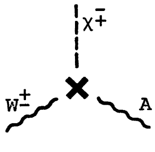
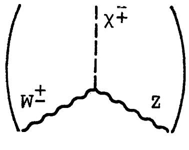
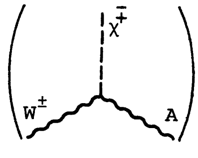


$$=YHZ_\chi^{1/2}Z_W^{1/2}Z_{ZA}^{1/2}$$



$$+YG_WZ_\chi^{1/2}Z_W^{1/2}Z_{AA}^{1/2}$$



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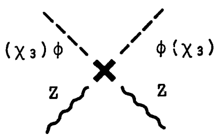
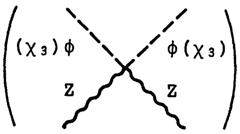
(2) Gauge-boson—Higgs four-point functions



$= Y^2G_2^2G_3^2Z_\phi(Z_{ZZ}^{1/2})^2$
$(Z_{\chi 3})$


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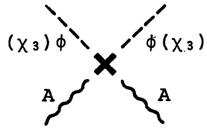
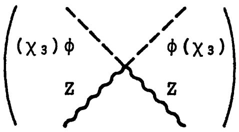


$= Y^2G_2^2G_3^2Z_\phi(Z_{ZA}^{1/2})^2$
$(Z_{\chi 3})$


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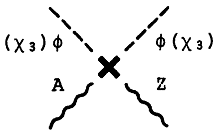
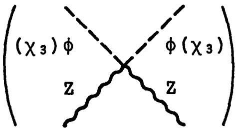


$= Y^2G_2^2G_3^2Z_\phi Z_{ZZ}^{1/2}Z_{ZA}^{1/2}$
$(Z_{\chi 3})$


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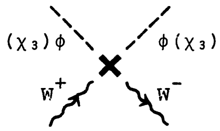
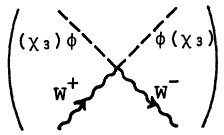


$= Y^2(G_2)^2Z_\phi Z_W$
$(Z_{\chi 3})$


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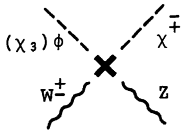
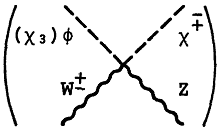
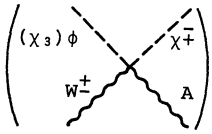


$= Y^2G_3Z_\chi^{1/2}Z_W^{1/2}Z_\phi^{1/2}Z_{ZZ}^{1/2}$
$(Z_{\chi 3}^{1/2})$


$+ Y^2G_2Z_\chi^{1/2}Z_W^{1/2}Z_\phi^{1/2}Z_{AZ}^{1/2}$
$(Z_{\chi 3}^{1/2})$


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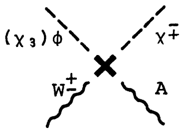
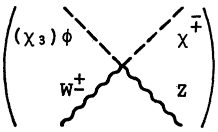
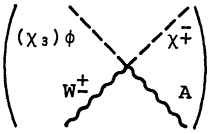


$= Y^2G_3Z_\chi^{1/2}Z_W^{1/2}Z_\phi^{1/2}Z_{ZA}^{1/2}$
$(Z_{\chi 3}^{1/2})$


$+ Y^2G_2Z_\chi^{1/2}Z_W^{1/2}Z_\phi^{1/2}Z_{AA}^{1/2}$
$(Z_{\chi 3}^{1/2})$


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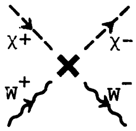
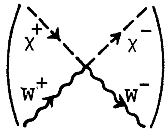


$= Y^2 (G_2)^2 Z_\chi Z_W$


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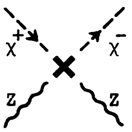
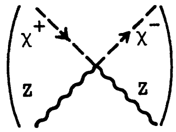


$= Y^2 (G_4)^2 Z_\chi (Z^{1/2}_{ZZ})^2$


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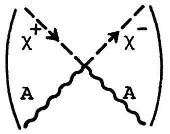
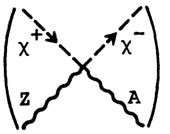
$+ Y^2 Z_\chi (Z^{1/2}_{AZ})^2$


$+ Y^2 G_4 Z_\chi Z^{1/2}_{ZZ} Z^{1/2}_{AZ}$


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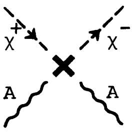
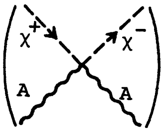
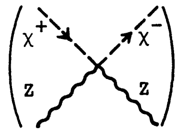
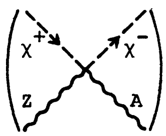


$= Y^2 Z_\chi (Z^{1/2}_{AA})^2$


$+ Y^2 (G_4)^2 Z_\chi (Z^{1/2}_{ZA})^2$


$+ Y^2 G_4 Z_\chi Z^{1/2}_{ZA} Z^{1/2}_{AA}$


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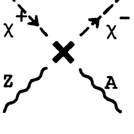
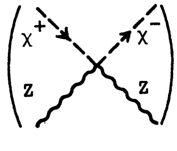
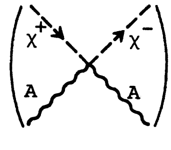
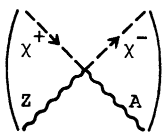


$= Y^2 (G_4)^2 Z_\chi Z^{1/2}_{ZA} Z^{1/2}_{ZZ}$


$+ Y^2 Z_\chi Z^{1/2}_{AZ} Z^{1/2}_{AA}$


$+ Y^2 G_4 Z_\chi (Z^{1/2}_{ZA} Z^{1/2}_{AZ} + Z^{1/2}_{ZZ} Z^{1/2}_{AA})$


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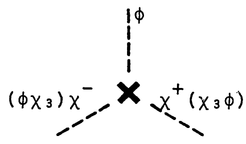
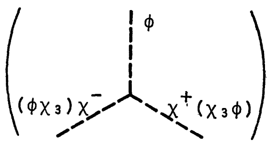
(3) Higgs three-point functions



$$=G_2G_W^{-1}G_\phi Z_\phi^{1/2}\underset{\begin{pmatrix}Z_{\chi3}\\ Z_\phi\end{pmatrix}}{Z_\chi}\left(1-\frac{YG_2T}{G_\phi G_W}\,\frac{eM_Z}{m_\phi^2M_W\sqrt{M_Z^2-M_W^2}}\right)$$

$\times$



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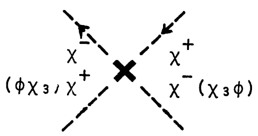
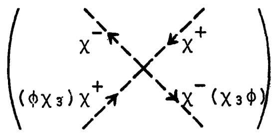
(4) Higgs four-point functions



$$=Y^2G_\phi(G_2)^2(G_W)^{-2}Z_\chi \underset{\begin{pmatrix}Z_{\chi3}\\ Z_\phi\end{pmatrix}}{Z_\chi}\left(1-\frac{YG_2T}{G_\phi G_W}\,\frac{eM_Z}{m_\phi^2M_W\sqrt{M_Z^2-M_W^2}}\right)$$

$\times$



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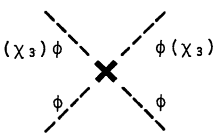
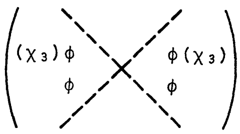


$$=Y^2G_\phi(G_2)^2(G_W)^{-2}Z_\phi \underset{(Z_{\chi3})}{Z_\phi}\left(1-\frac{YG_2T}{G_\phi G_W}\,\frac{eM_Z}{m_\phi^2M_W\sqrt{M_Z^2-M_W^2}}\right)$$

$\times$



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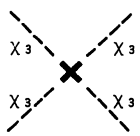
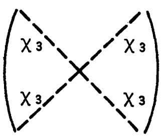


$$=Y^2G_\phi(G_2)^2(G_W)^{-2}Z_{\chi3}^2\left(1-\frac{YG_2T}{G_\phi G_W}\,\frac{eM_Z}{m_\phi^2M_W\sqrt{M_Z^2-M_W^2}}\right)$$

$\times$



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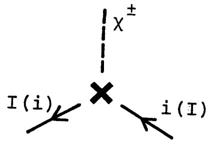
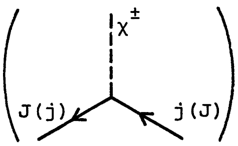
4.3.8 *Counter terms for three-point functions in $\mathcal{L}_M$*
Fermion-Higgs three-point functions

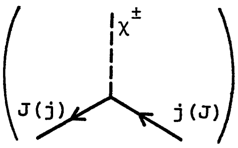


$$= Y G_W^{-1} G_2 Z_\chi^{1/2} (Z_R^{1/2\dagger})_{\substack{IJ\\(ij)}} G_{m_{\substack{J\\(j)}}}$$

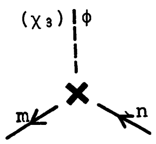


$$\Big)_{\text{left}} (Z_L^{1/2})_{\substack{ji\\(JI)}}$$

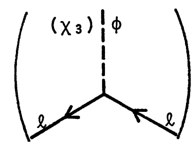
$$+ Y G_W^{-1} G_2 Z_\chi^{1/2} (Z_L^{1/2\dagger})_{\substack{IJ\\(ij)}} G_{m_{\substack{j\\(J)}}}$$



$$\Big)_{\text{right}} (Z_R^{1/2})_{\substack{ji\\(JI)}} \,.$$



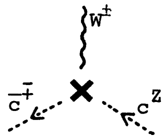
$$= Y G_W^{-1} G_2 \underset{(Z_{\chi 3}^{1/2})}{Z_\phi^{1/2}} (Z_\phi^{-1/2})_{ml} G_{m_l}$$



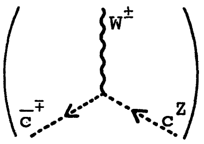
$$(Z_\phi^{1/2})_{ln} \,.$$

4.3.9 *Counter terms for three-point functions in* $\mathcal{L}_{\mathrm{FP}}$

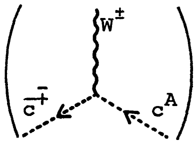
(1) Gauge-boson—ghost three-point functions



$=YG_1Z_W^{1/2}\widetilde{Z}_{ZZ}$



$+YZ_W^{1/2}\widetilde{Z}_{AZ}$



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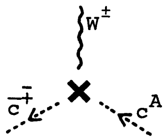
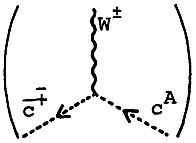
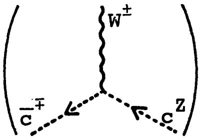


$=YZ_W^{1/2}\widetilde{Z}_{AA}$



$+YG_1Z_W^{1/2}\widetilde{Z}_{ZA}$



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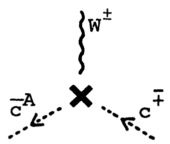
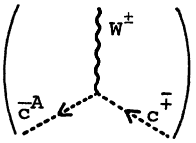


$=YZ_W^{1/2}\widetilde{Z}_3$



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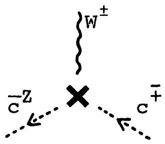
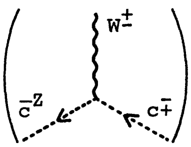


$=YG_1Z_W^{1/2}\widetilde{Z}_3$



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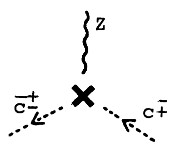
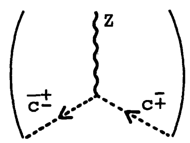
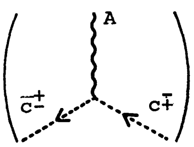


$=YG_1\widetilde{Z}_3Z_{ZZ}^{1/2}$



$+Y\widetilde{Z}_3Z_{AZ}^{1/2}$



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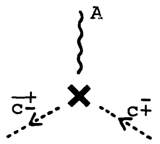
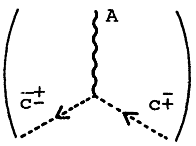
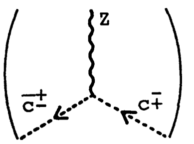


$=Y\widetilde{Z}_3Z_{AA}^{1/2}$



$+YG_1\widetilde{Z}_3Z_{ZA}^{1/2}$



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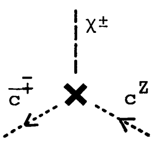
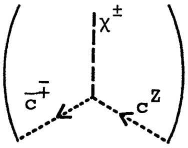
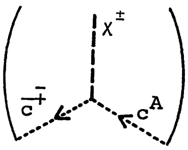
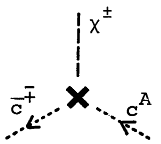
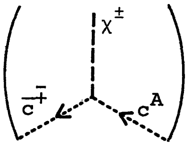
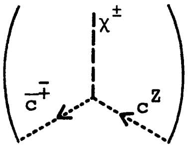
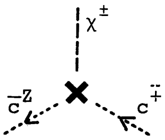
(2) Higgs-ghost three-point functions

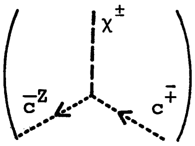


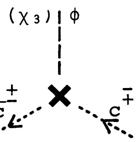
$= YG_4G_W^{-1}Z_W^{1/2}$ () $\tilde{Z}_{ZZ} + YZ_W^{1/2}$ () $\tilde{Z}_{AZ}$.

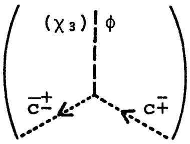


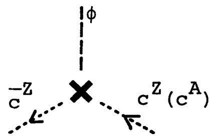
$= YZ_W^{1/2}$ () $\tilde{Z}_{AA} + YG_4G_W^{-1}Z_W^{1/2}$ () $\tilde{Z}_{ZA}$.

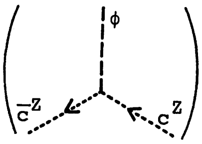


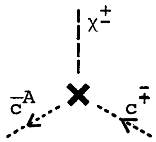
$= YG_2\tilde{Z}_3 Z_{ZZ}^{1/2} Z_\chi^{1/2} (Z_{\chi 3})^{-1/2}$ ().



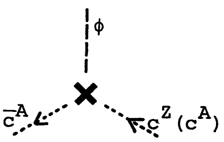
$= YG_2\tilde{Z}_3 \; \begin{matrix} Z_\phi^{1/2} \\ (Z_{\chi 3}^{1/2}) \end{matrix} \; Z_W^{1/2} Z_\chi^{-1/2}$ ().



$= YG_ZG_W^{-1}G_2Z_{ZZ}^{1/2}(Z_{\chi 3})^{-1/2}Z_\phi^{1/2}$ () $\begin{matrix} \tilde{Z}_{ZZ} \\ (\tilde{Z}_{ZA}) \end{matrix}$.



$$= \pm \frac{i}{2} YG_2 Z_{AZ}^{1/2} (Z_{\chi 3})^{-1/2} Z_\chi^{1/2} \tilde{Z}_3 \frac{e\alpha_Z M_Z^2}{\sqrt{M_Z^2 - M_W^2}} .$$



$$= -\frac{1}{2} YG_ZG_W^{-1}G_2Z_{AZ}^{1/2}(Z_{\chi 3})^{-1/2}Z_\phi^{1/2} \begin{matrix} \tilde{Z}_{ZZ} \\ (\tilde{Z}_{ZA}) \end{matrix} \frac{e\alpha_Z M_Z^3}{M_W\sqrt{M_Z^2 - M_W^2}} .$$

The last two counter terms are obtained from the corresponding couplings proportional to β in $\mathcal{L}_{\rm FP}^{(3)}$. (See Eqs. (4·31) and (4·37).) These couplings have no tree parts.

Chapter 5

Electroweak Radiative Corrections to Leptonic Processes

In this chapter we treat concrete examples of application of the on-shell renormalization procedure which has been explained in the foregoing chapters. We will calculate the electroweak radiative corrections of order α to the pure leptonic processes: the neutrino (anti-neutrino)-electron scattering, $\nu_\mu(\bar{\nu}_\mu)e^- \to \nu_\mu(\bar{\nu}_\mu)e^-$, the inverse muon decay, $\nu_\mu e^- \to \mu^- \nu_e$, and the muon decay, $\mu^- \to e^- \nu_\mu \bar{\nu}_e$. We mention briefly the outline of calculation in our renormalization scheme (§ 5. 2) and give the explicit expressions of results in every step of the calculation for the above processes (§§ 5. 3 and 5. 4).

§ 5. 1 Leptonic processes and the Weinberg-Salam theory

As we mentioned already on the occasion of setting the Weinberg-Salam model (§ 1. 2), the comparison of its predictions with various experiments gives almost the same values to the parameters of the Weinberg-Salam (WS) theory, the Weinberg angle θ_W and the coupling constant g (and consequently to the masses M_W, M_Z of the gauge bosons). The argument was, however, in the tree level; then it seems worthwhile to perform the systematic calculation of the higher order effects and to examine closely the aspect of the WS theory as the renormalizable field theory. This situation reminds us of the quantum electrodynamics (QED) in the fourties. In those days the renormalizability of QED was found and the investigations of the radiative corrections revealed out the abundant contents of the theory. Now we proceed similar calculations in the WS theory by making use of the on-shell renormalization procedure which we regard as the most natural extension of the renormalization prescription in QED.

In these calculations we will consider only the leptonic processes with non-hadronic targets which come out of the purely leptonic currents to avoid dynamical complications due to the relevant strong interactions as far as possible.

From this point of view the experimentally most feasible and well-defined process is the annihilation of e^+e^- into l^+l^- pair. This process is mediated by the photon in the tree level, and the Z boson exchange becomes effective in high energy region. The electroweak radiative corrections of order α to this process were given in [Passarino and Veltman 79] for $\mu^+\mu^-$ pair and [Consoli 79] for e^+e^- pair.

Next to the above process the leptonic reactions of our interests are the neutrino (anti-neutrino)-electron scattering

$$\nu_e(\bar{\nu}_e)e^- \rightarrow \nu_e(\bar{\nu}_e)e^-, \quad \nu_\mu(\bar{\nu}_\mu)e^- \rightarrow \nu_\mu(\bar{\nu}_\mu)e^-,$$

the inverse muon decay $\nu_\mu e^- \rightarrow \mu^- \nu_e (\bar{\nu}_e e^- \rightarrow \mu^- \bar{\nu}_\mu)$

and

the muon decay $\mu^-(\mu^+) \rightarrow e^- \nu_\mu \bar{\nu}_e (e^+ \bar{\nu}_\mu \nu_e)$.

These processes are all mediated through only the gauge boson exchange in the tree level.

The elastic scattering of the neutrino (anti-neutrino) on the electron is caused mainly by the neutral leptonic current which is one of the characteristics of the WS theory, and these elastic processes were early considered by 't Hooft as a test of the WS theory ['t Hooft 71a]. The electron-neutrino (anti-neutrino)-electron scattering is possible even in the usual charged current $V-A$ theory (the Feynman—Gell-Mann theory) but with different magnitudes of cross sections from those in the WS theory. The observed cross sections of $\bar{\nu}_e$-e scattering in the reactor experiment were in favor of the WS theory [Reines et al. 76]. The muon-neutrino (anti-neutrino)-electron scattering is mediated by the neutral leptonic current only and the confirmation of these reactions [Hasert et al. 73, see also Mo 82] was the most persuasive evidence for the existence of the neutral leptonic current and for the WS theory before the advent of the polarized electron experiment [Prescott et al. 78 and 79].

Then we will calculate the electroweak radiative corrections of order α to these processes $\nu_\mu e^- \rightarrow \nu_\mu e^-$ and $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ as examples of the neutral current processes [Salomonson and Ueda 75, Green and Veltman 80, Aoki et al. 81, Bardin et al. 82]. For the comparison with observations in the future, it may be relevant to calculate the processes for $\nu_\mu(\bar{\nu}_\mu)$-beams which are more easily feasible in accelerator experiments.

In the reactions $\nu_\mu e^- \rightarrow \mu^- \nu_e$ and $\bar{\nu}_e e^- \rightarrow \mu^- \bar{\nu}_\mu$ take part the charged leptonic currents.*) These reactions were discussed at the tree level by Jarlskog based on the Feynman—Gell-Mann theory [Jarlskog 70]. She suggested to observe the ratio of the forward and backward cross sections as a test of the $V-A$ structure of the charged leptonic current. It was, however, only recent years that these reactions were investigated experimentally because of their rather high threshold energies which amount to 10.9 GeV in the incident neutrino energy and of the exceedingly low cross sections [Armenise

*) The corresponding process induced by the muon-anti-neutrino $\bar{\nu}_\mu e^- \rightarrow \mu^- \bar{\nu}_e$ is possible only when one assumes the multiplicative conservation of the leptonic numbers [Feinberg and Weinberg 61, see also Derman and Jones 77, Derman 79]. The observation gives the ratio of cross sections $\sigma(\bar{\nu}_\mu e^- \rightarrow \mu^- \bar{\nu}_e)/\sigma(\nu_\mu e^- \rightarrow \mu^- \nu_e) < 0.09$ at 90% confidence level [Jonker et al. 80].

et al. 79, Jonker et al. 80]. As an example of charged current processes we will take up the reaction $\nu_\mu e^- \to \mu^- \nu_e$. Once we have the amplitude of this process, the amplitude of the muon decay can be easily obtained by making use of the crossing relation. In this way we present the electroweak radiative corrections of order α both to the reaction $\nu_\mu e^- \to \mu^- \nu_e$ [Aoki et al. 81, Aoki and Hioki 81, Hioki 82a] and to the muon decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ [Ross 73, Appelquist et al. 72 and 73, Hioki 82a and 82b].

§ 5.2 Scenario of calculation

When we intend to perform some calculation in the WS theory, we find with perplexed eyes that discouragingly great many Feynman diagrams should be taken into account in contrast with those in QED. This situation is brought about from the structure of the WS theory in which we have many fields to be taken into care. This is the case in our calculations. By adopting the relevant approximations explained in what follows, we can leave considerable numbers of diagrams out of account, but it is still desirable to arrange systematically the order of calculations. We give the outline of a plan for this purpose in Tables 5.1 and 5.2.

We will carry out the calculations of the electroweak radiative corrections to the cross sections of the above-mentioned leptonic processes in the one-loop order ($O(\alpha)$). In these calculations we assume the simplest version of the WS theory with three generations of quarks and leptons and with one Higgs scalar doublet. The masses of the electron- and muon-neutrino are considered as to be zero. For experimentally available energies E_ν of the incident neutrino in the laboratory frame, the squared momentum transfers q^2 in such processes are negligible in comparison with the square of gauge boson mass $M^2 = M_W^2$ or M_Z^2 (the mean value of the squared momentum transfer in our calculation is estimated, e.g., to be $\overline{q^2} = 4 \sim 5\,(\mathrm{GeV})^2$ at $E_\nu = 10^4\,\mathrm{GeV}$). This is the case for the lepton mass $m = m_e$ or m_μ compared with the gauge boson mass M. Then we neglect all the quantities of $O(q^2/M^2)$ and of $O(m/M)$ in our following calculations. As a result, we may leave out of consideration every Feynman diagram with the Higgs-lepton vertex from the beginning, because this coupling is proportional to the ratio m/M. The Feynman amplitudes associated with our leptonic processes in the one-loop order and the formulas on $1/M$ expansion of the amplitudes are found in Appendix B and Appendix C, respectively.

In these Feynman amplitudes we encounter inevitably both ultraviolet and infrared divergent integrals. In order to arrive at the well-defined cross sections we should calculate the divergent integrals by adopting relevant regularization method and separate out the finite parts of amplitudes. We adopt the method of dimensional regularization for the ultraviolet divergent integrals

Table 5.1. A scenario of one-loop calculation ($\nu e \to \nu e$).

Steps			What is to be calculated	Numbers of Feynman diagrams	Renormalization constants and counter terms to be determined	Renormalization conditions
UV-divergent (and IR-divergent for (*))	Self-energy corrections	1. Two-point functions				
		1–1	$A_{\mathrm{R}}^{Z}(q^2)$: Z boson self-energy part	13 (Fig. 5.3)	$\delta M_Z^{\,2}$, $Z_{ZZ}^{1/2}$; $A_{\mathrm{C}}{}^{Z}$	§4.3.2 (1)
		1–2	$A_{\mathrm{R}}^{ZA}(q^2)$: ZA transition self-energy part	9 (Fig. 5.4)	$Z_{ZA}^{1/2}$, $Z_{AZ}^{1/2}$; $A_{\mathrm{C}}{}^{ZA}$	§4.3.2 (1)
		1–3	$A_{\mathrm{R}}^{A}(q^2)$: Photon self-energy part	9 (Fig. 5.5)	$Z_{AA}^{1/2}$; $A_{\mathrm{C}}{}^{A}$	§4.3.2 (1)
		1–4	$A_{\mathrm{R}}^{W}(q^2)$: W boson self-energy part (*)	19 (Fig. 5.6)	$\delta M_W^{\,2}$; $A_{\mathrm{C}}{}^{W}$	§4.3.2 (1)
		1–5	$\Sigma_{\mathrm{R}}^{l}(q)$: Charged lepton self-energy part (*)	4 (Fig. 5.7)	δm_l, $Z_{\mathrm{L}}{}^{l}$, $Z_{\mathrm{R}}{}^{l}$; $\Sigma_{\mathrm{C}}{}^{l}$	§4.3.2 (2)
		1–6	$\Sigma_{\mathrm{R}}^{\nu}(q)$: Neutrino self-energy part	3 (Fig. 5.7 (2) (3))	$Z_{\mathrm{L}}{}^{\nu}$; $\Sigma_{\mathrm{C}}{}^{\nu}$	§4.3.2 (2)
	Vertex corrections	2. Three-point functions				
		2–1	$\Gamma_{\mathrm{R}\alpha}^{eeA}(p_e{}', p_e)$: eeA vertex function (*)	4 (Fig. 5.8)	Y; $\Gamma_{\mathrm{C}\alpha}^{eeA}$	§4.3.4
		2–2	$\Gamma_{\mathrm{R}\alpha}^{eeZ}(p_e{}', p_e)$: eeZ vertex function (*)	5 (Fig. 5.9)	$\Gamma_{\mathrm{C}\alpha}^{eeZ}$	§4.3.6
		2–3	$\Gamma_{\mathrm{R}\alpha}^{\nu\nu Z}(p_\nu{}', p_\nu)$: $\nu\nu Z$ vertex function	4 (Fig. 5.10)	$\Gamma_{\mathrm{C}\alpha}^{\nu\nu Z}$	§4.3.6
		2–4	$\Gamma_{\mathrm{R}\alpha}^{\nu\nu A}(p_\nu{}', p_\nu)$: $\nu\nu A$ vertex function	3 (Fig. 5.11)	$\Gamma_{\mathrm{C}\alpha}^{\nu\nu A}$	§4.3.6
		3. Four-point functions				
			Box diagrams	3 (Fig. 5.1 (7) (8) (9))	No UV- and IR-divergences	
IR-div.		4. Infrared divergences				
			Real photon emissions	2 (Fig. 5.12)		
			Soft photon effect			
			Hard photon effect			
		5. Cross sections		Arrangement of the invariant amplitudes →Numerical computation→$d\sigma/dt$		

['t Hooft and Veltman 72a]. For the infrared divergent integrals we calculate them by using both the dimensional regularization method and non-gauge invariant conventional method in which the fictitious photon mass λ is introduced. The infinite constants in the D-dimensional regularization are denoted as

$$C_{\mathrm{UV}} \equiv (4\pi\mu^2)^{\varepsilon_{\mathrm{U}}}\Gamma(\varepsilon_{\mathrm{U}}) \simeq (1/\varepsilon_{\mathrm{U}}) - \gamma + \ln(4\pi\mu^2), \tag{5·1}$$

$$C_{\mathrm{IR}} \equiv (4\pi\mu^2)^{\varepsilon_{\mathrm{I}}}\Gamma(\varepsilon_{\mathrm{I}}) \simeq (1/\varepsilon_{\mathrm{I}}) - \gamma + \ln(4\pi\mu^2), \tag{5·2}$$

Table 5.2. A scenario of one-loop calculation ($\nu e \to \mu\nu$).

Steps			What is to be calculated	Numbers of Feynman diagrams	Renormalization constants and counter terms to be determined	Renormalization conditions
UV-divergent (and IR-divergent for (*))	Self-energy corrections	1. Two-point functions				
		1-1	$A_R^Z(q^2)$: Z boson self-energy part	13 (Fig. 5.3)	δM_Z^2, A_C^Z	§4.3.2 (1)
		1-2	$A_R^W(q^2)$: W boson self-energy part (*)	19 (Fig. 5.6)	δM_W^2, Z_W; A_C^W	§4.3.2 (1)
		1-3	$A_R^{ZA}(q^2)$: ZA transition self-energy part	9 (Fig. 5.4)	$Z_{ZA}^{1/2}$, $Z_{AZ}^{1/2}$; A_C^{ZA}	§4.3.2 (1)
		1-4	$A_R^A(q^2)$: Photon self-energy part	9 (Fig. 5.5)	$Z_{AA}^{1/2}$; A_C^A	§4.3.2 (1)
		1-5	$\Sigma_R^l(q)$: Charged lepton self-energy part (*)	4 (Fig. 5.7)	$\delta m_l, Z_L^l, Z_R^l$; Σ_C^l	§4.3.2 (2)
		1-6	$\Sigma_R^\nu(q)$: Neutrino self-energy part	3 (Fig. 5.7 (2) (3))	Z_L^ν; Σ_C^ν	§4.3.2 (2)
	Vertex corrections	2. Three-point functions				
		2-1	$\Gamma_{R\alpha}^{eeA}(p_e', p_e)$: eeA vertex function (*)	4 (Fig. 5.8)	Y; $\Gamma_{C\alpha}^{eeA}$	§4.3.4
		2-2	$\Gamma_{R\alpha}^{\nu lW}(p_\nu', p_l)$: νlW vertex function ($l=e, \mu$)	5 (Fig. 5.13)	$\Gamma_{C\alpha}^{\nu lW}$	§4.3.6
IR-divergent		3. Four-point functions				
			Box diagrams	5 (Fig. 5.2, (5)$\sim$(9))	No UV-divergence	
		4. Infrared divergences				
			Real photon emissions	(3)$\to$2 (Fig. 5.14)		
			Soft photon effect			
			Hard photon effect			
		5. Cross sections		Arrangement of invariant amplitudes $\to$Numerical computation$\to d\sigma/dt$		

$$4-D\equiv 2\varepsilon_U \quad \text{or} \quad 2\varepsilon_I \tag{5·3}$$

for ultraviolet and infrared divergences, respectively, where γ is the Euler constant and μ is introduced on the dimensional reason. We have added the suffix U or I to ε as a convention to distinguish ultraviolet and infrared divergences. In our following calculations we choose $\mu^2=1$ without the loss of generality. As for the infrared divergent integrals which appear in our one-loop calculations, it can be seen that the correspondence

$$C_{IR} \leftrightarrow \ln(\lambda^2) \tag{5·4}$$

between the infinite constants obtained by the dimensional regularization method and those by the conventional method, and that for the finite parts of integrals both the methods give the completely same results (see also [Marciano and Sirlin 75]).

To the one-loop corrections for the $\nu_\mu(\bar{\nu}_\mu)$-e scattering contribute the Feynman diagrams of the types shown in Fig. 5.1, where diagram (1) is the tree diagram and the blobs in the other diagrams mean all possible one-loop corrections after reducing the number of diagrams by the above-mentioned approximation. These corrections consist of the self-energy corrections (two-point functions, Fig. 5.1 (2) (3)) and the vertex corrections which include the three-point functions (Fig. 5.1 (4) (5) (6)) and the four-point functions (Fig. 5.1 (7) (8) (9)). We must determine the renormalization constants to carry out our renormalization program on these functions. For the two-point functions the following renormalization constants come to be necessary as one can see from § 4.3.2 (1):

for the Z boson self-energy part (Fig. 5.1 (2)),

δM_Z^2, $Z_{ZZ}^{1/2}$ and $Z_{ZA}^{1/2}$;

for the ZA transition self-energy part (Fig. 5.1 (3)),

δM_Z^2, $Z_{ZZ}^{1/2}$, $Z_{ZA}^{1/2}$, $Z_{AZ}^{1/2}$ and $Z_{AA}^{1/2}$.

For the three-point functions we should have more renormalization constants in our hand (§ 4.3.6):

for eeZ vertex function (Fig. 5.1 (4)),*)

δM_Z^2, δM_W^2, $Z_{ZZ}^{1/2}$, $Z_{AZ}^{1/2}$, $Z_{\rm L}^e$, $Z_{\rm R}^e$ and Y;

for $\nu\nu Z$ vertex function (Fig. 5.1 (5)),

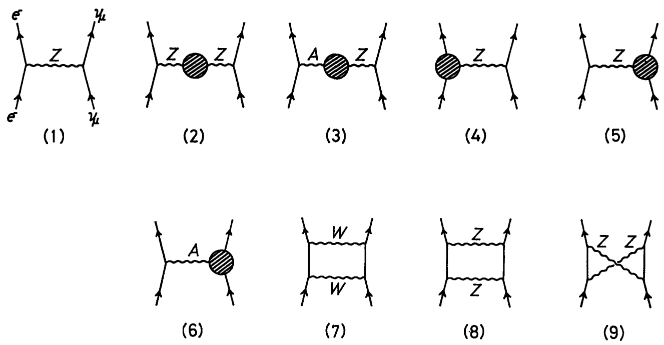


Fig. 5.1. Relevant diagrams of one-loop corrections for the processes $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$.

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*) In the present case, we need only the diagonal part of the renormalization constants of leptons $(Z_L^{1/2})_{nm}$ since we do not consider the lepton mixing (we set $m_\nu=0$). We express $(Z_L^{1/2})_{ll}$ as $(Z_L^l)^{1/2}$.

δM_Z^2, δM_W^2, $Z_{ZZ}^{1/2}$, Z_L^ν and Y;

for $\nu\nu A$ vertex function (Fig. 5.1 (6)),

δM_Z^2, δM_W^2, $Z_{ZA}^{1/2}$, Z_L^ν and Y.

First we determine the renormalization constants δM_Z^2, $Z_{ZZ}^{1/2}$, $Z_{ZA}^{1/2}$, $Z_{AZ}^{1/2}$ and $Z_{AA}^{1/2}$ in Steps 1–1∼1–3 of Table 5.1 by referring to the renormalization conditions in § 4.3.2 (1). For determining the charge renormalization constant Y from the eeA-vertex we need moreover the renormalization constants Z_L^e, Z_R^e and δM_W^2 in addition to those known in the above (§ 4.3.4). Then we prepare the renormalization constants δM_W^2 and Z_W in Step 1–4 (Z_W is for the later use) and Z_L^l, Z_R^l ($l=e$, μ) and Z_L^ν in Steps 1–5 and 1–6 in consideration of the renormalization of the above-mentioned vertex functions. Now we can determine the charge renormalization constant Y in Step 2–1 and obtain all the necessary renormalization constants. The explicit forms of renormalization constants are given in Appendix E. In terms of these constants we fix the counter terms for three-point functions in Steps 2–2∼2–4. For the four-point functions in Step 3 we have neither ultraviolet nor infrared divergences. In this way we arrive at the ultraviolet divergence free amplitude for the $\nu_\mu(\bar{\nu}_\mu)$-e scattering in the one-loop order.

In Step 4 we calculate the cross section for the real photon emission $\nu_\mu(\bar{\nu}_\mu)e\to\nu_\mu(\bar{\nu}_\mu)e\gamma$. It should be added to the cross section for the elastic scattering to cancel out the infrared divergences which remained in the transition amplitude. The work that sums up every part calculated up to this and makes the square is left to the computer's care in the final step 5 (Chapter 6).

The Feynman diagrams of the types seen in Fig. 5.2 contribute to the electroweak radiative corrections for the reaction $\nu_\mu e^-\to\mu^-\nu_e$ in the one-loop order, where the first one is the tree diagram. The renormalization constants which come to be necessary for our purpose in this case are

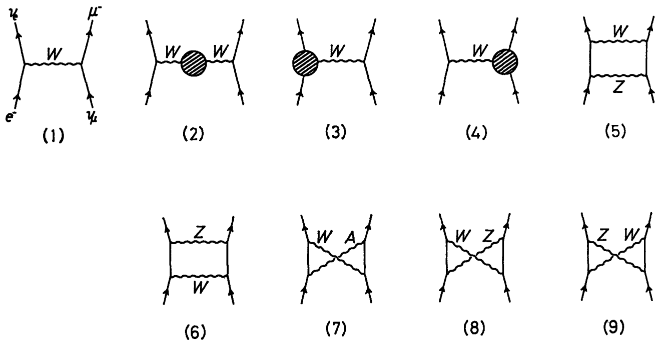

Fig. 5.2. Relevant diagrams of one-loop corrections for the process $\nu_\mu e\to\mu\nu_e$ (and $\mu\to e\nu_\mu\bar{\nu}_e$).

$$\delta M_W^2,\ Z_W;\ \delta M_Z^2,\ Z_L{}^l\ \ (l=e,\mu),\ Z_L{}^\nu \text{ and } Y\,.$$

The procedures which are necessary for determining these constants, i.e., the steps above the dotted line in Table 5.2 are already completed in the calculation of ν-e scattering. Then substantially new jobs are Step 2–2 (Fig. 5.2 (3) (4)) and Step 3 (Fig. 5.2 (5)$\sim$(9)) only. In the calculation of the four-point functions we should take care of the infrared divergence which comes out of diagram (7) in Fig. 5.2. In Step 4 we can neglect the contribution from diagram (2) in Fig. 5.15 in our approximation and apply the foregoing results to this case.

In what follows we will give the renormalized results in every step of our scenario.

§ 5.3 Neutral current processes: $\nu_\mu e^-\to\nu_\mu e^-$ and $\bar\nu_\mu e^-\to\bar\nu_\mu e^-$

The transition amplitudes for the processes $\nu_\mu e^-\to\nu_\mu e^-$ and $\bar\nu_\mu e^-\to\bar\nu_\mu e^-$ are of the general forms

$$\langle\nu_\mu e|T|\nu_\mu e\rangle=\bar u_e(p_e')\left[\gamma^\mu\{A(t)-B(t)\gamma_5\}+C(t)\frac{(p_e'+p_e)^\mu}{2m_e}\right]u_e(p_e)$$
$$\times\bar u_\nu(p_\nu')\gamma_\mu(1-\gamma_5)u_\nu(p_\nu) \tag{5·5a}$$

and

$$\langle\bar\nu_\mu e|T|\bar\nu_\mu e\rangle$$
$$=\text{the same expression as the above except the replacement } u_\nu\to v_\nu, \tag{5·5b}$$

respectively, where the transition amplitudes T is defined by Eq. (4·42) and we adopt the normalization $\bar u u=2m$. In terms of these invariant amplitudes A, B and C, which are the functions of the squared momentum transfer q^2 or $t\equiv(p_e'-p_e)^2$, the cross sections are written as

$$\frac{d\sigma}{dt}=\frac{1}{2\pi}\left[|A+\xi B|^2+|A-\xi B|^2\left(\frac{u-m_e^2}{s-m_e^2}\right)^2+2(|A|^2-|B|^2)\frac{m_e^2t}{(s-m_e^2)^2}\right.$$
$$\left.+4\mathrm{Re}(AC^*)\left\{1+\frac{st}{(s-m_e^2)^2}\right\}\right], \tag{5·6}$$

where $\xi=1$ for $\nu_\mu e$ scattering and $\xi=-1$ for $\bar\nu_\mu e$ scattering.

Then we must find the well-defined expressions of the invariant amplitudes A, B and C. We begin with the determination of the renormalization constants δM_Z^2, $Z_{ZZ}^{1/2}$, $Z_{ZA}^{1/2}$, $Z_{AZ}^{1/2}$ and $Z_{AA}^{1/2}$ which are defined by Eq. (4·27). We

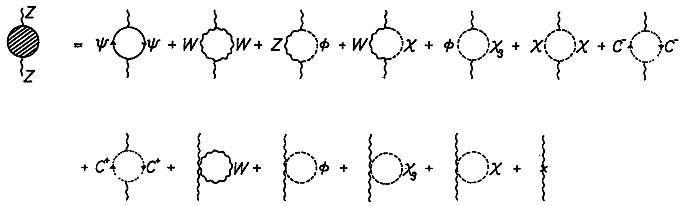

Fig. 5.3. The Z boson self-energy diagrams.

carry out the calculations in the Feynman gauge. The explicit forms of renormalization constants are all given collectively in Appendix E.

Step 1-1 *Z boson self-energy part*

For the blob of diagram (2) in Fig. 5.1 are thirteen Feynman diagrams shown in Fig. 5.3 where ψ stands for all relevant fermions and the last one expresses the counter term. By using formulas in Appendix B.1, we obtain the transverse part of the renormalized Z boson self-energy $A_{\mathrm{R}}{}^{Z}(q^2)$

$$A_{\mathrm{R}}{}^{Z}(q^2) = \sum_{n=1}^{12} A_n{}^{Z}(q^2) + A_{\mathrm{C}}{}^{Z}(q^2), \tag{5·7}$$

where $A_{\mathrm{C}}{}^{Z}(q^2)$ is the counter term corresponding to the last diagram in Fig. 5.3. The terms on the right-hand side are explicitly given by

$$\begin{aligned}
&\sum_{n=1}^{12} A_n{}^{Z}(q^2) \\
&\quad = \frac{e^2}{32\pi^2 M_W{}^2(M_Z{}^2 - M_W{}^2)}\Big[\sum_i M_Z{}^4\Big\{-\frac{1}{6}q^2(\eta_i{}^2+1)+m_i{}^2\Big\} \\
&\qquad -2M_Z{}^2(M_Z{}^4+2M_Z{}^2M_W{}^2-4M_W{}^4) \\
&\qquad -\frac{1}{3}q^2(M_Z{}^4-2M_Z{}^2M_W{}^2-18M_W{}^4)\Big]C_{\mathrm{UV}} \\
&\quad + \frac{e^2}{32\pi^2 M_W{}^2(M_Z{}^2 - M_W{}^2)}\Big[\sum_i M_Z{}^4\{(\eta_i{}^2+1)q^2F(m_i, m_i, q^2) \\
&\qquad -m_i{}^2F_0(m_i, m_i, q^2)\} - \frac{q^2}{3}(M_Z{}^4-2M_Z{}^2M_W{}^2+4M_W{}^4) \\
&\qquad +M_W{}^2(M_Z{}^4-4M_Z{}^2M_W{}^2+16M_W{}^4)\ln M_W{}^2 \\
&\qquad +\frac{1}{2}M_Z{}^4(M_Z{}^2\ln M_Z{}^2+m_\phi{}^2\ln m_\phi{}^2)-10M_W{}^4q^2F_0(M_W, M_W, q^2) \\
&\qquad +(M_Z{}^4-4M_Z{}^2M_W{}^2+24M_W{}^4)q^2F(M_W, M_W, q^2) \\
&\qquad +M_W{}^2(3M_Z{}^4-4M_Z{}^2M_W{}^2-16M_W{}^4)F_0(M_W, M_W, q^2)
\end{aligned}$$

$$+M_Z^4\{2M_Z^2F_0(m_\phi, M_Z, q^2)-M_Z^2F_1(m_\phi, M_Z, q^2)+q^2F(m_\phi, M_Z, q^2)\}$$

$$\left.-m_\phi^2M_Z^4F_1(M_Z, m_\phi, q^2)\right]. \tag{5·8}$$

In the one-loop order, the counter term given in § 4. 3. 2 (1) is reduced to

$$A_C^Z(q^2)=\delta M_Z^2+2Z_{ZZ}^{1/2}(M_Z^2-q^2). \tag{5·9}$$

On the right-hand side of Eq. (5·8), $\sum_i$ denotes the sum on the species of fermions and the sum over the color degrees of freedom must be included. The η_i's should be understood as

$$\begin{aligned}
&\eta_\nu\equiv 1\,,\\
&\eta_{e,\mu,\tau}\equiv(4M_W^2-3M_Z^2)/M_Z^2\,,\\
&\eta_{u,c,t}\equiv(8M_W^2-5M_Z^2)/3M_Z^2\,,\\
&\eta_{d,s,b}\equiv(4M_W^2-M_Z^2)/3M_Z^2\,.
\end{aligned} \tag{5·10}$$

The functions F_n $(n=0,1,2)$ and F are defined by

$$F_n(M_1, M_2, q^2)\equiv\int_0^1 x^n\ln\{M_1^2(1-x)+M_2^2x-q^2x(1-x)\}dx\,,$$

$$(n=0, 1, 2)$$

$$F(M_1, M_2, q^2)\equiv F_1(M_1, M_2, q^2)-F_2(M_1, M_2, q^2). \tag{5·11}$$

Their explicit forms are found in Appendix D.

The renormalization conditions in the one-loop order are

$$\sum_{n=1}^{12}A_n^Z(M_Z^2)+\delta M_Z^2=0 \tag{5·12}$$

and

$$\sum_{n=1}^{12}A_n^{Z\prime}(M_Z^2)-2Z_{ZZ}^{1/2}=0, \tag{5·13}$$

from which are determined the renormalization constants δM_Z^2 and $Z_{ZZ}^{1/2}$, respectively.

Step 1–2 *ZA transition self-energy part*

For diagram (3) in Fig. 5. 1 are nine Feynman diagrams which are shown in Fig. 5. 4. We call them the ZA transition self-energy part. The contributions to the transverse part from these diagrams are given by

$$A_R^{ZA}(q^2)=\sum_{n=1}^{8}A_n^{ZA}(q^2)+A_C^{ZA}(q^2), \tag{5·14}$$

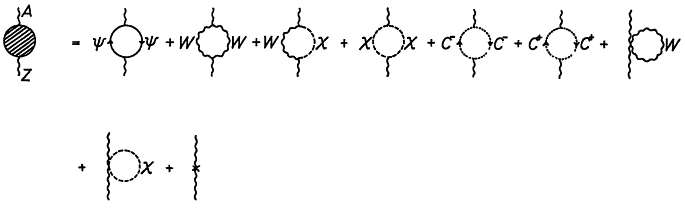

Fig. 5.4. The transition self-energy diagrams between the Z boson and the photon.

where

$$\sum_{n=1}^{8} A_n{}^{ZA}(q^2)$$

$$=\frac{e^2 M_W}{16\pi^2\sqrt{M_Z^2-M_W^2}}\Big[\Big\{\frac{q^2}{6}\Big(-\sum_i \zeta_i+\frac{M_Z^2}{M_W^2}+18\Big)+2M_Z^2\Big\}C_{\rm UV}$$

$$+(-M_Z^2+8M_W^2)\ln M_W^2-(M_Z^2+8M_W^2)F_0(M_W, M_W, q^2)$$

$$+q^2\Big\{\sum_i \zeta_i F(m_i, m_i, q^2)-\frac{M_Z^2-12M_W^2}{M_W^2}F(M_W, M_W, q^2)$$

$$-5F_0(M_W, M_W, q^2)+\frac{M_Z^2-4M_W^2}{6M_W^2}\Big\}\Big] \qquad (5\cdot15)$$

and

$$A_C{}^{ZA}(q^2)=Z_{ZA}^{1/2}(M_Z^2-q^2)-Z_{AZ}^{1/2}q^2 \qquad (5\cdot16)$$

is the counter term. In Eq. (5·15) the ζ_i's are

$$\zeta_\nu\equiv 0\,,$$

$$\zeta_{e,\mu,\tau}\equiv 2(4M_W^2-3M_Z^2)/M_W^2\,,$$

$$\zeta_{u,c,t}\equiv 4(8M_W^2-5M_Z^2)/9M_W^2$$

and

$$\zeta_{d,s,b}\equiv 2(4M_W^2-M_Z^2)/9M_W^2\,. \qquad (5\cdot17)$$

According to § 4.3.2 (1), the renormalization constants $Z_{AZ}^{1/2}$ and $Z_{ZA}^{1/2}$ are determined by the conditions

$$\sum_{n=1}^{8} A_n{}^{ZA}(M_Z^2)-M_Z^2 Z_{AZ}^{1/2}=0 \qquad (5\cdot18)$$

and

$$\sum_{n=1}^{8} A_n{}^{ZA}(0)+M_Z^2 Z_{ZA}^{1/2}=0\,, \qquad (5\cdot19)$$

respectively.

Step 1–3 *Photon self-energy part*

Relevant Feynman diagrams in this case are shown in Fig. 5.5.

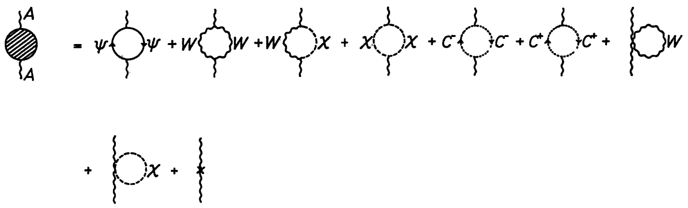

Fig. 5.5. The photon self-energy diagrams.

The transverse part of the renormalized photon self-energy $A_R{}^A(q^2)$ is given by

$$A_R{}^A(q^2) = \sum_{n=1}^{8} A_n{}^A(q^2) + A_C{}^A(q^2), \tag{5·20}$$

where

$$\begin{aligned}\sum_{n=1}^{8} A_n{}^A(q^2) = & \frac{e^2}{\pi^2}q^2\left(-\frac{2}{9}N+\frac{3}{16}\right)C_{\mathrm{UV}} \\ & +\frac{e^2}{16\pi^2}\Big[8\sum_i Q_i{}^2 q^2 F(m_i, m_i, q^2) \\ & +8M_W{}^2\{\ln M_W{}^2 - F_0(M_W, M_W, q^2)\} \\ & +q^2\left\{-\frac{2}{3}+12F(M_W, M_W, q^2)-5F_0(M_W, M_W, q^2)\right\}\Big]\end{aligned}$$

(N: the number of generations. We take $N=3$.) (5·21)

and the counter terms

$$A_C{}^A(q^2) = -2q^2 Z_{AA}^{1/2}. \tag{5·22}$$

It can be seen from Eqs. (5·21) and (5·22) that one of the renormalization conditions $A_R{}^A(0)=0$ is satisfied automatically, by observing $F_0(M_W, M_W, 0) = \ln M_W{}^2$. This is guaranteed by the remaining $U(1)$ symmetry in the theory. The other renormalization condition $A_R{}^{A\prime}(0)=0$ is reduced to

$$\sum_{n=1}^{8} A_n^{A\prime}(0) - 2Z_{AA}^{1/2}=0, \tag{5·23}$$

from which is determined the renormalization constant $Z_{AA}^{1/2}$. As seen in Steps 1–1 to 1–3, in the one-loop order we can determine independently every

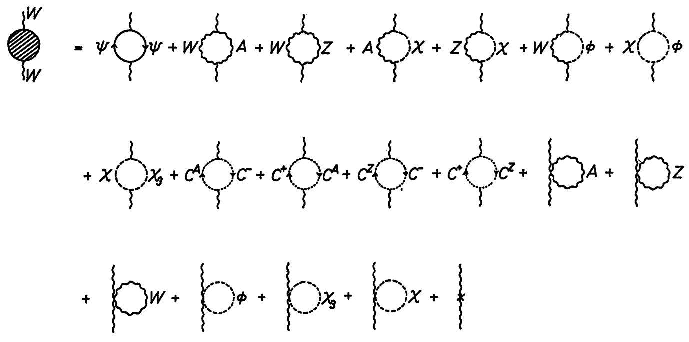

Fig. 5.6. The W boson self-energy diagrams.

renormalization constant in the neutral gauge boson sector. Then we go on the preparation to determine the charge renormalization constant Y, i.e., on the determination of the renormalization constants $\delta M_W{}^2$, Z_W, $Z_L{}^l$, $Z_R{}^l$ and $Z_L{}^\nu$.

Step 1-4 *W boson self-energy part*

The transverse part of the renormalized W boson self-energy $A_R^W(q^2)$ (see Fig. 5.6) is given by

$$A_R^W(q^2)=\sum_{n=1}^{18} A_n{}^W(q^2)+A_C^W(q^2). \tag{5·24}$$

The renormalization constants $\delta M_W{}^2$ and Z_W are determined by

$$\sum_{n=1}^{18} A_n{}^W(M_W^2)+\delta M_W{}^2=0\,, \tag{5·25}$$

$$\sum_{n=1}^{18} A_n^{W\prime}(M_W^2)-Z_W=0\,, \tag{5·26}$$

where

$$A_C^W(q^2)=\delta M_W^2+Z_W(M_W^2-q^2),$$

$$\sum_{n=1}^{18} A_n{}^W(q^2)=\frac{e^2M_Z^2}{32\pi^2(M_Z^2-M_W^2)}\left\{\sum_{(I,i)}|U_{Ii}|^2\left(m_i^2+m_I{}^2-\frac{2}{3}q^2\right)\right.$$

$$\left.-2(M_Z^2-2M_W^2)+\frac{19}{3}q^2\right\}C_{\mathrm{UV}}$$

$$+\frac{e^2}{32\pi^2(M_Z^2-M_W^2)}\left[2M_Z^2\sum_{(I,i)}|U_{Ii}|^2\{2q^2F(m_I,m_i,q^2)\right.$$

$$-m_i^2F_1(m_I,m_i,q^2)-m_I{}^2F_1(m_i,m_I,q^2)\}$$

$$-M_Z^2\left\{q^2-7M_W{}^2\ln M_W{}^2-\left(6M_W{}^2+\frac{M_Z^2}{2}\right)\ln M_Z^2-\frac{1}{2}m_\phi{}^2\ln m_\phi{}^2\right\}$$

$$
\begin{aligned}
&+(2M_Z^4-5M_Z^2M_W^2-14M_W^4)F_0(M_W, M_Z, q^2)\\
&-10M_W^2q^2F_0(M_W, M_Z, q^2)\\
&-(M_Z^4+15M_Z^2M_W^2-16M_W^4)F_1(M_W, M_Z, q^2)\\
&+(M_Z^2+20M_W^2)q^2F(M_W, M_Z, q^2)\\
&-2(M_Z^2-M_W^2)\{7M_W^2F_0(M_W, \lambda, q^2)+5q^2F_0(M_W, \lambda, q^2)\\
&-8M_W^2F_1(M_W, \lambda, q^2)-10q^2F(M_W, \lambda, q^2)\}\\
&-m_\phi{}^2M_Z^2\{F_0(m_\phi, M_W, q^2)-F_1(m_\phi, M_W, q^2)\}\\
&+M_Z^2M_W^2\{2F_0(m_\phi, M_W, q^2)-F_1(m_\phi, M_W, q^2)\}\\
&+M_Z^2q^2F(m_\phi, M_W, q^2)\Bigg],
\end{aligned}
\tag{5·27}
$$

where λ is the small photon mass introduced to regularize the infrared divergence, and U_{li} is the Cabibbo-like mixing matrix. (For the practical calculation we consider only the Cabibbo angle.)

Step 1-5 *Charged lepton self-energy part*

The relevant Feynman diagrams are shown in Fig. 5.7. The contributions of graphs (1) ~ (3) are

$$
\begin{aligned}
&\sum_{i=1}^{3}\Sigma_i{}^l(q)\\
&=\frac{e^2}{16\pi^2}\frac{1}{8M_W^2(M_Z^2-M_W^2)}[\{-16m_lM_Z^2(M_Z^2-M_W^2)\\
&+M_Z^2(5M_Z^2-2M_W^2)\not{q}+3M_Z^2(M_Z^2-2M_W^2)\not{q}\gamma_5\}C_{\mathrm{UV}}\\
&+8m_e(M_Z^2-M_W^2)\{M_Z^2+4M_W^2F_0(m_e, \lambda, q^2)\\
&+2(M_Z^2-2M_W^2)F_0(m_e, M_Z, q^2)\}\mathbf{1}\\
&-\{M_Z^2(5M_Z^2-2M_W^2)\\
&+16M_W^2(M_Z^2-M_W^2)F_1(m_e, \lambda, q^2)+4M_Z^2M_W^2F_1(0, M_W, q^2)\\
&+2(5M_Z^4-12M_Z^2M_W^2+8M_W^4)F_1(m_e, M_Z, q^2)\}\not{q}\\
&-M_Z^2\{3(M_Z^2-2M_W^2)-4M_W^2F_1(0, M_W, q^2)\\
&+2(3M_Z^2-4M_W^2)F_1(m_e, M_Z, q^2)\}\not{q}\gamma_5].
\end{aligned}
\tag{5·28}
$$

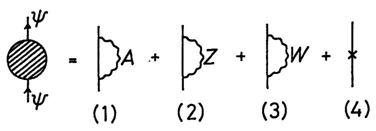

Fig. 5.7. The fermion self-energy diagrams.

By using the counter terms

$$\Sigma_{\rm C}{}^{l}(q) = -\delta m_l \mathbf{1} + \frac{1}{2}(Z_{\rm L}{}^{l}+Z_{\rm R}{}^{l})(\not{q}-m_l)\mathbf{1} - \frac{1}{2}(Z_{\rm L}{}^{l}-Z_{\rm R}{}^{l})\not{q}\gamma_5, \tag{5·29}$$

renormalization constants $Z_{\rm L}{}^{l}$, $Z_{\rm R}{}^{l}$ and δm_l are determined from the following conditions:

$$\begin{aligned} &\Sigma_1{}^{l}(m_l{}^2) + m_l \Sigma_\gamma{}^{l}(m_l{}^2) = 0, \\ &\Sigma_{5\gamma}^{l}(m_l{}^2) = 0, \\ &2m_l\{(\Sigma_1{}^{l\prime}(m_l{}^2) + m_l\Sigma_\gamma{}^{l\prime}(m_l{}^2)\} + \Sigma_\gamma{}^{l}(m_l{}^2) = 0. \end{aligned} \tag{5·30}$$

Here we have expressed the renormalized self-energy as

$$\Sigma_{\rm R}{}^{l}(q) \equiv \Sigma_1{}^{l}(q^2)\mathbf{1} + \Sigma_\gamma{}^{l}(q^2)\not{q} + \Sigma_{5\gamma}^{l}(q^2)\not{q}\gamma_5. \tag{5. 31}$$

Step 1–6 *Neutrino self-energy part*

The relevant diagrams for neutrino self-energy are the same as those in Fig 5. 7, but except for graph (1). Renormalized self-energy is

$$\Sigma_{\rm R}{}^{\nu}(q) = \sum_{n=2,3} \Sigma_n{}^{\nu}(q) + \Sigma_{\rm C}{}^{\nu}(q), \tag{5·32}$$

where

$$\begin{aligned} &\sum_{n=2,3} \Sigma_n{}^{\nu}(q) \\ &= \frac{e^2}{16\pi^2}\frac{M_Z^2}{8M_W^2(M_Z^2-M_W^2)}\{(M_Z^2+2M_W^2)C_{\rm UV} - (M_Z^2+2M_W^2) \\ &\qquad -4M_W^2F_1(m_l, M_W, q^2) - 2M_Z^2F_1(0, M_Z, q^2)\}\not{q}(1-\gamma_5), \\ &\Sigma_{\rm C}{}^{\nu}(q) = \frac{1}{2}Z_{\rm L}{}^{\nu}\not{q}(1-\gamma_5). \end{aligned} \tag{5·33}$$

$Z_{\rm L}{}^{\nu}$ is determined by

$$\Sigma_{\rm R}{}^{\nu}(q)|_{q^2=0} = 0. \tag{5·34}$$

Step 2–1 *eeA vertex function*

Using the renormalization constants which have so far been calculated, we determine the charge renormalization constants Y through *eeA* vertex.

The relevant diagrams are shown in Fig. 5. 8.

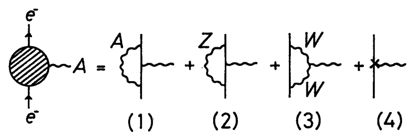

Fig. 5. 8. Relevant diagrams of *eeA* vertex.

$$\Gamma^{eeA}_{R\alpha}(p_e', p_e) = \sum_{n=1}^{3} \Gamma^{eeA}_{n\alpha}(p_e', p_e) + \Gamma^{eeA}_{C\alpha}, \tag{5·35}$$

where

$$\Gamma^{eeA}_{1\alpha}(p_e', p_e) = \frac{e^2}{16\pi^2}\left\{F(q^2)\gamma_\alpha + G(q^2)\frac{(p_e+p_e')_\alpha}{2m_e}\right\}, \tag{5·36}$$

$$F(q^2) = C_{\mathrm{UV}} - \ln m_e^2 + \frac{3}{a}\ln\frac{1+a}{1-a} - \frac{1+a^2}{a}$$
$$\times\left\{\frac{1}{2}\ln\frac{4}{1-a^2}\ln\frac{1+a}{1-a} - (C_{\mathrm{IR}} - \ln m_e^2)\ln\frac{1+a}{1-a} - \phi(a)\right\}, \tag{5·36a}$$

$$G(q^2) = -\frac{1-a^2}{a}\ln\frac{1+a}{1-a}. \tag{5·36b}$$

$$\sum_{n=2}^{3}\Gamma^{eeA}_{n\alpha}(p_e', p_e) = -\frac{e^2}{16\pi^2}\frac{eM_Z^2}{8M_W^2(M_Z^2-M_W^2)}$$
$$\times\left\{\left[\left(5M_Z^2 - 6M_W^2 + 8\frac{M_W^4}{M_Z^2}\right)\gamma_\alpha + (3M_Z^2 - 10M_W^2)\gamma_\alpha\gamma_5\right]C_{\mathrm{UV}}\right.$$
$$-\left[\left(\frac{5}{2}M_Z^2 - 5M_W^2 + 4\frac{M_W^4}{M_Z^2}\right) + 6M_W^2\ln M_W^2\right.$$
$$\left.+\left(5M_Z^2 - 12M_W^2 + 8\frac{M_W^4}{M_Z^2}\right)\ln M_Z^2\right]\gamma_\alpha$$
$$\left.+\left[\left(-\frac{3}{2}M_Z^2 + 3M_W^2\right) + 6M_W^2\ln M_W^2 - (3M_Z^2 - 4M_W^2)\ln M_Z^2\right]\gamma_\alpha\gamma_5\right\}, \tag{5·37}$$

$$\Gamma^{eeA}_{C\alpha} = -e\gamma_\alpha\left(Y + Z^{1/2}_{AA} + \frac{Z_L^e + Z_R^e}{2} - \frac{Z_L^e - Z_R^e}{2}\gamma_5\right)$$
$$+\frac{e}{4M_W\sqrt{M_Z^2 - M_W^2}}Z^{1/2}_{ZA}\gamma_\alpha\{(3M_Z^2 - 4M_W^2) + M_Z^2\gamma_5\}. \tag{5·38}$$

In Eq. (5·36a), we have used the following function,

$$\phi(a) \equiv \mathrm{Sp}\left(\frac{1+a}{2}\right) - \mathrm{Sp}\left(\frac{1-a}{2}\right), \tag{5·39}$$
$$\left(a \equiv \sqrt{\frac{-q^2}{-q^2+4m_e^2}}\right)$$

and

$$\mathrm{Sp}(x) \equiv -\int_0^x \frac{1}{t}\ln(1-t)\,dt. \quad \text{(Spence function)} \tag{5·40}$$

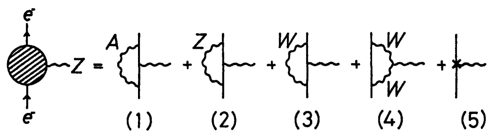

Fig. 5.9. The *eeZ* vertex diagrams.

Some useful formulas on the Spence function are given in Appendix F.

Step 2–2 *eeZ vertex function*

Now we can go on to evaluate the correction for *eeZ* vertex (Fig. 5.1 (4)) with the initial electron momentum p_e and the final one p_e'. Only five Feynman diagrams shown in Fig. 5.9 should be considered on our approximation. This correction is expressed as

$$\Gamma^{eeZ}_{R\alpha}(p_e', p_e) = \sum_{i=1}^{4} \Gamma^{eeZ}_{i\alpha}(p_e', p_e) + \Gamma^{eeZ}_{C\alpha}\,, \tag{5·41}$$

where $\Gamma^{eeZ}_{i\alpha}(p_e', p_e)$ are contributions from four diagrams ($i=1, 2, 3, 4$) respectively and $\Gamma^{eeZ}_{C\alpha}$ is the relevant counter terms. Their explicit forms are given as follows:

$$\begin{aligned}\Gamma^{eeZ}_{1\alpha}(p_e', p_e) &= \frac{e^3}{64\pi^2 M_W\sqrt{M_Z^2-M_W^2}}\\ &\times\Big[(3M_Z^2-4M_W^2)\Big\{F(q^2)\gamma_\alpha + G(q^2)\frac{(p_e'+p_e)_\alpha}{2m_e}\Big\}\\ &+M_Z^2\Big\{F_5(q^2)\gamma_\alpha\gamma_5 + G_5(q^2)\frac{(p_e'-p_e)_\alpha}{2m_e}\gamma_5\Big\}\Big].\end{aligned} \tag{5·42}$$

In Eq. (5·42), $F(q^2)$ and $G(q^2)$ are found in Eqs. (5·36a) and (5·36b) respectively,

$$\begin{aligned}F_5(q^2) &= C_{\rm UV} - \ln m_e^2 + \frac{1+2a^2}{a}\ln\Big(\frac{1+a}{1-a}\Big)\\ &-\frac{1+a^2}{a}\Big\{\frac{1}{2}\ln\Big(\frac{4}{1-a^2}\Big)\ln\Big(\frac{1+a}{1-a}\Big) - (C_{\rm IR}-\ln m_e^2)\ln\Big(\frac{1+a}{1-a}\Big) - \phi(a)\Big\}\end{aligned} \tag{5·42a}$$

and

$$G_5(q^2) = \frac{1-a^2}{a^2}\Big\{\frac{1+2a^2}{a}\ln\Big(\frac{1+a}{1-a}\Big) - 2\Big\}. \tag{5·42b}$$

$$\sum_{i=2}^{4} \Gamma^{eeZ}_{i\alpha}(p_e', p_e)$$

$$= \frac{e^3}{64\pi^2} \frac{M_Z^2}{(M_Z^2 - M_W^2)\sqrt{M_Z^2 - M_W^2}}$$

$$\times \left[\frac{-M_Z^4}{16 M_W^3 \xi^3} \left(-C_{\mathrm{UV}} + \frac{1}{2} + \ln M_Z^2 \right) \gamma_\alpha \{(1+3\xi^2) + (3\xi + \xi^3)\gamma_5\} \right.$$

$$+ \left\{ -\frac{M_Z^2}{2M_W} \left(-C_{\mathrm{UV}} + \frac{1}{2} + \ln M_W^2 \right) \right.$$

$$\left. \left. + M_W \left(-3C_{\mathrm{UV}} + \frac{1}{2} + 3\ln M_W^2 \right) \right\} \gamma_\alpha (1 - \gamma_5) \right],$$

$$\left(\xi \equiv \frac{M_Z^2}{3M_Z^2 - 4M_W^2} \right) \tag{5·43}$$

$$\Gamma^{eeZ}_{C\alpha} = -e Z_{AZ}^{1/2} \gamma_\alpha$$

$$- \frac{e M_Z^2}{4 M_W \sqrt{M_Z^2 - M_W^2}}$$

$$\times \left\{ Y + \frac{M_Z^2 - 2M_W^2}{2(M_Z^2 - M_W^2)} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) + Z_{ZZ}^{1/2} + Z_L^e \right\} \gamma_\alpha (1 - \gamma_5)$$

$$+ \frac{e}{M_W} \sqrt{M_Z^2 - M_W^2}\, \gamma_\alpha \left\{ Y + \frac{M_Z^2}{2(M_Z^2 - M_W^2)} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \right.$$

$$\left. + Z_{ZZ}^{1/2} + \frac{Z_L^e + Z_R^e}{2} - \frac{Z_L^e - Z_R^e}{2} \gamma_5 \right\}. \tag{5·44}$$

Steps 2–3 and 2–4 *$\nu\nu Z$ and $\nu\nu A$ vertex functions*

$\nu\nu Z$- and $\nu\nu A$-vertices which correspond to Figs. 5.10 and 5.11 are given as follows:

$$\Gamma^{\nu\nu Z}_{R\alpha}(p_\nu', p_\nu) = \sum_{i=1}^{3} \Gamma^{\nu\nu Z}_{i\alpha}(p_\nu', p_\nu) + \Gamma^{\nu\nu Z}_{C\alpha}, \tag{5·45}$$

$$\sum_{i=1}^{3} \Gamma^{\nu\nu Z}_{i\alpha}(p_\nu', p_\nu)$$

$$= \frac{-e^3 M_Z^2}{64\pi^2 (M_Z^2 - M_W^2)\sqrt{M_Z^2 - M_W^2}} \left\{ -\frac{(M_Z^2 + M_W^2)^2 + 7M_W^4}{4M_W^3} C_{\mathrm{UV}} \right.$$

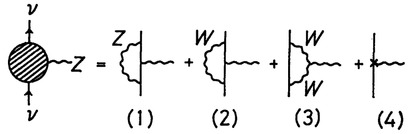

Fig. 5.10. The $\nu\nu Z$ vertex diagrams.

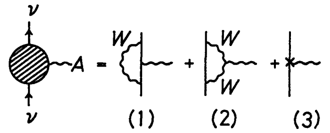

Fig. 5.11. The $\nu\nu A$ vertex diagrams.

$$+\frac{M_Z^4}{4M_W^3}\left(\frac{1}{2}+\ln M_Z^2\right)+\frac{M_Z^2-2M_W^2}{2M_W}\left(\frac{1}{2}+\ln M_W^2\right)$$

$$+M_W\left(\frac{1}{2}+3\ln M_W^2\right)\Big\}\gamma_\alpha(1-\gamma_5), \tag{5·46}$$

$$\Gamma^{\nu\nu Z}_{C\alpha}=\frac{eM_Z^2}{4M_W\sqrt{M_Z^2-M_W^2}}\Big\{Y+\frac{M_Z^2-2M_W^2}{2(M_Z^2-M_W^2)}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)$$

$$+Z_{ZZ}^{1/2}+Z_L^{\nu}\Big\}\gamma_\alpha(1-\gamma_5), \tag{5·47}$$

$$\Gamma^{\nu\nu A}_{R\alpha}(p_\nu', p_\nu)=\sum_{i=1}^{2}\Gamma^{\nu\nu A}_{i\alpha}(p_\nu', p_\nu)+\Gamma^{\nu\nu A}_{C\alpha}, \tag{5·48}$$

$$\sum_{i=1}^{2}\Gamma^{\nu\nu A}_{i\alpha}(p_\nu', p_\nu)=\frac{-e^3M_Z^2}{96\pi^2(M_Z^2-M_W^2)}$$

$$\times\Big[-3C_{\mathrm{UV}}+3\ln M_W^2+\frac{q^2}{M_W^2}\{1+\ln M_W^2$$

$$-6F(m_e, m_e, q^2)\}\Big]\gamma_\alpha(1-\gamma_5), \tag{5·49}$$

where q^2/M_W^2 terms must be maintained in connection with the $1/q^2$ term in the photon propagator. The counter term is

$$\Gamma^{\nu\nu A}_{C\alpha}=\frac{eM_Z^2}{4M_W\sqrt{M_Z^2-M_W^2}}Z_{ZA}^{1/2}\gamma_\alpha(1-\gamma_5). \tag{5·50}$$

Step 3 *Box diagrams*

The four-point functions in our processes come out of the three box diagrams in the one-loop calculation (in Fig. 5.1 (7) (8) (9)). They have no ultraviolet divergence and exchange gauge bosons with large masses, so that the results are very simple under the approximations we have adopted. Referring to Appendix B. 3, we have for the sum of contributions from these three diagrams

$$A(p_e', p_\nu'; p_e, p_\nu)_{(7)+(8)+(9)}$$

$$=\left(\frac{e^2}{4\pi}\right)^2\frac{M_Z^4}{64M_W^4(M_Z^2-M_W^2)^2}\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_\nu$$

$$\times\bar{u}_e\gamma_\mu\Big\{(28M_W^2-9M_Z^2)-\left(15M_Z^2-20M_W^2+\frac{24M_W^4}{M_Z^2}\right)\gamma_5\Big\}u_e. \tag{5·51}$$

In this way we have calculated every necessary part of one-loop corrections. Then we can arrive at the cross section for the $\nu_\mu(\bar{\nu}_\mu)e\to\nu_\mu(\bar{\nu}_\mu)e$ scattering free from ultraviolet divergence through Eq. (5·5).

Step 4 *Infrared divergence*

There still remains, however, a job of which we must make in order to obtain well-defined cross sections. Our amplitudes calculated in the foregoing steps have the infrared singularity and the collinear singularity, too. We can get rid of this infrared singularity by calculating cross sections for final states with finite energy resolution, that is, for those in which a photon of energy k smaller than some maximum energy ω (a soft photon) is emitted, and by adding them to the elastic cross sections (the well-known Kinoshita-Lee-Nauenberg theorem [Kinoshita 62, Lee and Nauenberg 64]; see also [Nakanishi 58]). In the same way one can regulate the collinear singularity by introducing an angular resolution on the direction of outgoing charged lepton and by computing transition rates to nearly degenerate states which include the charged lepton and a collinear photon (not necessarily soft) [Berman 58, Kinoshita and Sirlin 59, Tsai 60, 65, Nieuwenhuizen 71, Berends et al. 73a and 73b, Byers et al. 79, Green and Veltman 80, Aoki and Hioki 81, Hioki 82a, Paschos and Wirbel 82, Berends et al. 82]. (See also [Sterman and Weinberg 77] as an example in QCD.) The numerical values of the energy resolution and of the angular resolution should be determined depending on individual experimental apparatus.

For these purposes we make beforehand the square of the matrix element for bremsstrahlung $\nu e \to \nu l \gamma$. (For later convenience we deal with the neutral and charged current processes together.) The results corresponding to Fig. 5.12, and Fig. 5.15 (1) and (3) are

$$\begin{aligned} M(\nu e \to \nu l \gamma) &= G_B \bar{\nu}(p_\nu')\gamma^\rho(1-\gamma_5)\nu(p_\nu) \\ &\times \bar{l}(p_l')\left\{\not{\varepsilon}(k,\lambda)\frac{m_l+\not{k}+\not{p}_l'}{m_l^2-(k+p_l')^2}\gamma_\rho(\xi-\gamma_5)\right. \\ &\left.+\gamma_\rho(\xi-\gamma_5)\frac{m_e+\not{p}_e-\not{k}}{m_e^2-(p_e-k)^2}\not{\varepsilon}(k,\lambda)\right\}e(p_e), \qquad (5\cdot52) \end{aligned}$$

where $\varepsilon(k,\lambda)$ is the polarization vector of photon,

$$G_B = \frac{-e^3 M_Z^2}{16 M_W^2(M_Z^2-M_W^2)}, \quad \xi = \frac{4M_W^2-3M_Z^2}{M_Z^2}$$
$$\text{for} \quad \nu e \to \nu e \gamma \quad (l=e), \qquad (5\cdot53\text{a})$$

$$G_B = \frac{e^3 M_Z^2}{8 M_W^2(M_Z^2-M_W^2)}, \quad \xi = 1$$
$$\text{for} \quad \nu e \to \mu \nu \gamma \quad (l=\mu). \qquad (5\cdot53\text{b})$$

(We have neglected the contribution of Fig. 5.15 (2). See the comments after Eq. (5·64h).)

The process, $\nu e\to\mu\nu\gamma$, will be used in § 5.4. After taking a sum over final spin states and an average over initial spin states of the charged lepton, we have

$$\frac{1}{2}\sum_{s_1}\sum_{s_f}|M|^2=-32G_B{}^2\left[\frac{f_1}{(kp_l')^2}+\frac{f_2}{(kp_e)^2}-\frac{f_3}{(kp_l')(kp_e)}\right], \tag{5·54}$$

$$\begin{aligned}f_1=m_l{}^2f_0+m_l{}^2\{(1+\xi)^2(p_ep_\nu)(kp_\nu')+(1-\xi)^2(p_ep_\nu')(kp_\nu)\}\\ -(kp_l')[(1+\xi)^2(p_ep_\nu)\{(p_l'p_\nu')+(kp_\nu')\}\\ +(1-\xi)^2(p_ep_\nu')\{(p_l'p_\nu)+(kp_\nu)\}],\end{aligned} \tag{5·54a}$$

$$\begin{aligned}f_2=m_e{}^2f_0-m_e{}^2\{(1+\xi)^2(p_l'p_\nu')(kp_\nu)+(1-\xi)^2(p_l'p_\nu)(kp_\nu')\}\\ +(kp_e)[(1+\xi)^2(p_l'p_\nu')\{(p_ep_\nu)-(kp_\nu)\}\\ +(1-\xi)^2(p_l'p_\nu)\{(p_ep_\nu')-(kp_\nu')\}],\end{aligned} \tag{5·54b}$$

$$\begin{aligned}f_3=2(p_l'p_e)f_0\\ +2(1+\xi^2)\{(kp_e)(p_l'p_\nu)(p_l'p_\nu')-(kp_l')(p_ep_\nu)(p_ep_\nu')\}\\ +2(1-\xi^2)m_lm_e(kp_\nu')(kp_\nu)\\ +(p_l'p_e)[(1+\xi)^2\{(kp_\nu')(p_ep_\nu)-(kp_\nu)(p_l'p_\nu')\}\\ +(1-\xi)^2\{(kp_\nu)(p_ep_\nu')-(kp_\nu')(p_l'p_\nu)\}],\end{aligned} \tag{5·54c}$$

where

$$f_0=(1+\xi)^2(p_l'p_\nu')(p_ep_\nu)+(1-\xi)^2(p_l'p_\nu)(p_ep_\nu')+(1-\xi^2)m_lm_e(p_\nu'p_\nu) \tag{5·55}$$

is of the same form as the spin sum and average of the squared matrix element for the tree diagram (Fig. 5.1(1)):

$$\frac{1}{2}\sum_{s_1}\sum_{s_f}|M_{\text{tree}}|^2=(32G_B{}^2/e^2)f_0\,. \tag{5·56}$$

First we take the soft photon approximation in Eq. (5·28), i.e., we take the limit $k\to0$ in the f_i's $(i=1,2,3)$ and integrate over the two-body phase

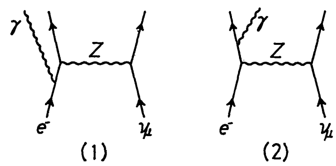

Fig. 5.12. Diagrams of a real-photon emission in the neutral current processes.

space restricted by the maximum photon energy ω. Then only the f_0 terms remain in Eqs. (5·54a)$\sim$(5·54c) and the cross section of soft photon emission is expressed by the factorized form

$$d\sigma^{\text{soft}} = -\frac{e^2}{16\pi^3}\int\frac{d^3k}{k_0}\left[\frac{m_l^2}{(kp_l')^2}+\frac{m_e^2}{(kp_e)^2}-\frac{2(p_l'p_e)}{(kp_l')(kp_e)}\right]\cdot d\sigma_0\,, \tag{5·57}$$

where $d\sigma_0$ is the Born cross section derived from Eq. (5·56). In the one-loop approximation the addition of the soft photon emission cross section is effectively equivalent to adding the finite part M_{soft} of Eq. (5·57) to the amplitude (5·4).*) M_{soft} is given by

$$M_{\text{soft}} = C_{\text{soft}} \times (\text{tree amplitude})\,, \tag{5·58}$$

where

$$\begin{aligned} C_{\text{soft}} &= \frac{e^2}{16\pi^2}\left\{\frac{s+m_l^2}{s-m_l^2}\ln\left(\frac{s}{m_l^2}\right)+\frac{s+m_e^2}{s-m_e^2}\ln\left(\frac{s}{m_e^2}\right)\right. \\ &\quad \left. -2(m_l^2+m_e^2-\eta)\int_0^1 dx\frac{1}{\varepsilon^2 a\sqrt{1-a}}\ln\frac{1+\sqrt{1-a}}{\sqrt{a}}\right\}, \end{aligned} \tag{5·58a}$$

$$\eta \equiv (p_l'-p_e)^2\,,$$

$$\varepsilon \equiv E_l'(1-x)+E_e x\,,$$

$$a \equiv \frac{1}{\varepsilon^2}\{m_l^2(1-x)+m_e^2 x-\eta x(1-x)\}\,,$$

in the center of mass frame, and

$$\begin{aligned} C_{\text{soft}} &= \frac{e^2}{8\pi^2}\left[1+\frac{E_l'}{|\boldsymbol{p}_l'|}\ln\left(\frac{E_l'+|\boldsymbol{p}_l'|}{m_l}\right)\left\{1-2\ln 2-\ln\frac{|\boldsymbol{p}_l'|^2}{m_l(E_l'+|\boldsymbol{p}_l'|)}\right\}\right. \\ &\quad \left. -\frac{E_l'}{|\boldsymbol{p}_l'|}\left\{\frac{\pi^2}{6}-\text{Sp}\left(\frac{E_l'-|\boldsymbol{p}_l'|}{E_l'+|\boldsymbol{p}_l'|}\right)\right\}\right], \end{aligned} \tag{5·58b}$$

in the laboratory frame.

The cross section calculated so far including soft photon emission effect is free from UV and IR divergence. However, it is still unrealistic in the sense that, in actual experiments, it is difficult to detect an extra photon with ω much smaller than the charged lepton mass. (Soft photon approximation is only valid under the condition $\omega \ll m_l$ in the Lab-frame.) Further, there remains theoretical dissatisfaction that the terms which diverge in the limit $m_l \to 0$ still exist in the results (collinear or mass singularity).

*) In the case where one regulates the soft photon divergence by introducing the fictitious photon mass λ, one should replace the photon mass λ in the expression (5·4) by twice the maximum photon energy, 2ω.

In order to eliminate them, we have to add the exact bremsstrahlung cross section,

$$\frac{d^4\sigma}{dE_l' d\cos\theta_1 d\cos\theta_2 d\phi}(\nu e\to\nu l\gamma)$$
$$=\frac{|\boldsymbol{p}_l'|E_\gamma}{32(2\pi)^4(s-m_e^2)(\sqrt{s}-E_l'+|\boldsymbol{p}_l'|\cos\theta_1)}\sum_{s_i}\sum_{s_f}|M|^2, \qquad (5\cdot59)$$

where the variables are shown in Fig. 5.13. Taking this hard photon contribution into account, we get the cross section which can directly be compared with experiments. Various numerical results are presented in the next chapter.

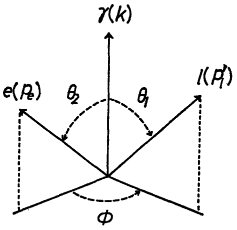

Fig. 5.13. Axes and angles used in Eq. (5·59).

§ 5.4 Charged current processes: $\nu_\mu e^-\to\mu^-\nu_e$ and $\mu^-\to e^-\nu_\mu\bar{\nu}_e$

We can calculate the $O(\alpha)$ correction to the charged current processes in a way similar to the neutral current processes. As mentioned in § 5.2, we need only to calculate νlW vertex function and box diagrams in Fig. 5.2. We again proceed according to Table 5.2.

Step 2–2 *νlW vertex function*

The relevant diagrams are in Fig. 5.14.

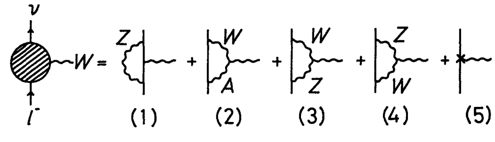

Fig. 5.14. The νlW vertex diagrams ($l=e, \mu$).

$$\Gamma^{\nu lW}_{R\alpha}(p_\nu', p_l)=\sum_{n=1}^{4}\Gamma^{\nu lW}_{n\alpha}(p_\nu', p_l)+\Gamma^{\nu lW}_{C\alpha}, \quad (l=e \text{ or } \mu) \qquad (5\cdot60)$$

$$\sum_{n=1}^{4} \Gamma_{n\alpha}^{\nu lW}(p_\nu', p_l)$$

$$= \frac{e^3 M_Z^3 (M_Z^2 + 10M_W^2)}{128\pi^2 M_W^2 (M_Z^2 - M_W^2)\sqrt{2(M_Z^2 - M_W^2)}} C_{\rm UV}$$

$$+ \frac{e^3 M_Z}{64\pi^2 \sqrt{2(M_Z^2 - M_W^2)}} \Big[5 - 6\ln M_W^2 + \frac{M_W^2}{(M_Z^2 - M_W^2)^2}$$

$$\times \{5(M_Z^2 - M_W^2) - 6(M_Z^2 \ln M_Z^2 - M_W^2 \ln M_W^2)\}$$

$$- \frac{M_Z^2(M_Z^2 - 2M_W^2)}{4M_W^2(M_Z^2 - M_W^2)}(1 + 2\ln M_Z^2)\Big] \gamma_\alpha (1 - \gamma_5), \quad (5\cdot 61)$$

$$\Gamma_{C\alpha}^{\nu lW} = \frac{eM_Z}{4\sqrt{2(M_Z^2 - M_W^2)}}$$

$$\times \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_Z^2 - \delta M_W^2}{M_Z^2 - M_W^2} + 2Y + Z_{\rm L}^l + Z_{\rm L}^\nu + Z_W \right) \gamma_\alpha (1 - \gamma_5)\ . \quad (5\cdot 62)$$

Step 3 *Box diagrams*

The contributions from box diagrams in Fig. 5.2 except the photon exchange graph are collected as

$$A(p_\mu', p_\nu'; p_e, p_\nu)_{(5)+(6)+(8)+(9)}$$

$$= \left(\frac{e^2}{4\pi}\right)^2 \frac{M_Z^2(3M_Z^4 - 6M_Z^2 M_W^2 - 2M_W^4)}{16 M_W^2 (M_Z^2 - M_W^2)^3} \ln\left(\frac{M_Z^2}{M_W^2}\right) \bar{u}_\mu \gamma_\lambda (1 - \gamma_5) u_\nu \cdot \bar{u}_\nu \gamma^\lambda (1 - \gamma_5) u_e\ .$$

$$(5\cdot 63)$$

The photon exchange box diagram (Fig. 5.2 (7)) gives, after the Fierz transformation, following contribution:

$$\bar{u}_\mu(p_\mu')[A(u)\gamma_\lambda(1 - \gamma_5) + B(u)\gamma_\lambda(1 + \gamma_5) + C(u)(1 - \gamma_5)(p_e)_\lambda$$

$$+ D(u)(1 + \gamma_5)] u_e(p_e)_\lambda \cdot \bar{u}_\nu(p_\nu')\gamma^\lambda(1 - \gamma_5) u_\nu(p_\nu), \quad (5\cdot 64)$$

$$A(u) = G\Big[\frac{m_\mu^2 + m_e^2 - u}{R} \Big\{ 2RT \ln\left(\frac{R}{2\lambda^2}\right) + \ln\left(\frac{4m_\mu^2}{-u}\right) \ln\left(\frac{m_e^2 - m_\mu^2 - u + R}{2m_\mu\sqrt{-u}}\right)$$

$$+ \ln\left(\frac{4m_e^2}{-u}\right) \ln\left(\frac{m_\mu^2 - m_e^2 - u + R}{2m_e\sqrt{-u}}\right) - \phi\left(\frac{R}{-u}, \frac{m_\mu^2 - m_e^2}{-u}\right) \Big\}$$

$$- \frac{1}{2} - \ln\left(\frac{M_W^2}{m_\mu m_e}\right) - \frac{m_\mu^2 - m_e^2}{u} \ln\left(\frac{m_\mu}{m_e}\right)$$

$$+ \frac{(m_\mu^2 - m_e^2)^2 - 4(m_\mu^2 + m_e^2)u + 3u^2}{u} T \Big], \quad (5\cdot 64\text{a})$$

$$B(u) = -4m_\mu m_e GT\ , \quad (5\cdot 64\text{b})$$

$$C(u) = \frac{2m_\mu G}{u}\left\{\ln\left(\frac{m_\mu}{m_e}\right) + (m_e^2 - m_\mu^2 + u)T\right\}, \tag{5·64c}$$

$$D(u) = \frac{2m_e G}{u}\left\{\ln\left(\frac{m_e}{m_\mu}\right) + (m_\mu^2 - m_e^2 + u)T\right\}, \tag{5·64d}$$

$$G \equiv \left(\frac{e^2}{4\pi}\right)^2 \frac{M_Z^2}{8M_W^2(M_Z^2 - M_W^2)}, \tag{5·64e}$$

$$R \equiv \sqrt{(m_\mu^2 - m_e^2 + u)^2 - 4m_\mu^2 u}\,, \tag{5·64f}$$

$$T \equiv \frac{1}{R}\ln\left(\frac{m_\mu^2 + m_e^2 - u + R}{2m_\mu m_e}\right), \tag{5·64g}$$

$$\phi(a, b) \equiv \mathrm{Sp}\left(\frac{a+1+b}{2a}\right) - \mathrm{Sp}\left(\frac{a-1+b}{2a}\right) + \mathrm{Sp}\left(\frac{a+1-b}{2a}\right) - \mathrm{Sp}\left(\frac{a-1-b}{2a}\right). \tag{5·64h}$$

Infrared divergence is removed by the soft-photon emission in the same way as in the case of the neutral current processes except the following point: In Fig. 5.15 graph (2) has no contribution to the infrared divergence and gives very small magnitude. We do not take account of it.

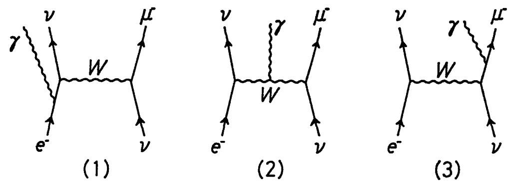

Fig. 5.15. Diagrams of a real-photon emission in the charged current processes.

Collecting all the parts, we obtain the one-loop corrected amplitudes of the form (5·63), where all the contributions except the photon exchange box diagram are collected into the term A. Then the cross section is written as

$$\frac{d\sigma}{dt} = \frac{2}{\pi(s-m_e^2)^2}[|A|^2(s-m_\mu^2)(s-m_e^2) + 2m_\mu m_e \mathrm{Re}(AB^*)u$$
$$+ (m_\mu^2 m_e^2 - st)\mathrm{Re}\, A^*(m_\mu C + m_e D)]. \tag{5·65}$$

So far, we have calculated the amplitude for $\nu_\mu e \to \mu\nu_e$, but the amplitude for μ decay can be obtained by the same function.

$$\frac{d^2\Gamma}{dE_e d\cos\theta} = \frac{|\boldsymbol{p}_e|(m_\mu^2 + m_e^2 - 2m_\mu E_e)^2}{4\pi^3(m_\mu - E_e + |\boldsymbol{p}_e|\cos\theta)^4}$$
$$\times [|A|^2\{(m_\mu^2 + m_e^2)E_e + (m_\mu^2 - m_e^2)|\boldsymbol{p}_e|\cos\theta - 2m_\mu m_e^2\}$$

$$-2m_e\mathrm{Re}(A^*B)(m_\mu-E_e+|\boldsymbol{p}_e|\cos\theta)^2$$
$$+m_\mu\mathrm{Re}\{A^*(m_\mu C+m_e D)\}|\boldsymbol{p}_e|^2\sin^2\theta]. \tag{5·66}$$

Finally, we obtain the complete cross section and decay width by adding the hard photon emission effect as done for the neutral current processes. The cross section, $\sigma(\nu e\to\mu\nu\gamma)$ has been evaluated in Eqs. (5·54) and (5·59). The width $\Gamma(\mu\to e\nu\bar{\nu}\gamma)$ can be calculated with slight modification of Eq. (5·54) as

$$\frac{d^3\Gamma}{dE_e dE_\gamma d\cos\theta}(\mu\to e\nu\bar{\nu}\gamma)=\frac{1}{32(2\pi)^6 m_\mu}E_\gamma|\boldsymbol{p}_e|I_{\rho\sigma}M^{\rho\sigma}, \tag{5·67}$$

(θ: angle between e and γ)

$$I_{\rho\sigma}=\frac{1}{6}\pi\{2Q_\rho Q_\sigma+g_{\rho\sigma}Q^2\}\qquad(Q\equiv p_\mu-p_e-k), \tag{5·67a}$$

$$M^{\rho\sigma}=-256G_B{}^2\Big[\frac{1}{(kp_\mu)^2}\{m_\mu{}^2+(kp_\mu)\}(p_e)^\rho(p_\mu-k)^\sigma$$
$$-\frac{1}{(kp_\mu)(kp_e)}\{2(p_\mu p_e)(p_e)^\rho(p_\mu)^\sigma+(p_\mu p_e)k^\rho(p_\mu-p_e)^\sigma$$
$$+(kp_\mu)(p_e)^\rho(p_e)^\sigma-(kp_e)(p_\mu)^\rho(p_\mu)^\sigma\}$$
$$+\frac{1}{(kp_e)^2}\{m_e^2-(kp_e)\}(p_\mu)^\rho(p_e+k)^\sigma\Big]. \tag{5·67b}$$

Chapter 6

Order α Corrections to Physical Quantities

We have calculated $O(\alpha)$ electroweak radiative corrections to $\nu_\mu e \to \nu_\mu e$, $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$, $\nu_\mu e \to \mu \nu_e$ and $\mu \to e \nu_\mu \bar{\nu}_e$ as functions of the renormalized parameters e, M_W, M_Z, m_f and m_ϕ. Our next step is to determine the values of these parameters from experimental data. After the determination of the values of the parameters, it becomes possible to give predictions for various quantities.

In this chapter, we go into these studies. At first we consider the relation between the higher order effects and predictions for $W^\pm$ and Z boson masses (§ 6.1). Next, we study the $O(\alpha)$ radiative corrections to the tree cross sections $\sigma(\nu_\mu e \to \nu_\mu e), \cdots$, and decay width $\Gamma(\mu \to e \nu_\mu \bar{\nu}_e)$, and present detailed numerical results for them (§ 6.2).

§ 6.1 Higher order effects and $W^\pm$, Z boson masses

Recent development of high energy accelerators has made it possible to expect that $W^\pm$ and Z bosons (and also other heavy particles) will be observed in the near future. As for the masses of $W^\pm$ and Z bosons, the values, $M_W \simeq 77$ (GeV) and $M_Z \simeq 88$ (GeV) have been predicted in the tree level analyses of low energy experimental data.

In this section, we consider the improvement for these predictions by including the $O(\alpha)$ correction. At the first step we summarize the tree level analyses.

According to the general discussion in § 3.1, we must first choose input data in order to determine the values of the parameters M_W and M_Z. Usually, experimental data on the muon decay Γ^{exp} and on a ratio of cross sections of ν and $\bar{\nu}$ induced neutral current processes (we take here $R^{\text{exp}} = \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) / \sigma(\nu_\mu e \to \nu_\mu e)$) are used. Then, the tree level values of $W^\pm$ and Z boson masses should be determined by

$$\begin{aligned} \Gamma^{(0)}(M_W, M_Z) &= \Gamma^{\text{exp}}, \\ R^{(0)}(M_W, M_Z) &= R^{\text{exp}}, \end{aligned} \qquad (6\cdot 1)$$

where $\Gamma^{(0)}$ and $R^{(0)}$ express the corresponding quantities calculated as functions of M_W and M_Z at the tree level in the Weinberg-Salam theory. The one-loop analyses are carried out in a similar way: Replace the left-hand side of Eq. (6·1) by the one-loop corrected quantities $\Gamma^{(1)}(M_W, M_Z)$ and $R^{(1)}(M_W, M_Z)$ as

$$\Gamma^{(1)}(M_W, M_Z) = \Gamma^{\text{exp}},$$

$$R^{(1)}(M_W, M_Z) = R^{\text{exp}}. \tag{6·2}$$

However, the traditional analyses on the muon decay are somewhat different from the above line of argument. The quantity which is used in the traditional analyses is not $\Gamma^{(0)}$ but $\Gamma_{\text{fermi}}\cdot(1+\Delta_{\text{EM}})$. Here Γ_{fermi} is the following muon decay width calculated in the framework of the four-fermion interaction,

$$\Gamma_{\text{fermi}} = \frac{G_F{}^2 m_\mu{}^5}{192\pi^3}\left(1-\frac{8m_e{}^2}{m_\mu{}^2}\right), \tag{6·3}$$

and Δ_{EM} is the QED-radiative correction to this interaction (Fig. 6.1 + soft and hard photon emission effects) [Berman 58, Kinoshita and Sirlin 59, Hioki 82a],

$$\Delta_{\text{EM}} = \frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right). \tag{6·4}$$

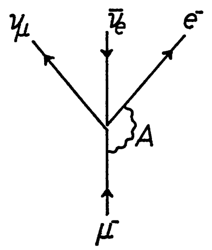

Fig. 6.1. The electromagnetic correction to the four-fermion interaction $\mu \to e\nu\bar{\nu}$.

That is, the following equation is used instead of the first equation in Eq. (6·1),

$$\Gamma_{\text{fermi}}\cdot(1+\Delta_{\text{EM}}) = \Gamma^{\text{exp}}. \tag{6·5}$$

Then, the parameters of the Weinberg-Salam theory M_W, M_Z are determined by

$$\Gamma^{(0)}(M_W, M_Z) = \Gamma_{\text{fermi}}(=\Gamma^{\text{exp}}\cdot(1-\Delta_{\text{EM}})), \tag{6·6a}$$

$$R^{(0)}(M_W, M_Z) = R^{\text{exp}}. \tag{6·6b}$$

We can summarize these as follows. There are two steps in the traditional analyses in order to determine M_W and M_Z at "tree level": First, the parameter of the four-fermion interaction G_F is determined, and the parameters M_W, M_Z are fixed by Eq. (6·6a, b). The well-known values $M_W \simeq 77$ (GeV) and $M_Z \simeq 88$ (GeV) are not the solutions of Eq. (6·1) but of Eq. (6·6). (Hereafter we express these solutions as $M_W{}^{(0)}$ and $M_Z{}^{(0)}$.) Such complicated determinations have been adopted from the historical reason that i) the QED radiative corrections to the four-fermion interaction $\mu \rightarrow e\nu\bar{\nu}$ (Fig. 6.1) had been found to be ultraviolet-convergent, and ii) the Weinberg-Salam theory was constructed to reproduce this four-fermion interaction effectively in a low-energy limit on the charged current sector. Let us call these analyses tree analyses here, although they are not "real tree" analyses.

These predictions ($M_W{}^{(0)} \simeq 77$ GeV and $M_Z{}^{(0)} \simeq 88$ GeV) are improved by including the $O(\alpha)$ electroweak corrections. The one-loop condition on the muon decay width is

$$\Gamma^{(1)}(M_W, M_Z)\ (\equiv \Gamma^{(0)}(M_W, M_Z)(1+\Delta_{\rm EW})) = \Gamma^{\rm exp}, \tag{6·7}$$

where $\Delta_{\rm EW}$ denotes "electroweak correction". By the use of Eq. (6·6a), Eq. (6·7) is reduced to

$$\Gamma^{(\rm W)}(M_W, M_Z) = \Gamma^{(0)}(M_W{}^{(0)}, M_Z{}^{(0)}), \tag{6·8a}$$

where

$$\Gamma^{(\rm W)}(M_W, M_Z) \equiv \Gamma^{(0)}(M_W, M_Z)(1+\Delta_{\rm W}),$$

$$\Delta_{\rm W} \equiv \Delta_{\rm EW} - \Delta_{\rm EM}. \tag{6·8b}$$

Therefore $\Delta_{\rm W}$ becomes necessary in the practical calculations.*) We conventionally call $\Delta_{\rm W}$ "weak" part and $\Delta_{\rm EM}$ "electromagnetic" or "QED" part respectively in the whole electroweak correction $\Delta_{\rm EW}$.

Next, let us consider R. Strictly speaking, one-loop-corrected R ($R^{(1)}(M_W, M_Z)$) depends on the incident $\nu(\bar{\nu})$ energy and consequently the average of $R^{(1)}$ over $\nu(\bar{\nu})$ energy spectrum is necessary. Fortunately, however, the energy

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*) In the Weinberg-Salam theory, it is not possible for the charged current interaction to define the QED part properly by selecting some diagrams in Fig. 5.2. It is a mere convention to call $\Delta_{\rm EM}$ the QED part of $\Delta_{\rm EW}$. Actually, however, the QED-corrected renormalized amplitude of the four-fermion interaction $M_{\rm fermi}$ (Fig. 6.1 + counter terms) has the following simple relation to the amplitude $M_{\rm box}$ corresponding to Fig. 5.2 (7),

$$M_{\rm fermi} = M_{\rm box} + \frac{1}{2}\left\{Z^e + Z^\mu - \frac{\alpha}{4\pi}(1+2\ln M_W^2)\right\} M_0 .$$

Here M_0 is the tree amplitude, and Z^l is (the finite part of) the photonic correction part of Z_L^l given in Appendix E,

$$Z^l = -\frac{\alpha}{4\pi}(4 - 3\ln m_l{}^2 + 2C_{\rm IR}).$$

(It should be noted that this type of definition of the QED part is no longer possible for the process, e.g., $\nu d \rightarrow \mu u$. For such processes, the photonic correction to the four-fermion interaction is not ultraviolet-convergent.)

On the other hand, it is possible to define the QED part in the electroweak correction for $\nu_\mu(\bar{\nu}_\mu)e \rightarrow \nu_\mu(\bar{\nu}_\mu)e$ processes by taking the contribution of Fig. 5.9 (1) + appropriate counter terms.

dependence of R and the pure QED correction to R are found to be negligible. (Concerning the energy dependence of $R^{(1)}$, see Δ_R in Table 6.3. The pure QED corrections to $\sigma(\nu_\mu e\to\nu_\mu e)$ and $\sigma(\bar\nu_\mu e\to\bar\nu_\mu e)$ (Fig. 5.9(1)) are non-negligible, but almost equal each other. See Table 6.2 or 6.3. Then the QED corrections to R become very small.) Therefore, we can take the condition on R similar to the case of Γ at some definite E_ν. (We take $E_\nu=5\,\mathrm{GeV}$.) That is, by denoting the ratio R including the weak one-loop correction as $R^{(\mathrm{W})}(M_W, M_Z)$, one-loopcorrected values $M_W{}^{(1)}$ and $M_Z{}^{(1)}$ are obtained as the solutions of the equations

$$\Gamma^{(\mathrm{W})}(M_W, M_Z)=\Gamma^{(0)}(M_W{}^{(0)}, M_Z{}^{(0)}),$$
$$R^{(\mathrm{W})}(M_W, M_Z)\big|_{E_\nu(\bar\nu)=5\,\mathrm{GeV}}=R^{(0)}(M_W{}^{(0)}, M_Z{}^{(0)}). \qquad (6\cdot9)$$

Here we can see the advantage of the on-shell renormalization scheme. That is, renormalized M_W and M_Z are always the real part of the poles of the W and Z propagators at any order of perturbation.

The analytical expression of the one-loop radiative correction is very complicated, so solving Eq. (6·9) numerically, we obtain

$$M_W{}^{(1)}\simeq 79.17\ \ (\mathrm{GeV}),$$
$$M_Z{}^{(1)}\simeq 90.54\ \ (\mathrm{GeV}). \qquad (6\cdot10)$$

(As for the other parameters, see Eq. (6·13).) These solutions give the answer to the question: In what energy region should the search for $W^\pm$ and Z bosons be performed? For example, concerning the Z boson search, the solutions (6·10) imply that *a prominent peak will be observed at* $\sqrt{s}\simeq 90.5\,(GeV)$, *not at* $\sqrt{s}\simeq 88\,(GeV)$ in e^+e^- collider (or dimuon invariant mass $M_{\mu\mu}\simeq 90.5\,(\mathrm{GeV})$ in hadron-hadron reactions). It may be rather surprising that there appear considerable deviations (2.8% correction to the tree prediction). This is due to the large logarithmic terms $\sim\alpha\ln(M^2_{W,Z}/(m_f^2$ or $q^2))$ which exist, e.g., in the charge renormalization constant, Y. It is possible to sum up these logarithmic terms by renormalization group equation as is well known in perturbative QCD. We will argue this method in Chapter 7.

Several authors have also predicted $W^\pm$ and Z boson masses at one-loop level. [Antonelli et al. 80 and 81a, Veltman 80, Aoki et al. 81, Sirlin and Marciano 81, Bardin et al. 82, Wheater and Llewellyn Smith 82]. Our results are consistent with the others although there are small differences which are considered to occur due to the different input parameters, e.g., quark masses.

Finally, let us consider another possibility to predict M_W once M_Z is experimentally measured. Among the input data used to determine the values of the parameters, $R^{\rm exp}$ includes a rather large experimental error. Present experimental uncertainties in $R^{\rm exp}$ lead to ambiguities at least $\sim\pm(2.5-3.0)$ GeV in the determination of $M_W^{(0)}$ and $M_Z^{(0)}$ (consequently, for $M_W^{(1)}$ and $M_Z^{(1)}$).

On the other hand, $M_Z^{\rm exp}$ is expected to be measured with high accuracy (within $\sim$0.1% [Davier 79]) in e^+e^- collider. Therefore, under such circumstances, it is worthwhile using the input data, (assumed) $M_Z^{\rm exp}$, instead of $R^{\rm exp}$. ([Hioki 82b]. This possibility was also indicated in [Sirlin 80].) The use of this new input will improve the prediction for $W^\pm$ boson mass. We examine this improvement.

The one-loop-corrected value $M_W^{(1)}$, and tree value $M_W^{(0)}$, are derived from

$$\Gamma^{(W)}(M_W^{(1)}, M_Z)\,|_{M_Z=M_Z{}^{\rm exp}}=\Gamma^{(0)}(M_W^{(0)}, M_Z)\,|_{M_Z=M_Z{}^{\rm exp}}$$
$$=\Gamma^{\rm exp}\cdot(1-\Delta_{\rm EM}). \qquad (6\cdot 11)$$

We present the results in Table 6.1. We should keep in mind that uncertainties in this case are small enough to clarify the higher order effects, $|M_W^{(1)}-M_W^{(0)}|$.

Table 6.1. Prediction for $W^\pm$ boson mass (in GeV unit) by using $M_Z^{\rm exp}$ and $\Gamma^{\rm exp}$.

$M_Z^{\rm exp}$	$M_W^{(0)}$	$M_W^{(1)}$
90.0	79.49	78.49
91.0	80.71	79.75
92.0	81.92	80.99
93.0	83.12	82.22
94.0	84.31	83.44
95.0	85.49	84.64
96.0	86.66	85.64

§ 6.2 Order α corrections to cross sections and decay width

In § 6.1 we have shown that there are rather large corrections for the predictions of $W^\pm$ and Z boson masses. In this section, we study the size of the correction to various cross sections and decay width. As mentioned in Chapter 1, the Weinberg-Salam theory has succeeded in explanation of various phenomena by using tree level approximation. Therefore, it is important to study how the above success of the Weinberg-Salam theory is affected by the inclusion of the higher order effects. We deal with the pure QED part and the weak part separately in order to clarify the effect of weak part which is important for the field theoretical test of the Weinberg-Salam theory.

As a first step of calculating the radiative correction, the values of the renormalized parameters, e, M_W, M_Z, m_f and m_ϕ have to be fixed. If all the values of the masses of $W^\pm$, Z bosons, fermions and Higgs boson are measured, it is quite natural and straightforward to substitute the experimental data $M_W{}^{\rm exp}$, $M_Z{}^{\rm exp}$, $m_f{}^{\rm exp}$ and $m_\phi{}^{\rm exp}$ for the corresponding parameters respectively. Another possibility is to use the input data, $\Gamma^{\rm exp}$ and $R^{\rm exp}$ instead of $M_W{}^{\rm exp}$ and $M_Z{}^{\rm exp}$ as mentioned in § 6.1. Below we study the radiative correction according to the above two cases.

a) (M_W, M_Z) *case*

In this case, we can immediately obtain the output (correction). We denote the weak radiative corrections as

$$\Delta_{\rm W}=\{\sigma^{\rm (W)}(M_W{}^{\rm exp}, M_Z{}^{\rm exp})-\sigma^{(0)}(M_W{}^{\rm exp}, M_Z{}^{\rm exp})\}/\sigma^{(0)}(M_W{}^{\rm exp}, M_Z{}^{\rm exp})\,. \tag{6·12}$$

Here we assume simply $M_W{}^{\rm exp}=77\,{\rm GeV}$ and $M_Z{}^{\rm exp}=88\,{\rm GeV}$. The results are shown in Table 6.2. (The results are found to change scarcely even if we take the values given in Eq. (6·10) for $M_W{}^{\rm exp}$ and $M_Z{}^{\rm exp}$.) As for the pure QED correction, we have shown there two kinds of corrections $\Delta_{\rm EM}^{\rm soft}$ and $\Delta_{\rm EM}$. $\Delta_{\rm EM}^{\rm soft}$ is the correction only with contribution of soft-photon emission, which depends on the maximum soft-photon energy, ω. $\Delta_{\rm EM}$ is the correction including not only soft-photon effect but also hard-photon contribution. A detailed description on these pure QED corrections is seen in [Byers et al. 79, Green and Veltman 80, Aoki and Hioki 81, Hioki 82a].

In the actual calculation we have used the following values for the parameters*) (in GeV unit for masses):

$$\begin{aligned}
&m_{\nu_e}=m_{\nu_\mu}=m_{\nu_\tau}=0\,,\quad m_e=5.11\times10^{-4}\,,\\
&m_\mu=1.057\times10^{-1}\,,\quad m_\tau=1.782\,,\quad m_u=m_d=m_s=0.1\,,\\
&m_c=1.5\,,\quad m_b=4.7\,,\quad m_t=30\quad\text{and}\quad m_\phi=10\,,\\
&\sin^2\theta_{\rm C}\ \text{(Cabibbo angle)}=0.0562\,.
\end{aligned} \tag{6·13}$$

We have neglected all the other mixings.

There appear large weak corrections. We, however, notice that these large corrections are almost universal and energy independent for all the cross sections and the decay width. This fact implies that the most part of these effects may be absorbed into the re-normalization of parameters which are related to the overall normalization of the cross sections and width, that is,

*) Concerning the values of $m_{u,d,s}$, see [Marciano and Sirlin 81].

Table 6.2. $O(\alpha)$ corrections with input $M_W{}^{\text{exp}}$ and $M_Z{}^{\text{exp}}$ to the tree cross sections $\sigma(\nu_\mu e\to\nu_\mu e)$, $\cdots$, and decay width $\Gamma(\mu\to e\nu_\mu\bar{\nu}_e)$. Δ_W is the weak correction, and $\Delta_{\text{EM}}(\Delta_{\text{EM}}^{\text{soft}})$ is the pure QED correction with (without) hard-photon contribution. $\Delta_{\text{EM}}^{\text{soft}}$ depends on ω, the maximum soft-photon energy.

$\nu_\mu e\to\nu_\mu e$

E_ν	Δ_W	$\Delta_{\text{EM}}^{\text{soft}}(\omega=1\text{ keV})$	$\Delta_{\text{EM}}^{\text{soft}}(\omega=100\text{ keV})$	Δ_{EM}
0.1 (GeV)	14.1 (%)	−14.2	− 6.0	−1.1
1	14.1	−25.7	−12.6	−1.4
5	14.1	−35.2	−18.7	−1.7
15	14.1	−42.4	−23.6	−1.9
50	14.1	−51.0	−29.5	−2.0
100	14.1	−56.2	−33.3	−2.1
500	14.2	−69.2	−42.8	−2.4
1000	14.2	−75.2	−47.3	−2.5
5000	14.2	−89.9	−58.6	−2.8
10000	14.3	−96.6	−63.8	−2.9

$\bar{\nu}_\mu e\to\bar{\nu}_\mu e$

$E_{\bar{\nu}}$	Δ_W	$\Delta_{\text{EM}}^{\text{soft}}(\omega=1\text{ keV})$	$\Delta_{\text{EM}}^{\text{soft}}(\omega=100\text{ keV})$	Δ_{EM}
0.1 (GeV)	13.2 (%)	−14.0	− 5.9	−1.0
1	13.2	−25.3	−12.4	−1.4
5	13.2	−34.8	−18.5	−1.6
15	13.2	−42.0	−23.4	−1.9
50	13.2	−50.6	−29.3	−2.0
100	13.2	−55.8	−33.0	−2.1
500	13.1	−68.8	−42.6	−2.4
1000	13.1	−74.7	−47.0	−2.5
5000	13.1	−89.4	−58.3	−2.7
10000	13.1	−96.1	−63.5	−2.9

$\nu_\mu e\to\mu\nu_e$

E_ν	Δ_W	$\Delta_{\text{EM}}^{\text{soft}}(\omega=1\text{ keV})$	$\Delta_{\text{EM}}^{\text{soft}}(\omega=100\text{ keV})$	Δ_{EM}
15 (GeV)	12.6 (%)	−21.7	−12.1	−4.5
50	12.6	−27.0	−15.7	−2.6
100	12.6	−30.6	−18.2	−2.4
500	12.6	−40.8	−25.3	−2.5
1000	12.6	−45.8	−28.9	−2.6
5000	12.6	−58.4	−38.1	−2.8
10000	12.6	−64.2	−42.5	−2.9

$\mu\to e\nu_\mu\bar{\nu}_e$

Δ_W	$\Delta_{\text{EM}}^{\text{soft}}(\omega=1\text{ keV})$	$\Delta_{\text{EM}}^{\text{soft}}(\omega=100\text{ keV})$	Δ_{EM}
12.6 (%)	−17.9	−9.5	−0.4

M_W and M_Z. This point has close connection to the calculation using the input data $\Gamma^{\exp}$ and $R^{\exp}$ as will be seen below.

b) (Γ, R) *case*

When we use the data $\Gamma^{\exp}$ and $R^{\exp}$ as input, we must first determine the values of the parameters, M_W and M_Z. This has been done in § 6.1. The results are

$$M_W{}^{(0)} \simeq 77 \ \text{(GeV)} ,$$
$$M_Z{}^{(0)} \simeq 88 \ \text{(GeV)} ,$$

at tree level, and

$$M_W{}^{(1)} \simeq 79.17 \ \text{(GeV)} ,$$
$$M_Z{}^{(1)} \simeq 90.54 \ \text{(GeV)} ,$$

at one-loop level.

Using these results, we can calculate the radiative correction as

$$\Delta_\mathrm{W} = \{\sigma^{(\mathrm{W})}(M_W{}^{(1)}, M_Z{}^{(1)}) - \sigma^{(0)}(M_W{}^{(0)}, M_Z{}^{(0)})\} / \sigma^{(0)}(M_W{}^{(0)}, M_Z{}^{(0)}), \qquad (6\cdot14)$$

where $M_{W,Z}^{(0)}$ and $M_{W,Z}^{(1)}$ satisfy the equations

$$\Gamma^{(\mathrm{W})}(M_W{}^{(1)}, M_Z{}^{(1)}) = \Gamma^{(0)}(M_W{}^{(0)}, M_Z{}^{(0)}),$$
$$R^{(\mathrm{W})}(M_W{}^{(1)}, M_Z{}^{(1)})\,|_{E_\nu = 5\,\mathrm{GeV}} = R^{(0)}(M_W{}^{(0)}, M_Z{}^{(0)}) \,. \qquad (6\cdot15)$$

The magnitudes of these corrections are very important for the test of the Weinberg-Salam theory because various tree predictions by using the input $\Gamma^{\exp}$ and $R^{\exp}$ are in good agreement with experimental data. If the Weinberg-Salam theory is the correct theory of electroweak interaction, the radiative corrections are expected to be small. We show the results in Table 6.3.

In the results, the weak corrections to μ decay width and $R \equiv \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) / \sigma(\nu_\mu e \to \nu_\mu e)$ at $E_{\nu,\bar{\nu}} = 5\,\mathrm{GeV}$ are exactly zero from the condition, Eq. (6·15). Concerning the weak corrections, we have given the numerical results up to second decimal place to clarify their energy dependence.

The results show that the absolute values of the weak corrections, $|\Delta_\mathrm{W}|$, are smaller than $|\Delta_\mathrm{EM}|$ in this case. That is, the large corrections in Table 6.2 have been actually absorbed into the shifts $\Delta M_{W,Z} = M_{W,Z}^{(1)} - M_{W,Z}^{(0)}$. This supports the validity of the tree approximation.

Δ_R denotes the weak correction to the ratio R. The fact that the energy dependence of Δ_R is weak shows that the calculations of $M_W{}^{(1)}$ and $M_Z{}^{(1)}$ (Eq. (6·9)) are not affected severely by the change of the energy where $R^{(1)}$ is calculated.

The experimental data of the neutral current processes are usually expressed in terms of the Weinberg angle, $\sin^2\theta_\mathrm{W}$. We have not used this parameter since it is not a convenient parameter once higher order effects are included. However, we can define E-dependent Weinberg angle, $\sin^2\theta_W(E)$ according to the tree relation

$$R^{(0)}=\frac{16\sin^4\theta_W-4\sin^2\theta_W+1}{16\sin^4\theta_W-12\sin^2\theta_W+3},$$

as follows:

$$R^{(1)}\equiv\frac{16\sin^4\theta_W(E)-4\sin^2\theta_W(E)+1}{16\sin^4\theta_W(E)-12\sin^2\theta_W(E)+3}. \tag{6·16}$$

Let us express the relation between $\sin^2\theta_W$ and $\sin^2\theta_W(E)$ as

$$\sin^2\theta_W(E)=\sin^2\theta_W\times(1+\Delta_{SW}). \tag{6·17}$$

That is, Δ_{SW} is the weak correction to $\sin^2\theta_W$. By combining Eq. (6·17) with

Table 6.3. $O(\alpha)$ corrections with input Γ^{exp} and R^{exp} to the tree cross sections and the width. Δ_R is the weak correction to the ratio $R\equiv\sigma(\bar{\nu}_\mu e\to\bar{\nu}_\mu e)/\sigma(\nu_\mu e\to\nu_\mu e)$. The meaning of Δ_{EM} is the same as those in Table 6.2.

$\nu_\mu e\to\nu_\mu e$

E_ν	Δ_W	Δ_{EM}	$\Delta_{EW}(=\Delta_{EM}+\Delta_W)$
0.1 (GeV)	0.85 (%)	−1.1	−0.2
1	0.86	−1.4	−0.5
5	0.86	−1.7	−0.8
15	0.87	−1.9	−1.0
50	0.88	−2.0	−1.1
100	0.89	−2.1	−1.2
500	0.92	−2.4	−1.5
1000	0.93	−2.5	−1.6
5000	0.97	−2.8	−1.8
10000	0.99	−2.9	−1.9

$\bar{\nu}_\mu e\to\bar{\nu}_\mu e$

$E_{\bar{\nu}}$	Δ_W	Δ_{EM}	$\Delta_{EW}(=_{EM}+\Delta_W)$
0.1 (GeV)	0.88 (%)	−1.0	−0.1
1	0.87	−1.4	−0.5
5	0.86	−1.6	−0.7
15	0.86	−1.9	−1.0
50	0.85	−2.0	−1.1
100	0.84	−2.1	−1.3
500	0.81	−2.4	−1.6
1000	0.80	−2.5	−1.7
5000	0.76	−2.7	−1.9
10000	0.74	−2.9	−2.2

(continued)

$E_{\nu,(\bar\nu)}$	Δ_R
0.1 (GeV)	0.03 (%)
1	0.01
5	0 (input)
15	−0.01
50	−0.03
100	−0.05
500	−0.11
1000	−0.13
5000	−0.21
10000	−0.25

$\nu_\mu e \to \mu \nu_e$

E_ν	Δ_W	Δ_{EM}	$\Delta_{EW}(=\Delta_{EM}+\Delta_W)$
15 (GeV)	0.00 (%)	−4.5	−4.5
50	0.00	−2.6	−2.6
100	0.00	−2.4	−2.4
500	0.00	−2.5	−2.5
1000	0.00	−2.6	−2.6
5000	0.00	−2.8	−2.8
10000	−0.01	−2.9	−2.9

$\mu \to e \nu_\mu \bar\nu_e$

Δ_W	Δ_{EM}	$\Delta_{EW}(=\Delta_{EM}+\Delta_W)$
0 (%) (input)	−0.4	−0.4

Eq. (6·16), Δ_{SW} is explicitly given in terms of Δ_R as

$$\Delta_{SW} = -\frac{(16\sin^4\theta_W - 4\sin^2\theta_W + 1)(16\sin^4\theta_W - 12\sin^2\theta_W + 3)}{64\sin^4\theta_W(2\sin^2\theta_W - 1)}\Delta_R$$

$$\simeq 0.54\Delta_R\,. \tag{6·18}$$

We can see that the effective Weinberg angle $\sin^2\theta_W(E)$ decreases as energy becomes higher. However, the magnitude of Δ_{SW} is so small that it will be difficult to check this behavior in ν-experiments. The effective Weinberg angle like this has been studied by many authors who have calculated the electroweak correction to the neutral current processes. (See § 7.1.)

Finally, concerning the dependence of the above results on unknown parameters, m_t and m_ϕ, we have examined the change of the results under the replacement, m_t: 30→100 (GeV) and m_ϕ: 10→100 (GeV). For example, $\Delta M_{W,Z}$ and the correction Δ_W to $\nu_\mu e \to \nu_\mu e$ at $E_\nu = 100$ (GeV) change as follows:

$$\Delta M_W = M_W^{(1)} - M_W^{(0)}: 2.167\,(\mathrm{GeV}) \to 2.221 \quad (m_t \to 100)$$
$$\to 2.224 \quad (m_\phi \to 100)\,,$$
$$\Delta M_Z = M_Z^{(1)} - M_Z^{(0)}: 2.535\,(\mathrm{GeV}) \to 2.416 \quad (m_t \to 100)$$
$$\to 2.634 \quad (m_\phi \to 100)\,,$$
$$\Delta_W: \quad 0.89\,(\%) \quad \to 1.41 \quad (m_t \to 100)$$
$$\to 0.82 \quad (m_\phi \to 100)\,.$$

In conclusion, we have shown that the success of the Weinberg-Salam theory at tree analyses is not affected by the inclusion of higher order effects.

Note added in proof: After the submission of this article for publication, the authors were informed that i) L. Maiani arrived at the same conclusion on the test of the electroweak higher order effects by using the input data, α^{exp}, Γ^{exp} and M_Z^{exp} (L. Maiani, Roma Univ. Preprint n304, July 1982), and ii) S. Sarantakos et al. calculated the electroweak one-loop correction to νe scattering and obtained consistent results on the E.M. correction (including hard photon contribution) with the present ones (S. Sarantokos, A. Sirlin and W. J. Marciano, Preprint October 1982). The authors are grateful to Professor L. Maiani and Professor A. Sirlin for their information.

Chapter 7

Brief Survey of Studies on Electroweak Radiative Corrections

In Chapters 5 and 6 we have studied electroweak higher order effects in purely leptonic weak processes. The study of such effects is, in fact, important as a precise test of the Weinberg-Salam theory. Similar studies have also been performed by other authors and the results are presented in a variety of renormalization schemes. In this chapter we summarize the present status of these studies in order to understand the interrelation among them.

We classify these studies on electroweak radiative corrections into two categories: (1) studies based on conventional perturbation in powers of a fixed coupling constant and (2) studies based on leading-log approximation in the renormalization group method. We shall, in the following, discuss these two approaches separately.

§ 7.1 Conventional perturbation

The leptonic processes which have been studied with higher order corrections in the conventional perturbation are listed as follows:

$\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ [Salomonson and Ueda 75, Veltman 80, Antonelli et al. 80, Green and Veltman 80, Aoki et al. 80, Aoki et al. 81, Antonelli et al. 81a, Bardin et al. 82]

$\nu_\mu e \to \mu \nu_e$ [Aoki et al. 81, Bardin et al. 82]

$\nu_\mu q \to \nu_\mu q$ [Marciano and Sirlin 80, Sakakibara 81, Llewellyn Smith and Wheater 81, Wheater and Llewellyn Smith 82]

$\nu_\mu q \to \mu q'$ [Sirlin and Marciano 81, Paschos and Wirbel 82]

$eq \to eq$ [Llewellyn Smith and Wheater 81, Wheater and Llewellyn Smith 82]

$e^+e^- \to e^+e^-$ [Consoli 79]

$e^+e^- \to \mu^+\mu^-$ [Passarino and Veltman 79]

$e^+e^- \to W^+W^-$ [Lemoine and Veltman 80, Phillippe 82]

$\mu^- \to e^- \nu_\mu \bar{\nu}_e$ [Ross 73, Appelquist et al. 72 and 73, Bardin et al. 82, Hioki 82b]

$W^- \to e\bar{\nu}_e$ [Inoue et al. 80]

$Z \to f\bar{f}$ [Antonelli et al. 81b]

H (Higgs boson) $\to \tau^+\tau^-$, W^+W^-, ZZ [Fleischer and Jegerlehner 81]

In these calculations diverse renormalization schemes are adopted and various choices of the set of independent parameters are made. Such differences in the calculational procedure, however, do not lead to essentially different physical predictions. In comparing the results of the calculations, the following two points should be clearly distinguished and carefully examined: (1) choice of independent parameters and (2) choice of input data needed to fix the values of independent parameters.

(1) *Independent parameters*

The choice of independent parameters is, in principle, arbitrary. It occurs, however, that some of the choices are more convenient in the sense that the definition of the parameters is unambiguous to all orders in perturbation series. Let us consider, as a typical example, the problem of the definition of the Weinberg angle θ_W.*) The parameter $\sin\theta_W$ is now widely used in analyzing neutral current phenomena. In fact $\sin\theta_W$ is convenient at the tree level in the sense that the structure of amplitudes relevant to the neutral current takes a simple universal expression in terms of this parameter. This simplicity, however, does not persist once higher order corrections are taken into account. With the higher order corrections included, there is no unique and proper definition of $\sin\theta_W$ and hence it is not convenient to adopt $\sin\theta_W$ as one of the independent parameters. In the following we explain, by taking two examples of different definitions of $\sin\theta_W$, how the simple tree relation $M_W=M_Z\cos\theta_W$ is modified by higher order corrections.

Example 1. A set of parameters**) e, g and M_W was first employed by Appelquist et al. [Appelquist et al. 72 and 73] and later by Salomonson and Ueda [Salomonson and Ueda 75]. Here e is the electric charge renormalized at $q^2=0$, M_W the physical W-boson mass corresponding to the pole of the renormalized W-boson propagator, and g is the $SU(2)$ coupling constant renormalized on the mass shell with the W-μ-ν vertex function. The Weinberg angle is defined by***)

$$\sin\theta_W=e/g\,. \tag{7·1}$$

The Z-boson mass is not an independent parameter and is calculated as a pole position in the Z-boson propagator, i.e., the solution of

$$q^2-M_{Z0}^2-A_0(q^2)=0\,, \tag{7·2}$$

where M_{Z0} is the bare mass and $A_0(q^2)$ is the unrenormalized Z-boson self-energy part. The counter term for the Z-boson self-energy part is settled

*) In our scheme this problem is irrelevant since we do not take this parameter (also in [Inoue et al. 80] and [Bardin et al. 82]).

**) We consider only three parameters in the present discussion and suppress all other independent parameters like μ^2 and f_l.

***) Note that $\cos^2\theta_W$ is denoted as R in [Appelquist et al. 72 and 73].

as a function of e, g and M_W and Eq. (7·2) reduces to the following form at the one-loop level:

$$q^2 - M_W^2/(1-e^2/g^2) - A(q^2) = 0 \tag{7·3}$$

with $A(q^2)$ the renormalized Z-boson self-energy part. Solving Eq. (7·3), one finds

$$M_Z = M_W/\sqrt{1-e^2/g^2} + \Delta(e, g, M_W), \tag{7·4}$$

where Δ is a function of e, g and M_W calculable once $A(q^2)$ is known and is of order e^2 (or g^2). Thus the relation $M_W = M_Z \cos\theta_W$ is violated in this renormalization scheme.

It should also be noted that the W-μ-ν vertex function on the mass shell includes the infrared divergence and hence g depends on the cutoff of photon energy after the cancellation of the infrared divergence. In this sense the scheme discussed in this example is rather inconvenient.

Example 2. In the renormalization scheme employed by Sirlin [Sirlin 80], g, g' and v are chosen as independent parameters, where g is the $SU(2)$ coupling constant as before, g' the $U(1)$ coupling constant and v the vacuum expectation value of the Higgs scalar. Other parameters θ_W, M_W, M_Z and e are defined as functions of g, g' v such that

$$\begin{aligned} &\tan\theta_W \equiv g'/g\,, \\ &M_W^2 \equiv v^2 g^2/4\,, \\ &M_Z^2 \equiv v^2(g^2+g'^2)/4\,, \\ &e \equiv gg'/(g^2+g'^2)^{1/2} = g\sin\theta_W\,. \end{aligned} \tag{7·5}$$

In dealing with loop corrections, counter terms for g, g' and v are determined by equating M_W and M_Z (as defined by Eq. (7·5)) to the pole positions in the W- and Z-boson propagators respectively. The constant e defined by Eq. (7·5) is identified with the electric charge. As a result the relation $M_W = M_Z \cos\theta_W$ holds to any order in the perturbation series.

With the above two examples we now recognize that the actual meaning of the parameter θ_W differs from scheme to scheme once the loop corrections are taken into account.

(2) *Input data*

The numerical values of independent parameters are determined by using experimental data on some measurable quantities. The resulting numerical values are affected by the choice of the quantities as experimental data as far as the truncated perturbation theory is concerned.

Let us assume that radiative corrections to cross sections of processes A and B, σ_A and σ_B, respectively in a theory with coupling constant g. Denote the tree cross section by $\sigma_A{}^0(g)$ $(\sigma_B{}^0(g))$, the cross section with one-loop correction by $\sigma_A{}^1(g)$ $(\sigma_B{}^1(g))$, and so on. One may think of two possibilities of determining the value of g.

One way is to take the experimental data on process A (denoted by $\sigma_A^{\rm exp}$) to fix the value of g in such a way that

$$\sigma_A{}^0(g_A{}^0)=\sigma_A{}^{\rm exp}\,,\quad \sigma_A{}^1(g_A{}^1)=\sigma_A{}^{\rm exp}\,,\ \cdots, \tag{7·6}$$

where $g_A{}^0$, $g_A{}^1$, $\cdots$ represent values of g determined by the condition (7·6). With this coupling constant determined by Eq. (7·6), we can calculate radiative corrections in process B, e.g.,

$$[\sigma_B{}^1(g_A{}^1)-\sigma_B{}^0(g_A{}^0)]/\sigma_B{}^0(g_A{}^0)\,. \tag{7·7}$$

This correction is, in general, nonzero while the correction for process A vanishes exactly because of Eq. (7·6).

The other way is to use the data on process B and the procedure is essentially the same as above if we exchange A for B. We denote by $g_B{}^0$, $g_B{}^1$, $\cdots$ the coupling constants corresponding to the previous ones $g_A{}^0$, $g_A{}^1$, $\cdots$.

The important observation here is that, as far as we rely on the truncated perturbation theory, the value of the coupling constant determined as above depends on how one determines it, i.e., $g_A{}^i\neq g_B{}^i$ $(i=0,1,2\cdots)$. The difference between $g_A{}^i$ and $g_B{}^i$ lies in higher order terms of order g^{i+1}.

§ 7.2 Leading-log calculation

In the presence of two hierarchically different mass scales μ and μ' with $\mu'\gg\mu$, large logarithmic terms of the form $(\alpha\ln(\mu'/\mu))^n$ show up in conventional perturbation series where α is an expansion parameter. A typical example of such large-log terms is already met in QCD, where summing up the logarithmic terms to all orders is necessary as the expansion parameter α is not so small and the renormalization group method is very useful to deal with this problem.

In electroweak theory we also have two hierarchically different sets of mass scales in low energy phenomena, i.e., (M_W, M_Z) and low energy variables like incident energy and momentum transfer. Here the coupling constant α is very small and so the logarithmic terms do not cause any serious problem. It is, however, still useful to use the renormalization group method to pick up dominant contributions. A study in this direction was put forward by Marciano in the discussion of electroweak higher order effects [Marciano 79]. The formulation based on operator product expansion was recently proposed by Rome group [Antonelli and Maiani 81, Bellucci et al. 81] and by Harvard group [Dawson et al. 81]. We shall briefly survey these works in the following.

The effective electroweak Hamiltonian for low energy processes characterized by mass scale μ may be expressed by

$$H(\mu)=\sum_i C_i\left(\frac{M}{\mu}, \alpha(\mu)\right)O_i, \tag{7·8}$$

where O_i are composite operators relevant to the process under consideration and M is a mass scale of order M_W (or M_Z) and $\alpha(\mu)$ is the fine structure constant defined at the scale μ. In Eq. (7·8) all corrections suppressed by inverse powers of M are neglected [Kazama and Yao 80a, 80b, 80c, 82]. As we consider here only the effective theory, the W- and Z-boson propagators are contracted and radiative corrections due to the photon propagator are relevant.

The coefficient function $C_i(M/\mu, \alpha(\mu))$ satisfies the renormalization group equation

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(\alpha)\frac{\partial}{\partial\alpha}-\gamma_i(\alpha)\right]C_i\left(\frac{M}{\mu}, \alpha(\mu)\right)=0, \tag{7·9}$$

where $\beta(\alpha)$ is the so-called β-function for the electromagnetic coupling and $\gamma_i(\alpha)$ is the anomalous dimension of operator O_i. They are defined as follows:

$$\beta(\alpha)=\mu\frac{d\alpha}{d\mu}, \quad \gamma_i(\alpha)=\frac{\mu}{Z_i}\frac{\partial Z_i}{\partial\mu}\bigg|_{\text{bare } \alpha \text{ fixed}} \tag{7·10}$$

with O_i(renormalized) $=Z_i^{-1}O_i$. Here we consider only the case where there is no operator mixing. Solving the renormalization group equation for the coefficient function $C_i(M/\mu, \alpha(\mu))$, we obtain

$$C_i\left(\frac{M}{\mu}, \alpha(\mu)\right)=C_i(1, \alpha(M))\exp\left[-\int_{\alpha(\mu)}^{\alpha(M)}dx\frac{\gamma_i(x)}{\beta(x)}\right]. \tag{7·11}$$

Calculating γ_i and β perturbatively

$$\gamma_i(\alpha)=\gamma_i^0\alpha+\gamma_i^1\alpha^2+\cdots,$$
$$\beta(\alpha)=\beta_0\alpha^2+\beta_1\alpha^3+\cdots, \tag{7·12}$$

we obtain an improved perturbation series for the coefficient function,

$$C_i\left(\frac{M}{\mu}, \alpha(\mu)\right)=C_i(1, \alpha(M))\left[\frac{\alpha(M)}{\alpha(\mu)}\right]^{-\gamma_i^0/\beta_0}$$
$$\times\left[1+\frac{\beta_1}{\beta_0}\left(\frac{\gamma_i^0}{\beta_0}-\frac{\gamma_i^1}{\beta_1}\right)\alpha(M)+\cdots\right]. \tag{7·13}$$

Here $C_i(1, \alpha(M))$ can also be calculated perturbatively in $\alpha(M)$ with no large logarithmic term.

In charged-current leptonic processes $\nu_\mu e\to\mu\nu_e$ and $\mu\to e\nu_\mu\bar{\nu}_e$, the relevant operators are

$$O_1=\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\nu_e\cdot\bar{e}\gamma^\lambda\mu\,,$$
$$O_2=\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\nu_e\cdot\bar{e}\gamma^\lambda\gamma_5\mu\,. \tag{7·14}$$

The coefficient functions corresponding to these operators are calculated to lowest order and the anomalous dimensions γ_1 and γ_2 to one-loop order. The relevant diagrams in one-loop order contributing to γ_1 and γ_2 are shown in Fig. 7.1. It is well-known that the contribution of Fig. 7.1 is ultraviolet convergent and hence there is no renormalization of operators O_1 and O_2. Accordingly $\gamma_1=\gamma_2=0$. Thus we have

$$C_i\Big(\frac{M}{\mu},\alpha(\mu)\Big)=C_i(1,\alpha(M))\simeq C_i(1,0)\,. \tag{7·15}$$

The explicit calculation shows that

$$C_1(1,0)=-C_2(1,0)=\frac{g^2(M)}{8M_W^2}=\frac{G_{\rm F}}{\sqrt{2}}\,. \tag{7·16}$$

In neutral-current processes $\nu_\mu f\to\nu_\mu f$ (f=fermions) and charged-current process $\nu_\mu d\to\mu u$, the anomalous dimensions are not necessarily zero.

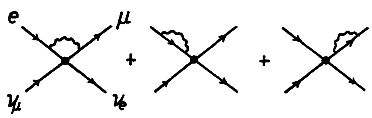

Fig. 7.1. The photonic correction for the four fermion interaction $\nu_\mu e\to\nu\mu_e$.

Let us consider the $\nu_\mu d\to\mu u$ process first. The relevant operators are

$$O_1=\bar{\mu}\gamma_\lambda(1-\gamma_5)\nu_\mu\cdot\bar{u}\gamma^\lambda d\,,$$
$$O_2=\bar{\mu}\gamma_\lambda(1-\gamma_5)\nu_\mu\cdot\bar{u}\gamma^\lambda\gamma_5 d\,. \tag{7·17}$$

Feynman diagrams contributing to the one-loop anomalous dimension are shown in Fig. 7.2. The full contribution of Fig. 7.2 is ultraviolet divergent and requires a renormalization.

In the case of the process $\nu_\mu f\to\nu_\mu f$, the relevant operators are

$$O^i=\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\nu_\mu\cdot\bar{f}\gamma^\lambda A^{(i)}f\,,$$
$$O_5^i=\bar{\nu}_\mu\gamma_\lambda(1-\gamma_5)\nu_\mu\cdot\bar{f}\gamma^\lambda\gamma_5 A_5^{(i)}f\,,$$

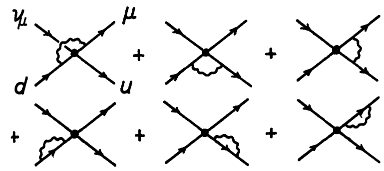

Fig. 7.2. The photonic correction for the four fermion interaction $\nu_\mu d \to \mu u$.

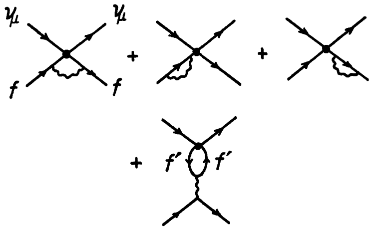

Fig. 7.3. The photonic correction for the four fermion interaction $\nu_\mu f \to \nu_\mu f$.

where $A^{(i)}$ and $A_5^{(i)}$ denote τ_3, Q, 1, etc. The relevant diagrams are shown in Fig. 7.3. In Fig. 7.3 the contributions of the first three diagrams are ultraviolet convergent while the final diagram creates a new ultraviolet divergence (the fourth diagram for operator O_5^i is convergent due to the presence of γ_5).

In order to see the significance of the leading-log summation in electroweak theory, we quote gauge-boson mass shifts calculated by Antonelli and Maiani in Table 7.1 [Antonelli and Maiani 81]. Here the leading-log correction ΔM_W(leading-log), is obviously very important while the correction of order α^2, $\Delta M_W(\alpha^2 \ln^2 M)$, is small and comparable to the non-logarithmic correction of order α, $\Delta M_W(\alpha)$. It should be mentioned here that to incorpolate the non-logarithmic terms into the renormalization group argument, one should deal with the next-to-leading terms as seen in Eq. (7.13) and calculate two-loop anomalous dimensions.

Table 7.1.

$M_W^{(0)}$	ΔM_W (leading-log)	$\Delta M_W(\alpha)$	$\Delta M_W(\alpha^2 \ln^2 M)$
79.48 (GeV)	3.50	−0.70	0.20
77.74	3.27	−0.65	0.18
76.10	3.07	−0.61	0.16

$M_Z^{(0)}$	ΔM_Z (leading-log)	$\Delta M_Z(\alpha)$	$\Delta M_Z(\alpha^2 \ln^2 M)$
90.00 (GeV)	3.83	−0.40	0.24
88.59	3.64	−0.36	0.21
87.29	3.46	−0.33	0.20

Chapter 8

Conclusions and Outlook

In the present review article, we have developed the on-shell renormalization procedure in the Weinberg-Salam theory and given a proof of the charge universality. The procedure has been applied to the study of higher order effects in purely leptonic weak processes $\nu_\mu e \to \nu_\mu e$, $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$, $\nu_\mu e \to \mu \nu_e$ and $\mu \to e \nu_\mu \bar{\nu}_e$. In the course of these investigations we have met with several interesting problems of theoretical and phenomenological importance. We shall briefly recapitulate these problems here.

(1) *On-shell renormalization* We have proposed the on-shell renormalization scheme of the Weinberg-Salam theory as a most convenient and physically significant scheme and have developed the renormalization procedure in this scheme. The consistency of this scheme has been explicitly proved by using the Ward-Takahashi identities especially in the neutral gauge boson sector. The structure of the theory has been clarified paying special attention to the relation between physical particles and corresponding Heisenberg fields.

(2) *Charge universality* While the proof of the charge universality is straightforward in quantum electrodynamics, the proof is nontrivial in gauge theories with spontaneous symmetry breakdown such as the Weinberg-Salam theory. One would naturally expect that the charge universality be satisfied in the Weinberg-Salam theory. By the lack of the proof, however, one could not safely perform higher order calculations. Hence it is worth spending a considerable amount of time to prove it. We have given a lengthy but transparent proof of the charge universality in § 3. 4 by the use of the Ward-Takahashi identity.

(3) *Scheme dependence* There is a variety of the renormalization schemes employed by many authors. The difference of the schemes should not lead to different physical predictions since the choice of a scheme is a matter of convention which is unphysical. In practical calculations, however, we truncate perturbation series and obtain expressions differing from each other by neglected higher-order terms. This situation creates some confusion when one tries to make a comparison of results obtained in different schemes. We have tried to clarify this point as much as possible in §§ 3. 1 and 7. 1. In particular there are two important ingredients in the problem of renormalization schemes: the definition of renormalized parameters and the choice of input data. In choosing independent parameters, one could pick up a parameter (or parameters) which is not directly measurable. For example, the Weinberg angle θ_{W} is not a directly measurable quantity although it is quite

a convenient parameter at the tree level. Beyond the tree level the definition of $\sin\theta_W$ is not unique and its definition must be clearly stated when two results obtained by different schemes are compared. We have adopted the on-shell renormalization scheme throughout the present review article with the choice of independent parameters α, M_W and M_Z (and also fermion and Higgs masses which we omit for a moment). Since our parameters are all directly measurable in experiments, we have no such ambiguity of the definition of parameters.

In order to fix these independent parameters, we must choose the sufficient number of input experimental data. If the data on M_W and M_Z were known, our scheme would be the most convenient to perform higher order calculations. In the present situation one is forced to take some other choices of input data. Different choices of input data may lead to quite a large discrepancy in the size of radiative corrections. This apparent disagreement can, however, be consistently explained by careful examination on the choice of parameters fixed in the calculation of radiative corrections. The careful analysis of this problem has been made in §§ 6.1 and 6.2.

(4) *Large logarithmic corrections* As discussed in Chapter 6, we have separated the electroweak radiative correction into two parts, the electromagnetic part and the rest. We have obtained, in our scheme, rather large corrections concerning the latter part. The origin of these large corrections may be traced back to the large logs of the type $\alpha\ln(M^2/q^2)$ and $\alpha\ln(M^2/m_f^2)$ where M stands for M_W or M_Z and m_f for light fermion masses. The large logs of this type are harmful to perturbative calculations and so they have to be summed to all orders. The renormalization group method is quite useful for this purpose just as it was in perturbative QCD. The renormalization group analyses of the leading log contribution to electroweak processes were carried out by some groups as described in § 7.2. According to the analysis by the Rome group mentioned there, the magnitudes of the leading log terms of order α, $[\alpha\ln(M^2/q^2(m_f^2))]$, those of the non-log terms of order α, $[\alpha]$, those of the leading log terms of order α^2, $[\alpha^2\ln^2(M^2/q^2(m_f^2))]$, etc., are in the following order,

$$[\alpha\ln(M^2/q^2(m_f^2))] > [\alpha] > [\alpha^2\ln^2(M^2/q^2(m_f^2))].$$

This means that the effect of the leading log sum to all orders is less important than that of non-log terms. Thus the conventional perturbation to one-loop order which takes care of terms $[\alpha\ln M]$ and $[\alpha]$ is complementary to the leading log sum by renormalization group method which takes into account terms of $[\alpha\ln M]$, $[\alpha^2\ln^2 M]$, $\cdots$. It would be interesting to estimate the next-to-leading log contribution in the renormalization group method where two-loop anomalous dimensions of relevant operators must be calculated.

(5) *Real photon emission* The calculation of radiative corrections to

exclusive electroweak processes necessarily includes infrared (soft) divergences due to the vanishing photon mass. There is another source of divergences if some of the charged lepton masses vanish. This is the so-called mass singularity. Actually the lepton has small but nonvanishing mass m_l and the singularity takes the form $\ln^2(q^2/m_l^2)$ and $\ln(q^2/m_l^2)$. As is well-known, both of the soft divergence and the mass singularity are removed by adding real photon emission effects. The soft divergence is cancelled by the low energy part of the real photon effect while the mass singularity is cancelled by the collinear configuration of the emitted real photon and the lepton. The hard photon effect, i.e., the collinear part of the real photon effect is more complicated to calculate than the low energy part of the real photon effect. In order to compare the calculated results with experimental data, however, it is indispensable to include the hard photon correction. We have performed this calculation in processes including neutrinos and presented the result in §6.2.

We have summarized here important problems some of which deserve further discussion. We believe that our investigation in the present review article is quite inclusive and presents all the necessary tools to discuss radiative corrections to electroweak processes.

By using low energy data, main independent parameters are determined to one-loop order. In fact the values of M_W and M_Z to one-loop order are now known,

$$M_W = 79.2\ \mathrm{GeV}\,,$$

$$M_Z = 90.5\ \mathrm{GeV}\,.$$

With a proper definition of the Weinberg angle θ_W, we can also fix the value of $\sin^2\theta_W$ at the mass scale M. Thus we have settled the necessary information on the values of relevant parameters at mass scale M in grand unification theories. Once the values of the parameters at mass scale M are fixed, the corresponding values at the grand unification mass scale are easily calculated by the renormalization group method.

Throughout the paper the Higgs particles are treated as elementary scalar fields. If the spontaneous breakdown of the $SU(2)\times U(1)$ symmetry is generated dynamically, the Higgs particles are composite systems of some fundamental object. This possibility has been studied by many authors and has led to models of composite quarks and leptons. If this is the case, our analysis in the present article is considered to be an effective low energy theory for electroweak interactions.

We have confined ourselves to the Weinberg-Salam theory of electroweak interactions and expected the existence of the W- and Z-boson. Experimentally it is of course an open problem whether the W- and Z-boson will be observed or not. Anticipated experiment in search for W and Z is a great challenge to the present electroweak theory. If W- and Z-boson will not be observed, there will emerge ages for theorists.

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Appendix A

Dimensional Regularization

Dimensional regularization ['t Hooft and Veltman 72a] is widely used in calculations of radiative corrections. Since an analytic continuation in the space-time dimensions is not unique, there is a variety of conventions in this method. We adopt the following convention: $\mathrm{Tr}(\mathbf{1})=4$, $\{\gamma_\mu, \gamma_5\}=0$ and $\int d^Dk/(2\pi)^D i$ in D-dimensional space. Below we list various formulas in our convention.

A. 1 γ matrix algebra in D-dimensional space

The basic algebra is

$$\{\gamma_\mu, \gamma_\nu\}=2g_{\mu\nu}\,. \tag{A·1}$$

Metric tensor $g_{\mu\nu}$ satisfies

$$g_{\mu\nu}g^{\mu\nu}=D\,. \tag{A·2}$$

Combining (A·1) and (A·2), we obtain

$$\gamma_\lambda\gamma^\lambda=D\,, \tag{A·3}$$

$$\gamma_\lambda\gamma_\mu\gamma^\lambda=(2-D)\gamma_\mu\,, \tag{A·4}$$

$$\gamma_\lambda\gamma_\mu\gamma_\nu\gamma^\lambda=4g_{\mu\nu}+(D-4)\gamma_\mu\gamma_\nu\,, \tag{A·5}$$

$$\gamma_\lambda\gamma_\mu\gamma_\nu\gamma_\rho\gamma^\lambda=-2\gamma_\rho\gamma_\nu\gamma_\mu+(4-D)\gamma_\mu\gamma_\nu\gamma_\rho\,. \tag{A·6}$$

Further, by using our convention on unit matrix $\mathbf{1}$,

$$\mathrm{Tr}(\mathbf{1})=4\,, \tag{A·7}$$

we find

$$\mathrm{Tr}(\gamma_\mu\gamma_\nu)=4g_{\mu\nu}\,, \tag{A·8}$$

$$\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho)=4(g_{\mu\nu}g_{\lambda\rho}+g_{\mu\rho}g_{\nu\lambda}-g_{\mu\lambda}g_{\nu\rho})\,. \tag{A·9}$$

As mentioned above, the γ_5 matrix is defined so that it satisfies

$$\{\gamma_\mu, \gamma_5\}=0\,. \tag{A·10}$$

There occurs no trouble concerning the γ_5 matrix in the present case, since the Weinberg-Salam theory is an anomaly-free theory.

A. 2 Momentum integral

Momentum integral is extended as

$$\int \frac{d^4k}{(2\pi)^4 i} \quad \rightarrow \quad \int \frac{d^D k}{(2\pi)^D i} \,. \tag{A·11}$$

This Minkowski-space integral is performed after Wick rotation into Euclidean space, $k_0 = iK_0$.

$$\begin{aligned}\int d^D K &= \int dK_0 dK_1 \cdots dK_{D-1} \\ &= \int K^{D-1} dK d\Omega_D \,, \quad (K^2 \equiv \sum_i K_i K^i)\end{aligned} \tag{A·12}$$

$$\begin{aligned}\int d\Omega_D &= \int_0^\pi d\theta_1 \sin^{D-2}\theta_1 \int_0^\pi d\theta_2 \sin^{D-3}\theta_2 \cdots \int_0^{2\pi} d\theta_{D-1} \\ &= 2\pi^{D/2} \left[\Gamma\left(\frac{D}{2}\right)\right]^{-1} .\end{aligned} \tag{A·13}$$

We list below several typical integral formulas:

$$\int d^D k \cdot k_\mu f(k^2) = 0 \,. \tag{A·14}$$

$$\int d^D k \cdot k_\mu k_\nu f(k^2) = \frac{1}{D} g_{\mu\nu} \int d^D k \cdot k^2 f(k^2) \,. \tag{A·15}$$

$$\int d^D k \frac{1}{(-k^2+Z)^\alpha} = i\pi^{D/2} \frac{\Gamma(\alpha-(D/2))}{\Gamma(\alpha)} Z^{((D/2)-\alpha)} . \tag{A·16}$$

$$\int d^D k \frac{-k^2}{(-k^2+Z)^\alpha} = i\pi^{D/2} \frac{D}{2} \frac{\Gamma(\alpha-1-(D/2))}{\Gamma(\alpha)} Z^{(1+(D/2)-\alpha)} \,. \tag{A·17}$$

$$\begin{aligned}\int d^D k \frac{1}{(-k^2)^\alpha \{-(k+q)^2\}^\beta} &= i\pi^{D/2} (-q^2)^{-\varepsilon+2-\alpha-\beta} \frac{\Gamma(\varepsilon-2+\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \\ &\quad \times B(2-\alpha-\varepsilon, 2-\beta-\varepsilon) \,.\end{aligned} \tag{A·18}$$

$$\begin{aligned}&\int d^D k \frac{k_\mu}{(-k^2)^\alpha \{-(k+q)^2\}^\beta} \\ &\quad = -i\pi^{D/2} q_\mu (-q^2)^{-\varepsilon+2-\alpha-\beta} \frac{\Gamma(\varepsilon-2+\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} B(3-\alpha-\varepsilon, 2-\beta-\varepsilon) \,.\end{aligned} \tag{A·19}$$

$$\int d^D k \frac{k_\mu k_\nu}{(-k^2)^\alpha \{-(k+q)^2\}^\beta} = i\pi^{D/2} (A g_{\mu\nu} + B q_\mu q_\nu) \,, \tag{A·20}$$

$$A=-(-q^2)^{-\varepsilon+3-\alpha-\beta}\frac{\Gamma(\varepsilon-3+\alpha+\beta)}{2\Gamma(\alpha)\Gamma(\beta)}B(3-\alpha-\varepsilon,3-\beta-\varepsilon), \tag{A·20a}$$

$$B=(-q^2)^{-\varepsilon+2-\alpha-\beta}\frac{\Gamma(\varepsilon-2+\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}B(4-\alpha-\varepsilon,2-\beta-\varepsilon). \tag{A·20b}$$

$\Gamma(x)$ and $B(x, y)$ represent the Gamma and Beta functions respectively.

$$\varepsilon\equiv\frac{1}{2}(4-D)\rightleftarrows\frac{D}{2}\equiv2-\varepsilon, \tag{A·21}$$

$$\Gamma(\varepsilon)=\frac{1}{\varepsilon}-\gamma+O(\varepsilon), \tag{A·22}$$

$$\Gamma(\varepsilon-1)=-\frac{1}{\varepsilon}-(1-\gamma)+O(\varepsilon), \tag{A·23}$$

(γ is the Euler constant)

$$B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}=\int_0^1 d\xi\,\xi^{x-1}(1-\xi)^{y-1}$$
$$=\int_0^\infty dt\frac{t^{y-1}}{(1+t)^{x+y}}=\int_0^\infty dt\frac{t^{x-1}}{(1+t)^{x+y}}. \tag{A·24}$$

Appendix B

Formulas for One-Loop Feynman Amplitudes

We give one-loop Feynman amplitudes expressed in the form of Feynman parameter integration. They are obtained by performing the loop momentum integral. We drop some overall factors in order to make formulas simple.

In the following, we express the ultraviolet divergent part as

$$C_{\mathrm{UV}}\equiv\frac{1}{\varepsilon}-\gamma+\ln 4\pi \tag{B·1}$$

and we set $D\to4$ for the convergent parts.

B.1 Two-point functions

Here we use the following function,

$$D_2(x)\equiv m_1^2(1-x)+m_2^2x-q^2x(1-x), \tag{B·2}$$

$$\gamma(A,B)\equiv(1+AB)-(A+B)\gamma_5, \tag{B·3}$$

where m_1, m_2, and q are shown in Fig. B.1.

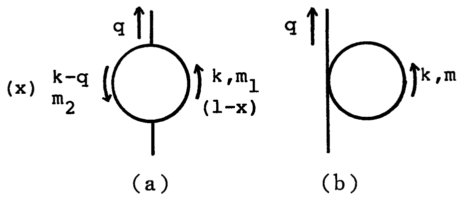
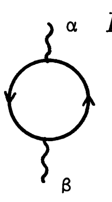


Fig. B. 1. Typical diagrams of two-point functions.
k is the loop momentum and q is the external momentum. m_1, m_2 and m denote the masses of particles of internal lines. x and $(1-x)$ in (a) show our choice of Feynman parameter.

$$\Sigma(q)$$
$$\sim \int \frac{d^D k}{(2\pi)^D i} \gamma_\alpha (1+A\gamma_5) \frac{m_2+\not{q}-\not{k}}{m_2^2-(q-k)^2} \gamma_\beta (1+B\gamma_5) \frac{g^{\alpha\beta}}{k^2-m_1^2}$$
$$= -\frac{1}{8\pi^2} m_2 \gamma(A, -B) \left\{ 2C_{\mathrm{UV}} - 1 - 2\int_0^1 dx \ln(D_2(x)) \right\}$$
$$+ \frac{1}{16\pi^2} \not{q} \gamma(-A, -B) \left\{ C_{\mathrm{UV}} - 1 - 2\int_0^1 dx (1-x) \ln(D_2(x)) \right\}. \tag{B·4}$$
*)

$$\Sigma(q)$$
$$\sim \int \frac{d^D k}{(2\pi)^D i} (1+A\gamma_5) \frac{m_2+\not{q}-\not{k}}{m_2^2-(q-k)^2} (1+B\gamma_5) \frac{1}{m_1^2-k^2}$$
$$= \frac{1}{16\pi^2} m_2 \gamma(-A, -B) \left\{ C_{\mathrm{UV}} - \int_0^1 dx \ln(D_2(x)) \right\}$$
$$+ \frac{1}{32\pi^2} \not{q} \gamma(A, -B) \left\{ C_{\mathrm{UV}} - 2\int_0^1 dx (1-x) \ln(D_2(x)) \right\}. \tag{B·5}$$
**)

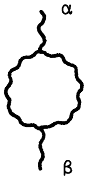


$$\Pi_{\alpha\beta}(q)$$
$$\sim (-1) \times \int \frac{d^D k}{(2\pi)^D i} \mathrm{Tr} \left\{ \gamma_\alpha (1+A\gamma_5) \frac{m_1+\not{k}}{m_1^2-k^2} \right.$$
$$\left. \times \gamma_\beta (1+B\gamma_5) \frac{m_2+\not{k}-\not{q}}{m_2^2-(k-q)^2} \right\}$$
$$= \frac{1}{2\pi^2} (1+AB)(g_{\alpha\beta} q^2 - q_\alpha q_\beta) \left\{ -\frac{1}{6} C_{\mathrm{UV}} + \int_0^1 dx\, x(1-x) \ln(D_2(x)) \right\}$$
$$- \frac{1}{4\pi^2} g_{\alpha\beta} C_{\mathrm{UV}} \left\{ m_1 m_2 (1-AB) - \frac{1}{2}(m_1^2+m_2^2)(1+AB) \right\}$$

*) A solid line and a wavy line represent a fermion and a gauge boson respectively.
**) A dashed line represents a scalar particle.

$$+\frac{1}{4\pi^2}g_{\alpha\beta}\int_0^1 dx[m_1m_2(1-AB)-(1+AB)\overline{m^2}]\ln(D_2(x)).$$

$$(\overline{m^2}\equiv m_1^2(1-x)+m_2^2x) \tag{B·6}$$



$\Pi_{\alpha\beta}(q)$

$$\sim\int\frac{d^Dk}{(2\pi)^Di}\{(-q+k-q)_\rho g_{\alpha\sigma}+(-k+q-k)_\alpha g_{\rho\sigma}+(k+q)_\sigma g_{\alpha\rho}\}$$

$$\times\frac{g^{\rho\rho'}}{k^2-m_1^2}\frac{g^{\sigma\sigma'}}{(k-q)^2-m_2^2}$$

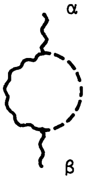
$$\times\{(q+k)_{\sigma'}g_{\beta\rho'}+(-k-k+q)_\beta g_{\rho'\sigma'}+(k-q-q)_{\rho'}g_{\sigma'\beta}\}$$

$$=\frac{-1}{48\pi^2}q_\alpha q_\beta(11C_{\text{UV}}+2)$$

$$+\frac{1}{16\pi^2}g_{\alpha\beta}\left\{\frac{3}{2}(m_1^2+m_2^2)(3C_{\text{UV}}+1)+\frac{1}{6}q^2(19C_{\text{UV}}-3)\right\}$$

$$+\frac{1}{8\pi^2}q_\alpha q_\beta\int_0^1 dx\{1+5x(1-x)\}\ln(D_2(x))$$

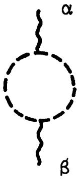
$$-\frac{1}{16\pi^2}g_{\alpha\beta}\int_0^1 dx\{9\overline{m^2}+q^2(11x^2-11x+5)\}\ln(D_2(x)). \tag{B·7}$$



$\Pi_{\alpha\beta}(q)$

$$\sim\int\frac{d^Dk}{(2\pi)^Di}g_{\alpha\sigma}\frac{g^{\sigma\rho}}{(k-q)^2-m_2^2}g_{\rho\beta}\frac{1}{m_1^2-k^2}$$

$$=\frac{-1}{16\pi^2}g_{\alpha\beta}\left\{C_{\text{UV}}-\int_0^1 dx\ln(D_2(x))\right\}. \tag{B·8}$$

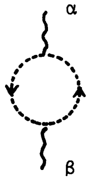


$\Pi_{\alpha\beta}(q)$

$$\sim\int\frac{d^Dk}{(2\pi)^Di}(-k+q-k)_\alpha\frac{1}{m_1^2-k^2}\frac{1}{m_2^2-(k-q)^2}(-k-k+q)_\beta$$

$$=\frac{1}{16\pi^2}\left\{\frac{1}{3}C_{\text{UV}}q_\alpha q_\beta+g_{\alpha\beta}(C_{\text{UV}}+1)\left(m_1^2+m_2^2-\frac{q^2}{3}\right)\right\}$$

$$-\frac{1}{16\pi^2}\int_0^1 dx\{(1-2x)^2q_\alpha q_\beta+2g_{\alpha\beta}D_2(x)\}\ln(D_2(x)). \tag{B·9}$$



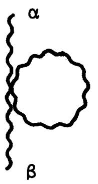
$\Pi_{\alpha\beta}(q)$

$$\sim(-1)\times\int\frac{d^Dk}{(2\pi)^Di}(-k+q)_\alpha\frac{1}{m_1^2-k^2}\frac{1}{m_2^2-(k-q)^2}(-k)_\beta$$

$$=\frac{-1}{16\pi^2}\left\{\frac{1}{4}(C_{\text{UV}}+1)g_{\alpha\beta}\left(m_1^2+m_2^2-\frac{q^2}{3}\right)-\frac{1}{6}C_{\text{UV}}q_\alpha q_\beta\right\}$$

$$-\frac{1}{16\pi^2}\int_0^1 dx\left\{-\frac{1}{2}g_{\alpha\beta}D_2(x)+x(1-x)\,q_\alpha q_\beta\right\}\ln(D_2(x)).$$

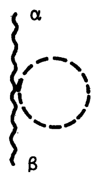
(B·10)*)



$$\Pi_{\alpha\beta}(q)$$

$$\sim\int\frac{d^Dk}{(2\pi)^D i}(g_{\beta\rho}g_{\alpha\sigma}+g_{\alpha\rho}g_{\beta\sigma}-2g_{\rho\sigma}g_{\alpha\beta})\frac{g^{\rho\sigma}}{k^2-m^2}$$

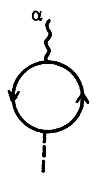
$$=\frac{-1}{8\pi^2}g_{\alpha\beta}m^2(3C_{\mathrm{UV}}+1-3\ln m^2). \tag{B·11}$$



$$\Pi_{\alpha\beta}(q)$$

$$\sim\int\frac{d^Dk}{(2\pi)^D i}g_{\alpha\beta}\frac{1}{m^2-k^2}$$

$$=-\frac{1}{16\pi^2}g_{\alpha\beta}m^2(C_{\mathrm{UV}}+1-\ln m^2). \tag{B·12}$$

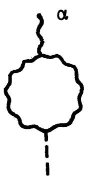


$$\Pi_\alpha(q)$$

$$\sim(-1)\times\int\frac{d^Dk}{(2\pi)^D i}\mathrm{Tr}\left\{\gamma_\alpha(1+A\gamma_5)\frac{m_1+\not k}{m_1{}^2-k^2}(1+B\gamma_5)\frac{m_2+\not k-\not q}{m_2{}^2-(k-q)^2}\right\}$$

$$=\frac{-1}{4\pi^2}q_\alpha\int_0^1 dx\{C_{\mathrm{UV}}-\ln(D_2(x))\}$$

$$\times\{(1-AB)\,m_2x-(1+AB)\,m_1(1-x)\}. \tag{B·13}$$

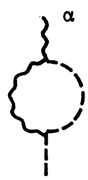


$$\Pi_\alpha(q)$$

$$\sim\int\frac{d^Dk}{(2\pi)^D i}\{(-q+k-q)_\rho g_{\alpha\sigma}+(-k+q-k)_\alpha g_{\rho\sigma}+(k+q)_\sigma g_{\alpha\rho}\}$$

$$\times\frac{g^{\rho\rho'}}{k^2-m_1{}^2}g_{\rho'\sigma'}\frac{g^{\sigma'\sigma}}{(k-q)^2-m_2{}^2}$$

$$=-\frac{3}{16\pi^2}q_\alpha\int_0^1 dx(1-2x)\ln(D_2(x)). \tag{B·14}$$

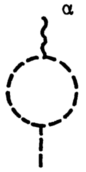


$$\Pi_\alpha(q)$$

$$\sim\int\frac{d^Dk}{(2\pi)^D i}g_{\alpha\rho}\frac{g^{\rho\sigma}}{(k-q)^2-m_2{}^2}(q+k)_\sigma\frac{1}{m_1{}^2-k^2}$$

$$=-\frac{1}{16\pi^2}q_\alpha\left\{\frac{3}{2}C_{\mathrm{UV}}-\int_0^1 dx(1+x)\ln(D_2(x))\right\}. \tag{B·15}$$

*) A dotted line represents an FP-ghost.



$$\Pi_\alpha(q)$$

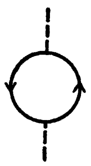
$$\sim \int \frac{d^D k}{(2\pi)^D i}(k+k-q)_\alpha \frac{1}{m_1^2-k^2}\frac{1}{m_2^2-(k-q)^2}$$

$$=\frac{1}{16\pi^2}q_\alpha \int_0^1 dx(1-2x)\ln(D_2(x)). \tag{B·16}$$

$$\Pi_\alpha(q)$$

$$\sim(-1)\times\int \frac{d^D k}{(2\pi)^D i}(-k+q)_\alpha \frac{1}{m_1^2-k^2}\frac{1}{m_2^2-(k-q)^2}$$

$$=\frac{-1}{16\pi^2}q_\alpha\left\{\frac{1}{2}C_{\rm UV}-\int_0^1 dx(1-x)\ln(D_2(x))\right\}. \tag{B·17}$$



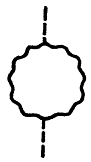
$$\Pi(q)$$

$$\sim(-1)\times\int \frac{d^D k}{(2\pi)^D i}{\rm Tr}\left\{(1+A\gamma_5)\frac{m_1+\not k}{m_1^2-k^2}(1+B\gamma_5)\frac{m_2+\not k-\not q}{m_2^2-(k-q)^2}\right\}$$

$$=\frac{-1}{4\pi^2}\int_0^1 dx[\{(1+AB)m_1m_2$$

$$-(1-AB)x(1-x)q^2\}\{C_{\rm UV}-\ln(D_2(x))\}$$

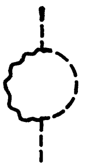
$$+(1-AB)\{2C_{\rm UV}+1-2\ln(D_2(x))\}D_2(x)]. \tag{B·18}$$



$$\Pi(q)$$

$$\sim\int \frac{d^D k}{(2\pi)^D i}g_{\rho\rho'}\frac{g^{\rho'\sigma'}}{(k-q)^2-m_2^2}g_{\sigma'\sigma}\frac{g^{\sigma\rho}}{k^2-m_1^2}$$

$$=\frac{1}{8\pi^2}\left\{2C_{\rm UV}-1-2\int_0^1 dx\ln(D_2(x))\right\}. \tag{B·19}$$

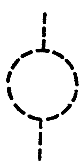


$$\Pi(q)$$

$$\sim\int \frac{d^D k}{(2\pi)^D i}(k+q)_\rho\frac{g^{\rho\sigma}}{(k-q)^2-m_2^2}(q+k)_\sigma\frac{1}{m_1^2-k^2}$$

$$=\frac{-1}{16\pi^2}\int_0^1 dx[q^2(1+x)^2\{C_{\rm UV}-\ln(D_2(x))\}$$

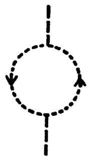
$$+D_2(x)\{2C_{\rm UV}+1-2\ln(D_2(x))\}]. \tag{B·20}$$



$$\Pi(q)$$

$$\sim\int \frac{d^D k}{(2\pi)^D i}\frac{1}{m_1^2-k^2}\frac{1}{m_2^2-(k-q)^2}$$

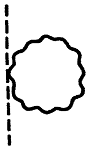
$$=\frac{1}{16\pi^2}\left\{C_{\rm UV}-\int_0^1 dx\ln(D_2(x))\right\}. \tag{B·21}$$



$$\Pi(q)$$

$$\sim(-1)\times\int\frac{d^Dk}{(2\pi)^Di}\frac{1}{m_1^2-k^2}\frac{1}{m_2^2-(k-q)^2}$$

$$=\frac{-1}{16\pi^2}\left\{C_{\mathrm{UV}}-\int_0^1 dx\ln(D_2(x))\right\}. \tag{B·22}$$



$$\Pi(q)$$

$$\sim\int\frac{d^Dk}{(2\pi)^Di}g_{\rho\sigma}\frac{g^{\sigma\rho}}{k^2-m^2}$$

$$=\frac{1}{8\pi^2}m^2(2C_{\mathrm{UV}}+1-2\ln m^2). \tag{B·23}$$

$$\Pi(q)$$

$$\sim\int\frac{d^Dk}{(2\pi)^Di}\frac{1}{m^2-k^2}$$

$$=\frac{-1}{16\pi^2}m^2(C_{\mathrm{UV}}+1-\ln m^2). \tag{B·24}$$

B. 2 Three-point functions

Momentum flow and masses are shown in Fig. B. 2.

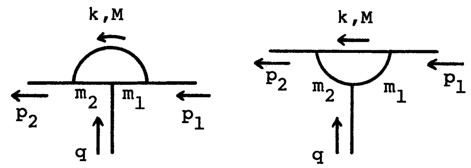


Fig. B. 2. Typical diagrams of three-point functions. k is the loop momentum and m_1, m_2 and M denote the masses of propagating particles.

We take the Feynman parametrization as

$$\frac{1}{ABC}=\int_0^1 dx\frac{1}{\{A(1-x)+Bx\}^2}\frac{1}{C}$$

$$=\int_0^1 dx\int_0^1 dy\frac{2y}{[\{A(1-x)+Bx\}y+C(1-y)]^3},$$

where

$$A\equiv m_1^2-(p_1-k)^2,$$

$$B\equiv m_2^2-(p_2-k)^2,$$

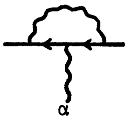
$$C\equiv M^2-k^2.$$

We use the following functions,

$$D_3(x, y) \equiv (1-y)M^2 + y\{\overline{m^2} - q^2x(1-x)\} - y(1-y)\bar{p}^2\,,$$

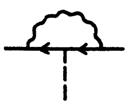
$$(\overline{m^2} \equiv (1-x)m_1^2 + xm_2^2, \quad \bar{p} \equiv (1-x)p_1 + xp_2) \qquad \text{(B·25)}$$

$$\gamma(A, B, C) \equiv (1+AB+BC+CA) - (A+B+C+ABC)\gamma_5\,. \qquad \text{(B·26)}$$



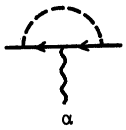
$$\begin{aligned}
&\Gamma_\alpha(p_2, p_1)\\
&\sim \int \frac{d^Dk}{(2\pi)^D i}\gamma_\rho(1+A\gamma_5)\frac{m_2+\not{p}_2-\not{k}}{m_2^2-(p_2-k)^2}\gamma_\alpha(1+B\gamma_5)\\
&\quad\times\frac{m_1+\not{p}_1-\not{k}}{m_1^2-(p_1-k)^2}\gamma_\sigma(1+C\gamma_5)\frac{g^{\rho\sigma}}{k^2-M^2}\\
&=\frac{1}{8\pi^2}m_1m_2\gamma(A, -B, C)\gamma_\alpha\int_0^1\int_0^1\frac{y}{D_3(x, y)}dxdy\\
&\quad-\frac{1}{4\pi^2}m_1\gamma(A, B, -C)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y(p_2-y\bar{p})_\alpha\\
&\quad-\frac{1}{4\pi^2}m_2\gamma(A, -B, -C)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y(p_1-y\bar{p})_\alpha\\
&\quad+\frac{1}{8\pi^2}\gamma(A, B, C)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y(\not{p}_1-y\bar{\not{p}})\gamma_\alpha(\not{p}_2-y\bar{\not{p}})\\
&\quad+\frac{1}{16\pi^2}\gamma(A, B, C)\gamma_\alpha\left\{C_{\mathrm{UV}}-2-2\int_0^1\int_0^1 dxdy\,y\ln(D_3(x, y))\right\}.
\end{aligned}$$

(B·27)



$$\begin{aligned}
&\Gamma(p_2, p_1)\\
&\sim \int \frac{d^Dk}{(2\pi)^D i}\gamma_\rho(1+A\gamma_5)\frac{m_2+\not{p}_2-\not{k}}{m_2^2-(p_2-k)^2}(1+B\gamma_5)\\
&\quad\times\frac{m_1+\not{p}_1-\not{k}}{m_1^2-(p_1-k)^2}\gamma_\sigma(1+C\gamma_5)\frac{g^{\rho\sigma}}{k^2-M^2}\\
&=\frac{-1}{4\pi^2}m_1m_2\gamma(A, B, -C)\int_0^1\int_0^1\frac{y}{D_3(x, y)}dxdy\\
&\quad+\frac{1}{8\pi^2}m_1\gamma(A, -B, C)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y(\not{p}_2-y\bar{\not{p}})\\
&\quad+\frac{1}{8\pi^2}m_2\gamma(A, B, C)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y(\not{p}_1-y\bar{\not{p}})\\
&\quad+\frac{1}{4\pi^2}\gamma(A, -B, -C)\Big[C_{\mathrm{UV}}-1
\end{aligned}$$

$$-\int_0^1\int_0^1 dxdy\ y\left\{\frac{(\not{p}_1-y\bar{\not{p}})(\not{p}_2-y\bar{\not{p}})}{D_3(x,y)}+2\ln(D_3(x,y))\right\}\Bigg]. \qquad (\mathrm{B}\cdot 28)$$



$$\Gamma_\alpha(p_2,p_1)$$

$$\sim\int\frac{d^Dk}{(2\pi)^D i}(1+A\gamma_5)\frac{m_2+\not{p}_2-\not{k}}{m_2^2-(p_2-k)^2}\gamma_\alpha(1+B\gamma_5)$$

$$\times\frac{m_1+\not{p}_1-\not{k}}{m_1^2-(p_1-k)^2}(1+C\gamma_5)\frac{1}{M^2-k^2}$$

$$=\frac{1}{16\pi^2}m_1m_2\gamma(-A,B,C)\gamma_\alpha\int_0^1\int_0^1\frac{y}{D_3(x,y)}dxdy$$

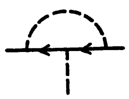
$$+\frac{1}{16\pi^2}m_1\gamma(-A,-B,-C)\int_0^1\int_0^1\frac{dxdy}{D_3(x,y)}y(\not{p}_2-y\bar{\not{p}})\gamma_\alpha$$

$$+\frac{1}{16\pi^2}m_2\gamma(-A,B,-C)\gamma_\alpha\int_0^1\int_0^1\frac{dxdy}{D_3(x,y)}y(\not{p}_1-y\bar{\not{p}})$$

$$+\frac{1}{16\pi^2}\gamma(-A,-B,C)\int_0^1\int_0^1\frac{dxdy}{D_3(x,y)}y(\not{p}_2-y\bar{\not{p}})\gamma_\alpha(\not{p}_1-y\bar{\not{p}})$$

$$+\frac{1}{32\pi^2}\gamma(-A,-B,C)\gamma_\alpha\left\{C_{\mathrm{UV}}-1-2\int_0^1\int_0^1 dxdy\ y\ln(D_3(x,y))\right\}.$$

$$(\mathrm{B}\cdot 29)$$



$$\Gamma(p_2,p_1)$$

$$\sim\int\frac{d^Dk}{(2\pi)^D i}(1+A\gamma_5)\frac{m_2+\not{p}_2-\not{k}}{m_2^2-(p_2-k)^2}(1+B\gamma_5)$$

$$\times\frac{m_1+\not{p}_1-\not{k}}{m_1^2-(p_1-k)^2}(1+C\gamma_5)\frac{1}{M^2-k^2}$$

$$=\frac{1}{16\pi^2}m_1m_2\gamma(-A,-B,-C)\int_0^1\int_0^1\frac{y}{D_3(x,y)}dxdy$$

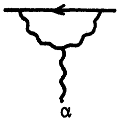
$$+\frac{1}{16\pi^2}m_1\gamma(-A,B,C)\int_0^1\int_0^1\frac{dxdy}{D_3(x,y)}y(\not{p}_2-y\bar{\not{p}})$$

$$+\frac{1}{16\pi^2}m_2\gamma(-A,-B,C)\int_0^1\int_0^1\frac{dxdy}{D_3(x,y)}y(\not{p}_1-y\bar{\not{p}})$$

$$+\frac{1}{16\pi^2}\gamma(-A,B,-C)\int_0^1\int_0^1\frac{dxdy}{D_3(x,y)}y(\not{p}_2-y\bar{\not{p}})(\not{p}_1-y\bar{\not{p}})$$

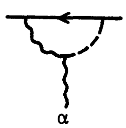
$$-\frac{1}{32\pi^2}\gamma(-A,B,-C)\left\{2C_{\mathrm{UV}}-1-4\int_0^1\int_0^1 dxdy\ y\ln(D_3(x,y))\right\}.$$

$$(\mathrm{B}\cdot 30)$$

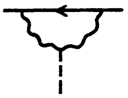


$$\begin{aligned}
&\Gamma_\alpha(p_2, p_1)\\
&\sim \int \frac{d^D k}{(2\pi)^D i}\gamma_\rho(1+A\gamma_5)\frac{M+\not k}{M^2-k^2}\gamma_\sigma(1+B\gamma_5)\\
&\times\frac{g^{\rho\rho'}}{(p_2-k)^2-m_2^2}\{(q-p_1+k)_{\rho'}g_{\alpha\sigma'}+(p_1-k+p_2-k)_\alpha g_{\rho'\sigma'}\\
&\qquad+(-p_2+k-q)_{\sigma'}g_{\alpha\rho'}\}\frac{g^{\sigma\sigma'}}{(p_1-k)^2-m_1^2}\\
&=\frac{3}{16\pi^2}M\gamma(A, -B)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y\{2(p_2-y\bar{p})_\alpha-\not q\gamma_\alpha\}\\
&+\frac{1}{16\pi^2}\gamma(A, B)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y^2\\
&\times\{(\not p_2-2\not p_1+y\bar{\not p})\bar{\not p}\gamma_\alpha-2(p_1+p_2-2y\bar{p})_\alpha\bar{\not p}+\gamma_\alpha\bar{\not p}(\not p_1-2\not p_2+y\bar{\not p})\}\\
&-\frac{1}{16\pi^2}\gamma(A, B)\gamma_\alpha\left\{3C_{\mathrm{UV}}-2-6\int_0^1\int_0^1 dxdyy\ln(D_3(x, y))\right\}.
\end{aligned}\tag{B·31}$$

(B·31)*)



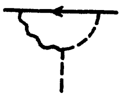
$$\begin{aligned}
&\Gamma_\alpha(p_2, p_1)\\
&\sim \int \frac{d^D k}{(2\pi)^D i}\gamma_\rho(1+A\gamma_5)\frac{M+\not k}{M^2-k^2}(1+B\gamma_5)\\
&\times\frac{g^{\rho\sigma}}{(p_2-k)^2-m_2^2}g_{\sigma\alpha}\frac{1}{m_1^2-(p_1-k)^2}\\
&=\frac{1}{16\pi^2}M\gamma(A, B)\gamma_\alpha\int_0^1\int_0^1\frac{y}{D_3(x, y)}dxdy\\
&+\frac{1}{16\pi^2}\gamma(A, -B)\gamma_\alpha\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y^2\bar{\not p}\,.
\end{aligned}\tag{B·32}$$



$$\begin{aligned}
&\Gamma(p_2, p_1)\\
&\sim \int \frac{d^D k}{(2\pi)^D i}\gamma_\rho(1+A\gamma_5)\frac{M+\not k}{M^2-k^2}\gamma_\sigma(1+B\gamma_5)\\
&\times\frac{g^{\rho\rho'}}{(p_2-k)^2-m_2^2}g_{\rho'\sigma'}\frac{g^{\sigma'\sigma}}{(p_1-k)^2-m_1^2}\\
&=\frac{1}{4\pi^2}M\gamma(A, -B)\int_0^1\int_0^1\frac{y}{D_3(x, y)}dxdy\\
&-\frac{1}{8\pi^2}\gamma(A, B)\int_0^1\int_0^1\frac{dxdy}{D_3(x, y)}y^2\bar{\not p}\,.
\end{aligned}\tag{B·33}$$

---

*) $\gamma(A,B)$ is defined in Eq. (B·3).



$\Gamma(p_2, p_1)$

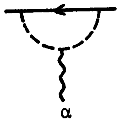
$$\sim \int \frac{d^D k}{(2\pi)^D i} \gamma_\rho (1+A\gamma_5) \frac{M+\not{k}}{M^2-k^2} (1+B\gamma_5)$$

$$\times \frac{g^{\rho\sigma}}{(p_2-k)^2-m_2^2} (p_1-k-q)_\sigma \frac{1}{m_1^2-(p_1-k)^2}$$

$$= \frac{-1}{16\pi^2} M\gamma(A, B) \int_0^1 \int_0^1 \frac{dxdy}{D_3(x, y)} y(2\not{p}_1-\not{p}_2-y\bar{\not{p}})$$

$$-\frac{1}{16\pi^2}\gamma(A, -B) \int_0^1 \int_0^1 \frac{dxdy}{D_3(x, y)} y^2(2\not{p}_1-\not{p}_2-y\bar{\not{p}})\bar{\not{p}}$$

$$-\frac{1}{32\pi^2}\gamma(A, -B) \left\{2C_{\mathrm{UV}}-1-4\int_0^1 \int_0^1 dxdy\, y\ln(D_3(x, y))\right\}. \tag{B·34}$$



$\Gamma_\alpha(p_2, p_1)$

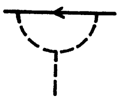
$$\sim \int \frac{d^D k}{(2\pi)^D i} (1+A\gamma_5) \frac{M+\not{k}}{M^2-k^2} (1+B\gamma_5)$$

$$\times \frac{1}{m_2^2-(p_2-k)^2} (p_1-k+p_2-k)_\alpha \frac{1}{m_1^2-(p_1-k)^2}$$

$$= \frac{1}{16\pi^2} M\gamma(-A, -B) \int_0^1 \int_0^1 \frac{dxdy}{D_3(x, y)} y(p_1+p_2-2y\bar{p})_\alpha$$

$$+\frac{1}{16\pi^2}\gamma(-A, B) \int_0^1 \int_0^1 \frac{dxdy}{D_3(x, y)} y^2(p_1+p_2-2y\bar{p})_\alpha \bar{\not{p}}$$

$$+\frac{1}{32\pi^2}\gamma(-A, B)\gamma_\alpha \left\{C_{\mathrm{UV}}-2\int_0^1 \int_0^1 dxdy\, y\ln(D_3(x, y))\right\}. \tag{B·35}$$



$\Gamma(p_2, p_1)$

$$\sim \int \frac{d^D k}{(2\pi)^D i} (1+A\gamma_5) \frac{M+\not{k}}{M^2-k^2} (1+B\gamma_5) \frac{1}{m_2^2-(p_2-k)^2} \frac{1}{m_1^2-(p_1-k)^2}$$

$$= \frac{1}{16\pi^2} M\gamma(-A, -B) \int_0^1 \int_0^1 \frac{y}{D_3(x, y)} dxdy$$

$$+\frac{1}{16\pi^2}\gamma(-A, B) \int_0^1 \int_0^1 \frac{dxdy}{D_3(x, y)} y^2\bar{\not{p}}\,. \tag{B·36}$$

B. 3 Four-point functions

Masses and momentum flow are shown in Fig. B. 3. Box type diagrams do not include UV divergence. Therefore we set $D\to4$ in the following.

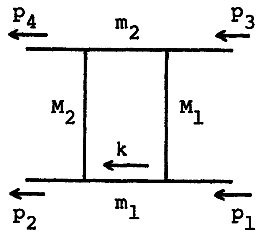


Fig. B.3. Typical diagram of four-point function. k is the loop momentum and $m_{1,2}$ and $M_{1,2}$ denote the masses of propagating particles.

Feynman parametrization is performed as

$$\frac{1}{ABCD}=\int_0^1 dx\int_0^1 dy\frac{1}{\{A(1-x)+Bx\}^2}\frac{1}{\{C(1-y)+Dy\}^2}$$
$$=\int_0^1 dx\int_0^1 dy\int_0^1 dz\frac{6z(1-z)}{[\{A(1-x)+Bx\}z+\{C(1-y)+Dy\}(1-z)]^4},$$

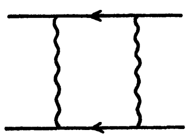
where

$$A\equiv M_1^2-(p_1-k)^2,$$
$$B\equiv M_2^2-(p_2-k)^2,$$
$$C\equiv m_1^2-k^2,$$
$$D\equiv m_2^2-(p_2+p_4-k)^2.$$

And we use the following function:

$$\begin{aligned}&D_4(x,y,z)\\&=\{(1-x)M_1^2+xM_2^2\}z+\{ym_2^2+(1-y)m_1^2\}(1-z)\\&\quad-(1-x)z(1-(1-x)z)p_1^2-\{xz+y(1-z)\}\{1-xz-y(1-z)\}p_2^2\\&\quad-y(1-z)\{1-y(1-z)\}p_4^2-2y(1-z)\{1-xz-y(1-z)\}p_2p_4\\&\quad+2z(1-x)\{y(1-z)+zx\}p_1p_2+2(1-x)yz(1-z)p_1p_4.\end{aligned}\tag{B·37}$$

In Eqs. (B·38) $\sim$ (B·40), spinors $\bar{u}(p)$, $u(p)$ are included in order to avoid confusion. However, on-shell relations, $\not{p}u(p)=mu(p)$, etc., have not been used in the derivation of these formulas.



$A(p_4, p_2; p_3, p_1)$

$$\sim\int\frac{d^4k}{(2\pi)^4 i}\bar{u}(p_4)\gamma_\rho(1+D\gamma_5)\frac{m_2-\not{k}+\not{p}_2+\not{p}_4}{m_2^2-(k-p_2-p_4)^2}\gamma_\sigma(1+C\gamma_5)u(p_3)$$

$$\times \frac{g^{\rho\rho'}}{(p_2-k)^2-M_2^2} \frac{g^{\sigma\sigma'}}{(p_1-k)^2-M_1^2}$$

$$\times \bar{u}(p_2)\gamma_{\rho'}(1+B\gamma_5)\frac{m_1+k\!\!\!/}{m_1^2-k^2}\gamma_{\sigma'}(1+A\gamma_5)u(p_1)$$

$$=\frac{1}{16\pi^2}\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{z(1-z)}{\{D_4(x,y,z)\}^2}$$

$$\times \bar{u}(p_4)\gamma^\rho[m_2\gamma(C,-D)-\gamma(-C,-D)\{z(1-x)p\!\!\!/_1$$

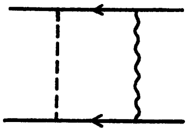
$$+(zx+y(1-z)-1)p\!\!\!/_2+(y(1-z)-1)p\!\!\!/_4\}]\gamma^\sigma u(p_3)$$

$$\times \bar{u}(p_2)\gamma_\rho[m_1\gamma(A,-B)+\gamma(-A,-B)\{z(1-x)p\!\!\!/_1$$

$$+(zx+y(1-z))p\!\!\!/_2+y(1-z)p\!\!\!/_4\}]\gamma_\sigma u(p_1)$$

$$+\frac{1}{32\pi^2}\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{z(1-z)}{D_4(x,y,z)}\bar{u}(p_4)\gamma(C,D)\gamma^\rho\gamma^\alpha\gamma^\sigma u(p_3)$$

$$\times \bar{u}(p_2)\gamma(A,B)\gamma_\rho\gamma_\alpha\gamma_\sigma u(p_1). \qquad (\text{B}\cdot 38)^{*)}$$



$A(p_4, p_2; p_3, p_1)$

$$\sim \int \frac{d^4k}{(2\pi)^4 i}\bar{u}(p_4)(1+D\gamma_5)\frac{m_2-k\!\!\!/+p\!\!\!/_2+p\!\!\!/_4}{m_2^2-(k-p_2-p_4)^2}\gamma_\rho(1+C\gamma_5)u(p_3)$$

$$\times \frac{1}{M_2^2-(k-p_2)^2}\frac{g^{\rho\sigma}}{(k-p_1)^2-M_1^2}$$

$$\times \bar{u}(p_2)(1+B\gamma_5)\frac{m_1+k\!\!\!/}{m_1^2-k^2}\gamma_\sigma(1+A\gamma_5)u(p_1)$$

$$=\frac{-1}{16\pi^2}\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{z(1-z)}{\{D_4(x,y,z)\}^2}$$

$$\times \bar{u}(p_4)[m_2\gamma(C,-D)-\gamma(-C,-D)\{z(1-x)p\!\!\!/_1$$

$$+(zx+y(1-z)-1)p\!\!\!/_2+(y(1-z)-1)p\!\!\!/_4\}]\gamma^\rho u(p_3)$$

$$\times \bar{u}(y_2)[m_1\gamma(A,-B)+\gamma(-A,-B)\{z(1-x)p\!\!\!/_1$$

$$+(zx+y(1-z))p\!\!\!/_2+y(1-z)p\!\!\!/_4\}]\gamma_\rho u(p_1)$$

$$-\frac{1}{32\pi^2}\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{z(1-z)}{D_4(x,y,z)}$$

$$\times \bar{u}(p_4)\gamma(-C,-D)\gamma^\rho\gamma^\sigma u(p_3)\cdot\bar{u}(p_2)\gamma(-A,-B)\gamma_\rho\gamma_\sigma u(p_1). \qquad (\text{B}\cdot 39)$$

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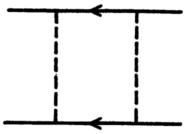
*) $\gamma(A,B)$ is defined at the beginning of B. 1, Eq. (B·3).



$A(p_4, p_2; p_3, p_1)$

$$\sim \int \frac{d^4k}{(2\pi)^4 i} \bar{u}(p_4)(1+D\gamma_5)\frac{m_2-\not{k}+\not{p}_2+\not{p}_4}{m_2^2-(k-p_2-p_4)^2}(1+C\gamma_5)u(p_3)$$

$$\times \frac{1}{M_2^2-(k-p_2)^2}\cdot\frac{1}{M_1^2-(k-p_1)^2}$$

$$\times \bar{u}(p_2)(1+B\gamma_5)\frac{m_1+\not{k}}{m_1^2-k^2}(1+A\gamma_5)u(p_1)$$

$$= \frac{1}{16\pi^2}\int_0^1 dx\int_0^1 dy\int_0^1 dz \frac{z(1-z)}{\{D_4(x,y,z)\}^2}$$

$$\times \bar{u}(p_4)[m_2\gamma(-C,-D)-\gamma(C,-D)\{z(1-x)\not{p}_1$$

$$+(zx+y(1-z)-1)\not{p}_2+(y(1-z)-1)\not{p}_4\}]u(p_3)$$

$$\times \bar{u}(p_2)[m_1\gamma(-A,-B)+\gamma(A,-B)\{z(1-x)\not{p}_1$$

$$+(zx+y(1-z))\not{p}_2+y(1-z)\not{p}_4\}]u(p_1)$$

$$+\frac{1}{32\pi^2}\int_0^1 dx\int_0^1 dy\int_0^1 dz \frac{z(1-z)}{D_4(x,y,z)}$$

$$\times \bar{u}(p_4)\gamma(-C,D)\gamma^\rho u(p_3)\cdot\bar{u}(p_2)\gamma(A,-B)\gamma_\rho u(p_1). \tag{B·40}$$

Appendix C

1/*M* Expansion of Some Feynman Amplitudes

One-loop Feynman amplitudes relevant to our calculation consist of the two-point (self-energy part), three-point (vertex part) and four-point (box diagram) functions. These functions, denoted by A_2, A_3 and A_4 respectively, are expressed by the following integral forms,

$$A_2=\int_0^1 dx\, I_2(x, M, m, q), \tag{C·1}$$

$$A_3=\int_0^1 dxdy\, I_3(x, y, M, m, p_1, p_2), \tag{C·2}$$

$$A_4=\int_0^1 dxdydz\, I_4(x, y, z, M, m, p_1, p_2, p_3), \tag{C·3}$$

where M and m generically represent large masses (like M_W and M_Z) and small masses respectively, and q, p_1, p_2 and p_3 are momenta of external lines.

It is well-known that in one-loop order the Feynman integrals can be performed analytically except for an unintegrable special function, Spence function (or dilogarithm) ['t Hooft and Veltman 79, Lewin 58]. Actually the integral (C·1) may be performed explicitly in terms of elementary functions while the integrals (C·2) and (C·3) are expressed in terms of elementary functions and the Spence function. It is, however, important to note that $M \gg m$, $|q|$, $|p_i|$ $(i=1,2,3)$ as far as low energy phenomena are concerned and hence useful approximate formulas may be derived for Eqs. (C·1), (C·2) and (C·3).

We consider the amplitudes (C·1), (C·2) and (C·3) for Feynman diagrams including at least one large-mass particle with mass M and derive the formulas in which these amplitudes are expanded in powers of $1/M$. The leading term in this expansion will be found to give a good approximation to the full amplitude and to have a simple form.

C. 1 Two-point functions (self-energy part)

The integrals for which we wish to derive the $1/M$ expansion formulas have the following forms,

$$\int_0^1 dx\, x^n \ln D_2\,, \tag{C·4}$$

$$\int_0^1 dx\, x^n \ln D_2'\,, \tag{C·5}$$

where $n=0$, 1, 2 and

$$D_2 = M^2(1-x) + m^2 x - q^2 x(1-x)\,, \tag{C·6}$$

$$D_2' = M_1^2(1-x) + M_2^2 x - q^2 x(1-x) \tag{C·7}$$

with two different large masses M_1 and M_2.

If one expands the integral (C·4) in powers of $1/M$ by separating

$$\ln D_2 = \ln M^2 + \ln(1-x) + \ln\left[1 + \frac{m^2 x - q^2 x(1-x)}{M^2(1-x)}\right],$$

one immediately see that the coefficients of the expansion in $1/M$ are divergent integrals (divergent at $x=1$). Hence we use the following form:

$$D_2 = [M^2(1-x) + m^2 x](1+f), \tag{C·8}$$

where

$$f = \frac{-q^2 x(1-x)}{M^2(1-x) + m^2 x}\,, \tag{C·9}$$

and expand $\ln D_2$ in powers of f. This expansion in f is uniformly convergent and so the term-by-term integration is justified,

$$\begin{aligned}&\int_0^1 dx\, x^n \ln D_2\\ &= \int_0^1 dx\, x^n \ln[M^2(1-x)+m^2x] - \int_0^1 dx\, x^n \frac{x(1-x)q^2}{M^2(1-x)+m^2x}\\ &\quad + \int_0^1 dx\, x^n \frac{x^2(1-x)^2q^4}{2[M^2(1-x)+m^2x]^2} + \cdots . \end{aligned} \tag{C·10}$$

We easily see that the l-th order term in Eq. (C·10) is of order $(1/M^2)^{l-1}$ up to $\ln(m^2/M^2)$. Hence Eq. (C·10) is the desired expansion. We list here the explicit expansion formulas for $n=0$, 1, 2 to order $1/M^2$,

$$\int_0^1 dx \ln D_2 = \ln M^2 - 1 - \frac{1}{M^2}\left(\frac{q^2}{2} + m^2 \ln\frac{m^2}{M^2}\right), \tag{C·11}$$

$$\int_0^1 dx\, x \ln D_2 = \frac{1}{2}\ln M^2 - \frac{3}{4} - \frac{1}{M^2}\left(\frac{q^2}{3} + \frac{m^2}{2} + m^2 \ln\frac{m^2}{M^2}\right), \tag{C·12}$$

$$\int_0^1 dx\, x^2 \ln D_2 = \frac{1}{3}\ln M^2 - \frac{11}{18} - \frac{1}{M^2}\left(\frac{q^2}{4} + \frac{5}{6}m^2 + m^2 \ln\frac{m^2}{M^2}\right). \tag{C·13}$$

For the integrals of type (C·5), we use the device

$$\begin{aligned} D_2' &= [M_1^2(1-x)+M_2^2x](1+f'),\\ f' &= \frac{-q^2x(1-x)}{M_1^2(1-x)+M_2^2x}, \end{aligned} \tag{C·14}$$

and expand the integrals in powers of f'. We find the following formulas,

$$\begin{aligned}\int_0^1 dx \ln D_2' &= \ln M_2^2 - \frac{M_1^2}{M_1^2-M_2^2}\ln\frac{M_2^2}{M_1^2} - 1\\ &\quad + \frac{q^2}{M_1^2-M_2^2}\left[\frac{1}{2} - \frac{M_1^2}{M_1^2-M_2^2} - \frac{M_1^2M_2^2}{(M_1^2-M_2^2)^2}\ln\frac{M_2^2}{M_1^2}\right], \end{aligned} \tag{C·15}$$

$$\begin{aligned}\int_0^1 dx\, x \ln D_2' &= \frac{1}{2}\ln M_2^2 - \frac{1}{4} - \frac{1}{2}\left(\frac{M_1^2}{M_1^2-M_2^2}\right)^2 \ln\frac{M_2^2}{M_1^2} - \frac{M_1^2}{2(M_1^2-M_2^2)}\\ &\quad + \frac{q^2}{M_1^2-M_2^2}\left[\frac{1}{6} + \frac{M_1^2}{2(M_1^2-M_2^2)} - \left(\frac{M_1^2}{M_1^2-M_2^2}\right)^2\right.\\ &\quad \left. - \frac{M_1^4M_2^2}{(M_1^2-M_2^2)^3}\ln\frac{M_2^2}{M_1^2}\right], \end{aligned} \tag{C·16}$$

$$\int_0^1 dx\, x^2 \ln D_2' = \frac{1}{3}\ln M_2^2 - \frac{1}{9} - \frac{1}{3}\left(\frac{M_1^2}{M_1^2-M_2^2}\right)^3 \ln\frac{M_2^2}{M_1^2}$$

$$-\frac{M_1^2}{6(M_1^2-M_2^2)}-\frac{1}{3}\left(\frac{M_1^2}{M_1^2-M_2^2}\right)^2$$

$$+\frac{q^2}{M_1^2-M_2^2}\left[\frac{1}{12}+\frac{M_1^2}{6(M_1^2-M_2^2)}+\frac{1}{2}\left(\frac{M_1^2}{M_1^2-M_2^2}\right)^2\right.$$

$$\left.-\left(\frac{M_1^2}{M_1^2-M_2^2}\right)^3-\frac{M_1^6M_2^2}{(M_1^2-M_2^2)^4}\ln\frac{M_2^2}{M_1^2}\right]. \qquad (C\cdot 17)$$

C. 2 Three-point functions (vertex part)

We wish to derive the $1/M$ expansion formulas for the integrals of the form,

$$\int_0^1 dx\int_0^1 dy\ y^n \ln D_3^{(i)}, \quad (n=0,1) \qquad (C\cdot 18)$$

$$\int_0^1 dx\int_0^1 dy\ y^n/D_3^{(i)}, \quad (n=0,1,2,3) \qquad (C\cdot 19)$$

where $i=1$ or 2 and

$$D_3^{(1)}=(1-y)M^2+y[\overline{m^2}-q^2x(1-x)]-\bar{p}^2y(1-y), \qquad (C\cdot 20)$$

$$D_3^{(2)}=(1-y)m^2+y[\overline{M^2}-q^2x(1-x)]-\bar{p}^2y(1-y) \qquad (C\cdot 21)$$

with

$$\overline{m^2}\equiv(1-x)m_1^2+xm_2^2,$$

$$\overline{M^2}\equiv(1-x)M_1^2+xM_2^2,$$

$$\bar{p}\equiv(1-x)p_1+xp_2. \qquad (C\cdot 22)$$

For this purpose it is sufficient to consider the case where the $D_3^{(i)}$'s are replaced by

$$D=(1-y)C+yB-y(1-y)A \qquad (C\cdot 23)$$

with $C\gg B$, A. The formulas for the case $B\gg C$, A may be obtained from the above case by changing the integration variable y to $1-y$.

The formulas for the integral (C·18) are essentially the same as Eqs. (C·11) and (C·12). The $1/M$ expansion formulas for the first integration of the integral of type (C·19) are given by

$$\int_0^1 dy\frac{1}{D}=-\frac{1}{C}\ln\frac{B}{C}-\frac{1}{C^2}\left[B\ln\frac{B}{C}+A\left(2+\ln\frac{B}{C}\right)\right], \qquad (C\cdot 24)$$

$$\int_0^1 dy\frac{y}{D}=-\frac{1}{C}\left[1+\ln\frac{B}{C}\right]-\frac{1}{C^2}\left[B\left(1+2\ln\frac{B}{C}\right)+A\left(\frac{5}{2}+\ln\frac{B}{C}\right)\right], \qquad (C\cdot 25)$$

$$\int_0^1 dy \frac{y^2}{D} = -\frac{1}{C}\left[\frac{3}{2}+\ln\frac{B}{C}\right] - \frac{1}{C^2}\left[B\left(\frac{5}{2}+3\ln\frac{B}{C}\right)+A\left(\frac{17}{6}+\ln\frac{B}{C}\right)\right], \tag{C·26}$$

$$\int_0^1 dy \frac{y^3}{D} = -\frac{1}{C}\left[\frac{11}{6}+\ln\frac{B}{C}\right] - \frac{1}{C^2}\left[B\left(\frac{13}{3}+4\ln\frac{B}{C}\right)+A\left(\frac{37}{12}+\ln\frac{B}{C}\right)\right]. \tag{C·27}$$

The remaining x-integration can be performed by applying the formulas already obtained.

C. 3 Four-point functions (box diagram)

We consider the function A_4 in Eq. (C·3) and pay attention to the z-integration. The relevant integrals are of the form,

$$\int_0^1 dz \frac{z^n}{D_4}, \quad \int_0^1 dz \frac{z^n}{D_4^2}, \tag{C·28}$$

where

$$D_4 = Cz + B + Az \tag{C·29}$$

with $C \gg B$, A and

$$C = (1-x)M_1^2 + xM_2^2. \tag{C·30}$$

We expand Eq. (C·28) in powers of $1/C$. For this purpose we use the following form of D_4,

$$D_4 = (Cz+B)(1+f),$$
$$f = \frac{Az}{Cz+B}. \tag{C·31}$$

Expanding the integrands in Eq. (C·28) in powers of f, we obtain for $n \geq 1$

$$\int_0^1 dz \frac{z^n}{D_4} = \begin{cases} \dfrac{1}{nC} - \dfrac{A}{nC^2} - \dfrac{1}{n-1}\dfrac{B}{C^2}, & (n \geq 2) \\ \dfrac{1}{C} - \dfrac{A}{C^2} + \dfrac{B}{C^2}\ln\dfrac{B}{C}, & (n=1) \end{cases} \tag{C·32}$$

$$\int_0^1 dz \frac{z^n}{D_4^2} = \begin{cases} \dfrac{1}{n-1}\dfrac{1}{C^2}, & (n \geq 2) \\ -\dfrac{1}{C^2} - \dfrac{1}{C^2}\ln\dfrac{B}{C}. & (n=1) \end{cases} \tag{C·33}$$

The remaining x- and y-integration may be performed in the same way as before.

For the box diagram with the photon and W boson as internal lines (see Fig. 5.2), the function D_4 takes the form,

$$D_4 = Czx + B + Azx\,, \tag{C·34}$$

where $C = M^2 \gg B,\ A$. As before we rewrite D_4 such that

$$D_4 = (Czx + B)(1 + f) \tag{C·35}$$

with

$$f = \frac{Azx}{Czx + B}\,. \tag{C·36}$$

We expand the necessary integrals in powers of f and perform the z-integrations term by term. The remaining x- and y-integration, however, are not so simple. The full expression is given in Eq. (5·64).

Appendix D

F_n and G_n Functions

As has already been mentioned in Appendix C, the integration appearing in the calculation of the two-point functions may be performed explicitly. It is convenient to define the following functions F_n and F:

$$F_n(M_1, M_2, q^2) \equiv \int_0^1 dx\, x^n \ln[M_1^2(1-x) + M_2^2 x - q^2 x(1-x)], \tag{D·1}$$

$$F(M_1, M_2, q^2) \equiv F_1(M_1, M_2, q^2) - F_2(M_1, M_2, q^2)\,, \tag{D·2}$$

where $n = 0,\ 1,\ 2$. The analytic expressions for F_0, F_1 and F_2 are given by

$$F_0(M_1, M_2, q^2) = \ln M_2^2 - 2 - \frac{1}{2}(1+\delta)\ln\left(\frac{M_2^2}{M_1^2}\right) + \frac{1}{2} r \ln \rho\,, \tag{D·3}$$

$$\begin{aligned} F_1(M_1, M_2, q^2) = {} & \frac{1}{2}\ln M_2^2 - 1 - \frac{1}{2}\left\{\frac{1}{2}(1+\delta)^2 - \frac{M_1^2}{q^2}\right\}\ln\left(\frac{M_2^2}{M_1^2}\right) \\ & - \frac{1}{2}\delta + \frac{1}{4}(1+\delta)\, r \ln \rho, \end{aligned} \tag{D·4}$$

$$\begin{aligned} F_2(M_1, M_2, q^2) = {} & \frac{1}{3}\ln M_2^2 - \frac{2}{9} - \frac{1}{6}(1+\delta)\left\{(1+\delta)^2 - \frac{3M_1^2}{q^2}\right\}\ln\left(\frac{M_2^2}{M_1^2}\right) \\ & - \frac{1}{6}(1+\delta)(3+2\delta) + \frac{2M_1^2}{3q^2} \end{aligned}$$

$$+\frac{1}{6}\left\{(1+\delta)^2-\frac{M_1^2}{q^2}\right\}r\ln\rho\,, \tag{D·5}$$

where

$$\delta\equiv(M_1^2-M_2^2)/q^2\,, \tag{D·6}$$

$$r\equiv\sqrt{(1+\delta)^2-4M_1^2/q^2}\,, \tag{D·7}$$

$$\rho\equiv\frac{M_1^2+M_2^2-(1+r)q^2}{M_1^2+M_2^2-(1-r)q^2}\,. \tag{D·8}$$

The function G_n is defined by

$$\begin{aligned}G_n(M_1,M_2,q^2)&\equiv\frac{\partial}{\partial q^2}F_n(M_1,M_2,q^2)\,,\\&=-\int_0^1 dx\,x^{n+1}(1-x)/[M_1^2(1-x)+M_2^2x-q^2x(1-x)]\end{aligned} \tag{D·9}$$

Performing the integration (D·9), we find for G_0, G_1 and G_2,

$$G_0(M_1,M_2,q^2)=\frac{1}{q^2}\left[1+\frac{1}{2}\delta\ln\frac{M_2^2}{M_1^2}-\frac{1}{2}\left(\delta^2-\frac{M_1^2+M_2^2}{q^2}\right)\frac{\ln\rho}{r}\right], \tag{D·10}$$

$$\begin{aligned}G_1(M_1,M_2,q^2)=\frac{1}{q^2}\Bigg[&\frac{1}{2}+\delta+\frac{1}{2}\left(\delta^2-\frac{M_2^2}{q^2}\right)\ln\frac{M_2^2}{M_1^2}\\&+\frac{1}{2}\left(\frac{M_2^2}{q^2}+\frac{M_1^2+2M_2^2}{q^2}\delta-\delta^3\right)\frac{\ln\rho}{r}\Bigg],\end{aligned} \tag{D·11}$$

$$\begin{aligned}G_2(M_1,M_2,q^2)=\frac{1}{q^2}\Bigg[&\frac{1}{3}+\delta(1+\delta)-\frac{M_1^2+M_2^2}{2q^2}\\&+\frac{1}{2}\left\{\delta(1+\delta)^2-\frac{M_1^2}{q^2}(1+2\delta)\right\}\ln\frac{M_2^2}{M_1^2}\\&+\frac{1}{2}\left\{\frac{M_1^2}{q^2}(1+\delta)(1+4\delta)-\delta(1+\delta)^3-\frac{2M_1^4}{q^4}\right\}\frac{\ln q}{r}\Bigg].\end{aligned} \tag{D·12}$$

Appendix E

Renormalization Constants

We first summarize all the relevant renormalization constants defined in § 4. 3:

$$\begin{aligned}
&Z_{ij}^{1/2}\ (i,j=A,Z)\ ,\quad \delta M_Z^2\ ,\\
&Z_W\ ,\quad \delta M_W^2\ ,\quad T\ ,\\
&(Z_{\mathrm{L}}{}^{1/2})_{nm},\ (Z_{\mathrm{R}}{}^{1/2})_{nm},\ \delta m_{i,I},\\
&Z_\chi\ ,\quad Z_{\chi_3}\ ,\quad Z_\phi\ ,\quad \delta m_\phi{}^2\ ,\\
&\tilde{Z}_{ij}\ (i,j=A,Z)\ ,\quad \tilde{Z}_3\ ,\quad Y\ .
\end{aligned} \tag{E·1}$$

These constants are determined on the mass shell by the use of the renormalization conditions described in § 4. 3. In actual calculations in Chapter 5, we need only the following subset of the renormalization constants (E·1) according to our approximations and assumptions:

$$\begin{aligned}
&Z_{ij}^{1/2}\ ,\quad \delta M_Z^2\ ,\quad Z_W\ ,\quad \delta M_W^2\ ,\quad (Z_{\mathrm{L,R}}^l)^{1/2}\ \ (\equiv (Z_{\mathrm{L,R}}^{1/2})_{ll})\ ,\\
&\delta m_l\ \ (l=e\ \text{or}\ \mu),\quad Y\ .
\end{aligned} \tag{E·2}$$

We here present the explicit expressions for the above constants (E·2) in one-loop order in the on-shell renormalization scheme.

E. 1 Gauge-boson field renormalization constants

The constants $Z_{ZZ}^{1/2}$, $Z_{AZ}^{1/2}$, $Z_{ZA}^{1/2}$, $Z_{AA}^{1/2}$ and Z_W are calculated using the Feynman diagrams Figs. 5. 3, 5. 4, 5. 5 in § 5. 3 and Fig. 5. 6 in § 5. 4 respectively. We list the results in the following:

$$\begin{aligned}
Z_{ZZ}^{1/2}=1-\frac{e^2}{24\pi^2}\frac{C_{\mathrm{UV}}}{M_W^2(M_Z^2-M_W^2)}\Big[&\frac{N}{3}(8M_W^4-10M_W^2M_Z^2+5M_Z^4)\\
&-\frac{1}{8}(18M_W^4+2M_W^2M_Z^2-M_Z^4)\Big]\ ,
\end{aligned} \tag{E·3}$$

$$Z_{AZ}^{1/2}=\frac{e^2}{12\pi^2}\frac{C_{\mathrm{UV}}}{M_W\sqrt{M_Z^2-M_W^2}}\Big[-\frac{N}{3}(8M_W^2-5M_Z^2)+\frac{1}{8}(30M_W^2+M_Z^2)\Big]\ , \tag{E·4}$$

$$Z_{ZA}^{1/2}=-\frac{e^2}{8\pi^2}\frac{M_W}{\sqrt{M_Z^2-M_W^2}}(C_{\mathrm{UV}}-\ln M_W^2)\ , \tag{E·5}$$

$$Z_{AA}^{1/2}=1-\frac{e^2}{8\pi^2}\Big[\frac{1}{3}\sum_i Q_i^2(C_{\mathrm{UV}}-\ln m_i^2)-\frac{3}{4}(C_{\mathrm{UV}}-\ln M_W^2)-\frac{1}{6}\Big]\ , \tag{E·6}$$

$$Z_W=1+\frac{e^2}{12\pi^2}\frac{M_Z^2}{M_Z^2-M_W^2}C_{\mathrm{UV}}\Big(-N+\frac{19}{8}\Big)\ , \tag{E·7}$$

where the summation on i runs over all fermions and color degrees of freedom, N is the number of generations and $C_{\rm UV}$ is given in Eq. (5·1) with $\mu=m$: $C_{\rm UV}=1/\varepsilon-\gamma+\ln 4\pi$. It should be noted here that we have discarded all finite pieces in Eqs. (E·3), (E·4) and (E·7). The reason for this is that $Z_{ZZ}^{1/2}$, $Z_{AZ}^{1/2}$ and Z_W are cancelled out when all the Feynman amplitudes are summed up and only the divergent pieces are necessary to get finite amplitudes in the intermediate stage. The renormalization constant $Z_{AA}^{1/2}$ is necessary only in determining the constant Y.

E. 2 Gauge-boson mass shifts

The mass shifts δM_Z^2 and δM_W^2 are obtained by considering Fig. 5. 3 in § 5 3 and Fig. 5. 6 in § 5. 4 respectively. Utilizing the F_n and F functions defined in Appendix D and η_i defined in Eq. (5·10), we present the results for δM_Z^2 and δM_W^2 in the following:

$$\delta M_Z^2=\frac{e^2M_Z^2}{16\pi^2M_W^2(M_Z^2-M_W^2)}\Big[\frac{4}{9}N(5M_Z^4-10M_Z^2M_W^2+8M_W^4)$$
$$-\frac{1}{2}M_Z^2\sum_i m_i^2+\frac{1}{6}(7M_Z^4+10M_Z^2M_W^2-42M_W^4)\Big]C_{\rm UV}+\delta M_{Zf}^2, \tag{E·8}$$

$$\delta M_{Zf}^2=\frac{-e^2}{32\pi^2M_W^2(M_Z^2-M_W^2)}\Big[\sum_i M_Z^4\{M_Z^2(\eta_i^2+1)F(m_i,m_i,M_Z^2)$$
$$-m_i^2F_0(m_i,m_i,M_Z^2)\}-\frac{M_Z^2}{3}(M_Z^4-2M_Z^2M_W^2+4M_W^4)$$
$$+\frac{1}{2}M_Z^6\ln M_Z^2+M_W^2(M_Z^4-4M_Z^2M_W^2+16M_W^4)\ln M_W^2$$
$$+M_Z^2(M_Z^4-4M_Z^2M_W^2+24M_W^4)F(M_W,M_W,M_Z^2)$$
$$+M_W^2(3M_Z^4-14M_Z^2M_W^2-16M_W^4)F_0(M_W,M_W,M_Z^2)$$
$$+M_Z^6\{2F_0(m_\phi,M_Z,M_Z^2)-F_2(m_\phi,M_Z,M_Z^2)\}$$
$$-m_\phi^2M_Z^4\Big\{F_1(M_Z,m_\phi,M_Z^2)-\frac{1}{2}\ln m_\phi^2\Big\}\Big], \tag{E·9}$$

$$\delta M_W^2=-\frac{e^2M_Z^2}{96\pi^2(M_Z^2-M_W^2)}\Big[\sum_{(I,i)}|U_{Ii}|^2(3m_I^2+3m_i^2-2M_W^2)$$
$$+31M_W^2-6M_Z^2\Big]C_{\rm UV}+\delta M_{Wf}^2, \tag{E·10}$$

$$\delta M_{Wf}^2=\frac{-e^2}{32\pi^2(M_Z^2-M_W^2)}\Big[2M_Z^2\sum_{(I,i)}|U_{Ii}|^2\{2M_W^2F(m_I,m_i,M_W^2)$$
$$-m_i^2F_1(m_I,m_i,M_W^2)-m_I^2F_1(m_i,m_I,M_W^2)\}$$

$$-M_Z^2M_W^2+7M_Z^2M_W^2\ln M_W^2+M_Z^2\left(6M_W^2+\frac{M_Z^2}{2}\right)\ln M_Z^2$$

$$+\{(M_Z^4-20M_Z^2M_W^2-8M_W^4)F_0(M_Z, M_W, M_W^2)$$

$$+(M_Z^4+16M_Z^2M_W^2+4M_W^4)F_1(M_Z, M_W, M_W^2)$$

$$-M_W^2(M_Z^2+20M_W^2)F_2(M_Z, M_W, M_W^2)\}$$

$$+4M_W^2(M_Z^2-M_W^2)\{-2F_0(0, M_W, M_W^2)$$

$$+F_1(0, M_W, M_W^2)-5F_2(0, M_W, M_W^2)\}$$

$$+m_\phi^2M_Z^2\left\{\frac{1}{2}\ln m_\phi^2-F_1(M_W, m_\phi, M_W^2)\right\}$$

$$+M_Z^2M_W^2\{2F_0(m_\phi, M_W, M_W^2)-F_2(m_\phi, M_W, M_W^2)\Big\}\Big]. \tag{E·11}$$

E. 3 Lepton field renormalization constants

The constants Z_L^ν, Z_L^l and Z_R^l are calculated through the Feynman diagrams Fig. 5. 7 in § 5. 3. In the calculation we have neglected the Higgs contribution since its effect is of order m_l/M_W. The result is as follows:

$$Z_L^\nu=1-\frac{e^2}{16\pi^2}\frac{M_Z^2}{4M_W^2(M_Z^2-M_W^2)}[(2M_W^2+M_Z^2)(C_{UV}-1)$$

$$-4M_W^2F_1(m_l, M_W, 0)-2M_Z^2F_1(0, M_Z, 0)], \tag{E·12}$$

$$\simeq1-\frac{e^2}{16\pi^2}\frac{M_Z^2}{4M_W^2(M_Z^2-M_W^2)}\left[(2M_W^2+M_Z^2)\left(C_{UV}-\frac{1}{2}\right)\right.$$

$$\left.-2M_W^2\ln M_W^2-M_Z^2\ln M_Z^2\right], \tag{E·13}$$

$$Z_L^l=1-\frac{e^2}{16\pi^2}[C_{UV}-1-2F_1(m_l, 0, m_l^2)$$

$$+4m_l^2\{2G_0(m_l, \lambda, m_l^2)-G_1(m_l, \lambda, m_l^2)\}]$$

$$-\frac{e^2}{16\pi^2}\frac{M_Z^2}{2(M_Z^2-M_W^2)}[C_{UV}-1-2F_1(0, M_W, m_l^2)$$

$$-2m_l^2\cdot G_1(0, M_W, m_l^2)]$$

$$-\frac{e^2}{16\pi^2}\left[\frac{(M_Z^2-2M_W^2)^2}{4M_W^2(M_Z^2-M_W^2)}\{C_{UV}-1-2F_1(m_l, M_Z, m_l^2)\}\right.$$

$$+4m_l^2\left\{\frac{M_Z^2-2M_W^2}{M_W^2}G_0(m_l, M_Z, m_l^2)\right.$$

$$\left.\left.-\frac{8(M_Z^2-M_W^2)^2+M_Z^2(4M_W^2-3M_Z^2)}{8M_W^2(M_Z^2-M_W^2)}G_1(m_l, M_Z, m_l^2)\right\}\right], \tag{E·14}$$

$$\simeq 1-\frac{e^2}{16\pi^2}\Big[C_{\rm UV}+2C_{\rm IR}+4-3\ln m_l^2+\frac{1}{2}\frac{M_Z^2}{M_Z^2-M_W^2}\Big(C_{\rm UV}-\frac{1}{2}-\ln M_W^2\Big)$$

$$+\frac{1}{4}\frac{(M_Z^2-2M_W^2)^2}{M_W^2(M_Z^2-M_W^2)}\Big(C_{\rm UV}-\frac{1}{2}-\ln M_Z^2\Big)\Big], \qquad (\text{E}\cdot 15)$$

$$Z_{\rm R}^l=1-\frac{e^2}{16\pi^2}[C_{\rm UV}-1-2F_1(m_l,0,m_l^2)$$

$$+4m_l^2\{2G_0(m_l,\lambda,m_l^2)-G_1(m_l,\lambda,m_l^2)\}]$$

$$+\frac{e^2}{16\pi^2}\frac{M_Z^2}{M_Z^2-M_W^2}m_l^2G_1(0,M_W,m_l^2)$$

$$-\frac{e^2}{16\pi^2}\Big[\frac{M_Z^2-M_W^2}{M_W^2}\{C_{\rm UV}-1-2F_1(m_l,M_Z,m_l^2)\}$$

$$+4m_l^2\Big\{\frac{M_Z^2-2M_W^2}{M_W^2}G_0(m_l,M_Z,m_l^2)$$

$$-\frac{8(M_Z^2-M_W^2)^2+M_Z^2(4M_W^2-3M_Z^2)}{8M_W^2(M_Z^2-M_W^2)}G_1(m_l,M_Z,m_l^2)\Big\}\Big], \qquad (\text{E}\cdot 16)$$

$$\simeq 1-\frac{e^2}{16\pi^2}\Big[C_{\rm UV}+2C_{\rm IR}+4-3\ln m_l^2+\frac{M_Z^2-M_W^2}{M_W^2}\Big(C_{\rm UV}-\frac{1}{2}-\ln M_Z^2\Big)\Big]. \qquad (\text{E}\cdot 17)$$

E. 4 Lepton mass shifts

The lepton mass-renormalization constant δm_l is given by

$$\delta m_l=\frac{e^2}{16\pi^2}\frac{m_l}{8M_W^2(M_Z^2-M_W^2)}[-(11M_Z^4-14M_Z^2M_W^2)C_{\rm UV}$$

$$+3M_Z^4-6M_Z^2M_W^2+16M_W^2(M_Z^2-M_W^2)\{2F_0(m_l,0,m_l^2)$$

$$-F_1(m_l,0,m_l^2)\}-4M_Z^2M_W^2F_1(0,M_W,m_l^2)$$

$$+16(M_Z^2-M_W^2)(M_Z^2-2M_W^2)F_0(m_l,M_Z,m_l^2)$$

$$-2(5M_Z^4-12M_Z^2M_W^2+8M_W^4)F_1(m_l,M_Z,m_l^2)]. \qquad (\text{E}\cdot 18)$$

E. 5 Vertex renormalization constant Y

To calculate the constant Y, we first evaluate the eeA vertex function at $q^2=0$ (see Fig. 5. 8 in § 5. 3) and then add the counter term found in § 4. 3 with $Z_{AA}^{1/2}$, $Z_{ZA}^{1/2}$, $Z_{\rm L}^e$ and $Z_{\rm R}^e$ as given in E. 1 and E. 3. We find

$$Y=1-\frac{e^2}{16\pi^2}\Big[\frac{7}{2}(C_{\rm UV}-\ln M_W^2)+\frac{1}{3}-\frac{2}{3}\sum_i Q_i^2(C_{\rm UV}-\ln m_i^2)\Big]. \qquad (\text{E}\cdot 19)$$

Appendix F

Spence Function

Feynman parameter integrals appearing in one-loop amplitudes can be performed analytically leaving one special function unintegrated ['t Hooft and Veltman 79]. The special function is called the dilogarithm or the Spence function [Lewin 58]. In this appendix we list some useful properties of the Spence function. The Spence function is defined by Eq. (5·40), i.e.,

$$\mathrm{Sp}(z) = -\int_0^z dt \frac{\ln(1-t)}{t}. \qquad (\mathrm{F}\cdot 1)$$

It is an analytic function of complex z with a branch cut from $z=1$ to ∞. To see this property more clearly, we reexpress Eq. (F·1) by making integration by parts with a suitable change of the integration variable:

$$\mathrm{Sp}(z) = \int_0^1 dt \frac{\ln t}{t-1/z}. \qquad (\mathrm{F}\cdot 2)$$

Obviously in Eq. (F·2) we observe that the branch cut lies in the interval $0 \leq 1/z \leq 1$.

There are some useful relations among the Spence functions with different arguments:

$$\mathrm{Sp}(z) + \mathrm{Sp}(1-z) = \frac{\pi^2}{6} - \ln z \ln(1-z), \qquad (\mathrm{F}\cdot 3)$$

$$\mathrm{Sp}\left(-\frac{1}{z}\right) + \mathrm{Sp}(-z) = -\frac{\pi^2}{6} - \frac{1}{2}(\ln z)^2, \qquad (\mathrm{F}\cdot 4)$$

$$\mathrm{Sp}(x) + \mathrm{Sp}\left(-\frac{x}{1-x}\right) = -\frac{1}{2}(\ln(1-x))^2, \quad x<1. \qquad (\mathrm{F}\cdot 5)$$

The series expansion of $\mathrm{Sp}(z)$ in powers of z reads

$$\mathrm{Sp}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2}, \quad |z| \leq 1. \qquad (\mathrm{F}\cdot 6)$$

At some special values of z, $\mathrm{Sp}(z)$ can be expressed in terms of the known constants:

$$\mathrm{Sp}(1) = \pi^2/6, \qquad (\mathrm{F}\cdot 7)$$

$$\mathrm{Sp}(-1) = -\pi^2/12, \qquad (\mathrm{F}\cdot 8)$$

$$\mathrm{Sp}(1/2) = \pi^2/12 - (1/2)(\ln 2)^2. \qquad (\mathrm{F}\cdot 9)$$

Appendix G

Formulas for the Generating Functional and the Effective Action

We derive formulas for the generating functional $W[J, K]$ defined in Eq. (2·95) and the effective action $\Gamma[\phi, K]$ defined in Eq. (2·99). We deal with both commuting and anti-commuting field variables simultaneously and give a compact proof of formulas [Aoki 79].

We start by recalling the definition of the generating functional $W[J, K]$,

$$\exp iW[J, K] \equiv \langle 0|\mathrm{T}\exp iS[J, K]|0\rangle\,, \tag{G·1}$$

$$S[J, K] \equiv \int d^4x[J_i(x)\hat{\phi}_i(x) + K_i(x)\delta^{\mathrm{BRS}}\hat{\phi}_i(x)], \tag{G·2}$$

where $\hat{\phi}_i(x)$ represent field operators and $J_i(x)$ are corresponding c-number sources with the same commutativity as that of $\hat{\phi}_i(x)$. The BRS sources K_i are also introduced and its commutativity is the same as that of corresponding $\delta^{\mathrm{BRS}}\hat{\phi}_i(x)$.

First of all we introduce the connected part of a Green function. The connected part is defined graphically with the aid of the corresponding Feynman diagram. The vacuum expectation values of operators should be subtracted in the connected part. We give some examples. It is convenient to introduce $\bar{O}$ defined by

$$\bar{O}_i \equiv O_i - \langle 0|O_i|0\rangle\,. \tag{G·3}$$

With the aid of these operators $\bar{O}_i$, we have

$$\begin{aligned}\langle 0|\mathrm{T}O_1O_2|0\rangle^c &= \langle 0|\mathrm{T}O_1O_2|0\rangle - \langle 0|O_1|0\rangle\langle 0|O_2|0\rangle \\ &= \langle 0|\mathrm{T}\bar{O}_1\bar{O}_2|0\rangle\,,\end{aligned} \tag{G·4a}$$

$$\langle 0|\mathrm{T}O_1O_2O_3|0\rangle^c = \langle 0|\mathrm{T}\bar{O}_1\bar{O}_2\bar{O}_3|0\rangle\,, \tag{G·4b}$$

$$\begin{aligned}\langle 0|\mathrm{T}O_1O_2O_3O_4|0\rangle^c = \langle 0|\mathrm{T}\bar{O}_1\bar{O}_2\bar{O}_3\bar{O}_4|0\rangle & \\ -\langle 0|\mathrm{T}\bar{O}_1\bar{O}_2|0\rangle\langle 0|\mathrm{T}\bar{O}_3\bar{O}_4|0\rangle & \\ -\langle 0|\mathrm{T}\bar{O}_1\bar{O}_3|0\rangle\langle 0|\mathrm{T}\bar{O}_2\bar{O}_4|0\rangle & \\ -\langle 0|\mathrm{T}\bar{O}_1\bar{O}_4|0\rangle\langle 0|\mathrm{T}\bar{O}_2\bar{O}_3|0\rangle\,, &\end{aligned} \tag{G·4c}$$

where we have assumed that O_i are commuting operators.

Consider the term with S^n in the expansion of the right-hand side of Eq. (G·1), $\langle 0|(1/n!)\mathrm{T}(iS)^n|0\rangle$. This n-point amplitude is a sum of various products of connected amplitudes:

$$\langle 0|\frac{1}{n!}\mathrm{T}(iS)^n|0\rangle = \sum_{\{\{n_k\}|\sum_{k=0}^{\infty}kn_k=n,\, n_k\geqq 0\}} \frac{1}{n!}\mathrm{P}(\{n_k\})\prod_{k=0}^{\infty}(\langle 0|\mathrm{T}(iS)^k|0\rangle^c)^{n_k}\,. \tag{G·5}$$

We characterize a product of connected amplitudes by a set $\{n_k\}$ where n_k is the number of k-point amplitudes in the product of connected amplitudes. The factor $\mathrm{P}(\{n_k\})$ in Eq. (G·5) is the number of combinations to separate n points into parts where the number of k points is n_k set. This factor is evaluated as

$$\mathrm{P}(\{n_k\}) = n!/\left(\prod_{k=0}^{\infty} (k!)^{n_k} n_k!\right). \tag{G·6}$$

Substituting Eqs. (G·6) and (G·5) into Eq. (G·1), we have

$$\begin{aligned}
\exp iW &= \sum_{n=0}^{\infty} \langle 0|\frac{1}{n!}\mathrm{T}\,(iS)^n|0\rangle \\
&= \sum_{n=0}^{\infty} \sum_{\{\{n_k\}|\sum_{k=0}^{\infty} kn_k=n,\, n_k\geq 0\}} \prod_{k=0}^{\infty} \frac{1}{n_k!}\left(\langle 0|\frac{1}{k!}\mathrm{T}\,(iS)^k|0\rangle^{\mathrm{c}}\right)^{n_k} \\
&= \prod_{k=0}^{\infty} \sum_{n_k=0}^{\infty} \frac{1}{n_k!}\left(\langle 0|\frac{1}{k!}\mathrm{T}\,(iS)^k|0\rangle^{\mathrm{c}}\right)^{n_k} \\
&= \prod_{k=0}^{\infty} \exp\langle 0|\frac{1}{k!}\mathrm{T}\,(iS)^k|0\rangle^{\mathrm{c}} \\
&= \exp\langle 0|\mathrm{T}\exp iS|0\rangle^{\mathrm{c}}.
\end{aligned} \tag{G·7}$$

Thus the functional iW is the connected part of the Green function,

$$iW = \langle 0|\mathrm{T}\exp iS|0\rangle^{\mathrm{c}}. \tag{G·8}$$

Before proceeding to the formulas for differentiation of W, it is necessary to fix a convention for the differentiation with respect to anticommuting variables. We adopt the so-called left-differentiation. In this convention the distribution formula is

$$\frac{\delta}{\delta\phi}(AB) = \left(\frac{\delta A}{\delta\phi}\right)B + (-1)^A_\phi A\left(\frac{\delta B}{\delta\phi}\right), \tag{G·9}$$

where $(-1)^A_\phi$ is $+1$ (-1) when A is commutative (anticommutative) with ϕ.

The formulas for differentiation of W with respect to sources are

$$\frac{1}{i^{n-1}}\frac{\delta^n W}{\delta S_1\delta S_2\cdots\delta S_n} = \langle 0|\mathrm{T}O_1O_2\cdots O_n \exp iS|0\rangle^{\mathrm{c}}, \tag{G·10}$$

where we have used the following simplified notations:

$$S[J,k] \equiv J_i\cdot\hat{\phi}_i + K_i\cdot\delta^{\mathrm{BRS}}\hat{\phi}_i \equiv S_i\cdot O_i, \tag{G·11a}$$

$$S_i \equiv \{J, K\}, \quad O_i \equiv \{\hat{\phi}, \delta^{\mathrm{BRS}}\hat{\phi}\}. \tag{G·11b}$$

It should be mentioned that the order of differentiation and that of operators are correlated with each other. Setting all sources to be zero, we have

$$\frac{1}{i^{n-1}}\left.\frac{\delta^n W}{\delta S_1 \delta S_2 \cdots \delta S_n}\right|_0 = \langle 0|\mathrm{T}O_1 O_2 \cdots O_n|0\rangle^c . \tag{G·12}$$

Now let us proceed to the effective action defined by

$$\Gamma[\phi, K] \equiv W[J, K] - J_i \cdot \phi_i , \tag{G·13}$$

$$\phi_i \equiv \frac{\delta}{\delta J_i} W[J, K] . \tag{G·14}$$

In Eq. (G·13) J_i is understood to be a functional of ϕ (and K). In fact J_i is expressed as

$$J_i = \varepsilon^i \frac{\delta \Gamma}{\delta \phi_i} , \tag{G·15}$$

where ε^i is a sign factor defined by

$$\varepsilon^i = -1(+1) \quad \text{for commuting } \phi_i \text{ (anticommuting } \phi_i\text{)}. \tag{G·16}$$

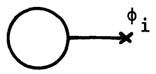
We start with the following identity:

$$\frac{\delta \phi_i}{\delta \phi_k} = \delta_{ik} . \tag{G·17}$$

By using the definition (G·14) and the formula (G·10), we have

$$\frac{\delta}{\delta \phi_k} \frac{\delta W}{\delta J_i} = \frac{\delta}{\delta \phi_k} \langle 0|\mathrm{T}\hat{\phi}_i \exp iS|0\rangle^c = \delta_{ik} . \tag{G·18}$$

This is expressed graphically as

$\dfrac{\delta}{\delta \phi_k} \cdot$ 

$= \delta_{ik}$. (G·19)

Thus the operator $\delta/\delta\phi_k$ amputates the "leg" $\langle 0|\mathrm{T}\hat{\phi}_i \exp iS|0\rangle^c$.

We rewrite Eq. (G·18) as

$$\delta_{ik} = \frac{\delta J_j}{\delta \phi_k} \frac{\delta^2 W}{\delta J_j \delta J_i} = \varepsilon^j \frac{\delta^2 \Gamma}{\delta \phi_k \delta \phi_j} \frac{\delta^2 W}{\delta J_j \delta J_i}$$

$$= \varepsilon^j \frac{i\delta^2 \Gamma}{\delta \phi_k \delta \phi_j} \langle 0|\mathrm{T}\hat{\phi}_j \hat{\phi}_i \exp iS|0\rangle^c . \tag{G·20}$$

By setting all field (source) variables to their vacuum expectation values (zero): $\phi_i = \langle 0|\hat{\phi}_i|0\rangle$, $K_i = J_i = 0$ (this setting is represented by the symbol $|_0$), we have

$$\frac{1}{i}\left.\frac{\delta^2 \Gamma}{\delta \phi_j \delta \phi_k}\right|_0 \cdot \langle 0|\mathrm{T}\hat{\phi}_j \hat{\phi}_i|0\rangle^c = \delta_{ik} , \tag{G·21}$$

where use has been made of

$$\varepsilon^j \frac{\delta^2 \Gamma}{\delta\phi_k \delta\phi_j}\bigg|_0 = -\frac{\delta^2 \Gamma}{\delta\phi_j \delta\phi_k}\bigg|_0 . \tag{G·22}$$

This relation is obtained on the assumption that any anticommuting field has vanishing vacuum expectation value. Equations (G·20) and (G·21) indicate that the second derivative of Γ is the inverse propagator.

We make further differentiation of Eq. (G·20) with respect to ϕ_l and we have

$$\varepsilon^j \frac{\delta^3 \Gamma}{\delta\phi_l \delta\phi_k \delta\phi_j} \cdot \frac{\delta^2 W}{\delta J_j \delta J_i} + \varepsilon_l{}^{kj} \varepsilon^j \frac{\delta^2 \Gamma}{\delta\phi_k \delta\phi_j} \cdot \frac{\delta}{\delta\phi_l} \frac{\delta^2 W}{\delta J_j \delta J_i} = 0 . \tag{G·23}$$

The factor $\varepsilon_l{}^{kj}$ is the sign factor depending on the commutativity of ϕ_l with $\phi_k \phi_j$,

$$\varepsilon_l{}^{kj} = \begin{cases} +1 & \text{for } \phi_l \text{ commutative with the product } \phi_k\phi_j , \\ -1 & \text{for } \phi_l \text{ anticommutative with the product } \phi_k\phi_j . \end{cases} \tag{G·24}$$

In Eq. (G·23) we have used the fact that W and Γ are commutative quantities. We have another useful relation,

$$\frac{\delta^2 W}{\delta J_k \delta J_j} \frac{\delta^2 \Gamma}{\delta\phi_j \delta\phi_i} \varepsilon^i = \delta_{ik} , \tag{G·25}$$

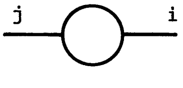
which is obtained from the identity

$$\frac{\delta J_i}{\delta J_k} = \delta_{ik} . \tag{G·26}$$

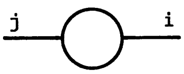
By the use of Eq. (G·25), we rewrite Eq. (G·23) as

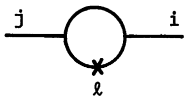
$$\frac{i\delta^3 \Gamma}{\delta\phi_l \delta\phi_k \delta\phi_m} = -\varepsilon_l{}^{kj} \varepsilon^j \frac{i\delta^2 \Gamma}{\delta\phi_k \delta\phi_j} \left(\frac{\delta}{\delta\phi_l} \frac{\delta^2 W}{i\delta J_j \delta J_i} \right) \frac{i\delta^2 \Gamma}{\delta\phi_i \delta\phi_m} . \tag{G·27}$$

The right-hand side of Eq. (G·27) is understood to be the amputated three-point vertex with the aid of graphic considerations as follows:

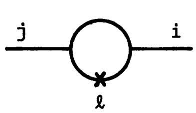
$$\frac{\delta^2 W}{i\delta J_j \delta J_i} =$$ 

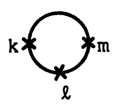
, (G·28a)

$$\frac{\delta}{\delta\phi_l} \cdot$$ 

$=$ 

, (G·28b)

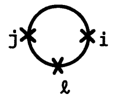
$$\varepsilon_l^{kj} \varepsilon^j \frac{i\delta^2 \Gamma}{\delta\phi_k \delta\phi_j} \cdot$$ 

$$\cdot \frac{i\delta^2 \Gamma}{\delta\phi_i \delta\phi_m} = \varepsilon^m$$ 

. (G·28c)

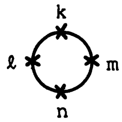
Thus we have

$$\frac{i\delta^3 \Gamma}{\delta\phi_l \delta\phi_k \delta\phi_m} = -\varepsilon^m \quad . \qquad \text{(G·29)}$$



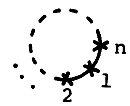
The further differentiation $\delta/\delta\phi_n$ of Eq. (G·29) gives,

$$\frac{i\delta^4 \Gamma}{\delta\phi_n \delta\phi_l \delta\phi_k \delta\phi_m} = -\varepsilon^m \quad . \qquad \text{(G·30)}$$



Generally we have

$$\frac{i\delta^n \Gamma}{\delta\phi_1 \delta\phi_2 \cdots \delta\phi_n} = -\varepsilon^n \quad . \qquad \text{(G·31)}$$



The n-th derivative of Γ is the one-particle irreducible n-point functions. Setting all the sources equal to zero and treating the sign factors including ε^n carefully, we obtain the final result:

$$\langle 0|\mathrm{T}\hat{\phi}_i \hat{\phi}_{i'}|0\rangle^c \langle 0|\mathrm{T}\hat{\phi}_j \hat{\phi}_{j'}|0\rangle^c \cdots \langle 0|\mathrm{T}\hat{\phi}_n \hat{\phi}_{n'}|0\rangle^c \left.\frac{i\delta^N \Gamma}{\delta\phi_{i'}\delta\phi_{j'}\cdots\delta\phi_{n'}}\right|_0$$
$$= \langle 0|\mathrm{T}\hat{\phi}_i \hat{\phi}_j \cdots \hat{\phi}_n|0\rangle^c_{\text{proper}}, \quad N \geq 3, \qquad \text{(G·32)}$$

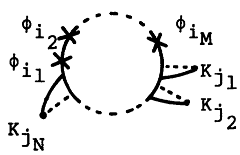
where we have used Eq. (G·22). It should be noted that Eq. (G·32) with $N=2$ is not equal to Eq. (G·21).

The differentiation of Γ with respect to sources K is easily transformed into that of W:

$$\left.\frac{\delta\Gamma}{\delta K_i}\right|_{\phi\ \text{fixed}} = \left.\frac{\delta W}{\delta K_i}\right|_{\phi\ \text{fixed}} - \left.\frac{\delta J}{\delta K_i}\right|_{\phi\ \text{fixed}} \cdot \phi$$
$$= \left.\frac{\delta W}{\delta K_i}\right|_{J\ \text{fixed}}$$
$$= \langle 0|\mathrm{T}\ \delta^{\mathrm{BRS}}\hat{\phi}_i \exp iS|0\rangle\,. \qquad \text{(G·33)}$$

The further differentiation is evaluated as follows:

$$\frac{i^{-N+1}\delta^{M+N}\Gamma}{\delta\phi_{i_1}\delta\phi_{i_2}\cdots\delta\phi_{i_M}\delta K_{j_1}\delta K_{j_2}\cdots\delta K_{j_N}} = \quad , \qquad \text{(G·34)}$$



$$
\begin{aligned}
&\langle 0|\mathrm{T}\hat{\phi}_{i_1}\hat{\phi}_{i_1'}|0\rangle^c \langle 0|\mathrm{T}\hat{\phi}_{i_2}\hat{\phi}_{i_2'}|0\rangle^c \cdots \langle 0|\mathrm{T}\hat{\phi}_{i_M}\hat{\phi}_{i_M'}|0\rangle^c \\
&\qquad\qquad \times \frac{i^{-N+1}\delta^{M+N}\Gamma}{\delta\phi_{i_1'}\delta\phi_{i_2'}\cdots\delta\phi_{i_M'}\delta K_{j_1'}\delta K_{j_2'}\cdots\delta K_{j_N'}} \\
&\qquad = \langle 0|\mathrm{T}\hat{\phi}_{i_1}\hat{\phi}_{i_2}\cdots\hat{\phi}_{i_M}\delta^{\mathrm{BRS}}\hat{\phi}_{j_1}\delta^{\mathrm{BRS}}\hat{\phi}_{j_2}\cdots\delta^{\mathrm{BRS}}\hat{\phi}_{j_M}|0\rangle^c_{\mathrm{proper}}\,.
\end{aligned} \tag{G·35}
$$

It should be mentioned that Eqs. (G·34) and (G·35) do not hold for the linear part of fields in $\delta^{\mathrm{BRS}}\hat{\phi}$. For example, we take

$$
\delta^{\mathrm{BRS}}\hat{\phi}_i \equiv \hat{\phi}_j + \cdots\,. \tag{G·36}
$$

In this case the differentiation of Γ with respect to K_i is

$$
\begin{aligned}
\frac{\delta\Gamma}{\delta K_i} &= \langle 0|\mathrm{T}\hat{\phi}_j \exp iS|0\rangle^c + \cdots \\
&= \frac{\delta W}{\delta J_j} + \cdots \\
&= \phi_j + \cdots\,.
\end{aligned} \tag{G·37}
$$

For the first term on the right-hand side, further differentiation gives trivial results,

$$
\frac{\delta}{\delta\phi_k}\phi_j = \delta_{kj}\,, \qquad \frac{\delta}{\delta K_k}\phi_j = 0\,. \tag{G·38}
$$

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